PHILOSOPHICAL TRANSACTIONS.

Giving Some

ACCOUNT

OF THE

Present Undertakings, Studies and Labours

OF THE

INGENIOUS,

In Many

Considerable Parts of the WORLD.

VOL. XXX. For the Years 1717. 1718. 1719.

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Jos: Banks

To the Right Honourable

THOMAS

Lord PARKER,

Baron of *Macclesfield*;

Lord High Chancellor

Of *Great-Britain*:

By INCLINATION as well as OFFICE,

First PATRON of Useful Arts and Discoveries.

This THIRTIETH Volume

OF THE

Philosophical Transactions

(As a *small* Acknowledgment for *very Great* Favours)

Is Humbly Inscrib'd by

His Lordship's most obliged Servant,

Edmund Halley, R. S. Secr.

Jos: Banks

PHILOSOPHICAL TRANSACTIONS.

For the Months of *Januar. Febr.* and *March*, 1717.

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I. *Obſervationes Stellæ fixæ in* Geminis *à corpore* Jovis *occultatæ*, Januarii 11*mo. St. vet.* 1717. *& Tranſitus arctiſſimi* Martis *infra Borealem in fronte* Scorpii Febr. 5. *mane.*

ANTE biennium in *Tranſact. Philoſ.* No. 344. *pag.* 294, rerum cœleſtium ſtudioſis indicavimus, *Jovem* corpore ſuo ſtellam quandam fixam obtegere debere, eoſque ad obſervationem Phænomeni rariſſimi, & magno in Aſtronomicis uſui futuri, invitavimus, ſignantes diem *Januarii* hujus anni decimum. *Jove* autem pene Stationario, & paulo amplius in orientem quam per Tabulas noſtras provecto, non ante undecimum incidit prædicta Occultatio; quam quidem *Londini* ob Nubes non contigit ex voto obſervare.

Nec tamen fruſtra invigilarunt Aſtronomi noſtri. D. *Martinus Folkes Londini*, præſentibus aliis nonnullis e Societate Regia, *Jan.* undecimo 8*h.* P. M. vidit *Jovis* centrum una diametro corporis ejus Fixam ſequi, quæ dicto centro Borealior erat quaſi dodrante ſemidiametri Jovis. Poſtea Nubes Jovem occuparunt, ſed, habita ratione motus Jovis, paulo poſt medium Noctis ſtellam Jovi conjunctam fuiſſe, & a Borea diſci ejus parte occultatam, concluſit.

Reverendus Dominus *J. Theoph. Deſaguliers*, R. S. S. & D^nus^. *Stephanus Grey*, *Weſtmonaſterii*, viderunt Fixam, Horâ Sextâ veſpertina, integra Jovis diametro diſtare a limbo ejus, Corum verſus. Unde & ex ſequentium dierum Obſervationibus, circa medium noctis incidiſſe conjunctionem evincitur

Reverendus quoque D *J. Pound*, apud *Wanſted*, infraſcriptas nactus eſt obſervationes, quas utique accuratiſſimas

tissimas, Tubo scil. prælongo & Micrometro captas, huc transcribere non pigebit.

Itaque *Januarii* Quinto 5^h. 6′. T. æq. Jovis centrum distabat a dictâ Fixâ 31′. 49″. quam 5^h. 38′. sequebatur 34′. 12″ Ascensionis rectæ: simulque limbus *Jovis* Australis eandem habuit Declinationem cum stella.

Die autem Nono sequente 6^h. 6′. Jovis centrum distabat a stella 10′. 49″; & post octo minuta erat differentia Ascensionum rectarum 11′. 32″: & tum centrum Planetæ, tantillo, ita ut vix perciperetur, erat Stellâ Australius.

Die Undecimo 5^h. 30′. T. æq. erat distantia centrorum 1′. 24′. simulq; visa est stella quasi, quadrante diametri Jovis Borealior centro ejus. Diameter autem minima *Jovis* inventa est 0′ 43″. Deinde Nubes.

Die vero Duodecimo 5^h. 17′. erat distantia centrorum 3′. 7″; ac 5^h. 50′. Jupiter stellam præcedebat 3′. 30″. Ascen. Rect. Eodemq; tempore limbus Jovis Boreus eandem habuit Declinationem quam Fixa accurate.

Collatis autem his Observationibus manifestum est Fixam hanc *Jovi* conjunctam, *Januarii* undecimo 13^h. circiter, non nisi 17″ vel 18″ centro ejus Borealiorem fuisse, ac proinde occultatam.

Fixa hæc, etiamsi nulli Catalogo hactenus ascripta, Locum tunc habuit ♊ 22°. 13′. cum Lat. Aust. 0°. $13' \frac{1}{2}$; Comitemque habet 17 min. eam præcedentem & 7 min. Borealiorem, sive in ♊ 21°. 56′ cum Lat. Aust. 0°. $6' \frac{1}{2}$, cui *Jupiter* conjungi visus est *Jan.* 16. 6^h. 30′. vesperi.

Sic spatio minus bimestri *Jupiter* corporaliter eclipsavit duas Fixas, cujus rei ne singulare quidem exemplum ab invento Telescopio extat: proinde hæc observata inter pretiosissima Uraniæ κειμήλια, in usum Posterorum, merito reponenda sunt.

No-

Noſtra autem ſtellula anno 1634. *Feb.* 6. *Jovi* Stationario conjuncta, tribus ejus diametris Auſtralior erat, obſervante *Gaſſendo*: unde conſtabit, calculo rite inſtituto, *Jovis* Nodos quoad ſenſum immobiles hæſiſſe, per 83 annos ultimo elapſos, idque ad $2^s.8^o.35'.$ a 1^a ✱ ♈. Ad alteram autem Obſervationem Tranſitûs *Martis* prope *Boream Frontis Scorpii* non minus inſignem, iidem Aſtronomi eadem cum cura invigilarunt omnes. *Mars* autem, *Februarii* Quinto Mane, vel Quarto $16^h.$ viſus eſt adeo vicinus ſtellæ dictæ, ut ea nudis oculis non conſpiceretur; ſed per Teleſcopium inventa eſt ſupra & ad ortum, adeoque *Mars* nondum ei conjunctus. Hora $16^h.$ $10'.$ T. app. *Mars* erat in recta cum Boreâ frontis & Teleſcopicâ quæ eam ſequitur ad Boream, ad diſtantiam octo circiter minutorum. $16^h.$ $35'$, *Mars* intermedius erat in recta cum Boreâ & Mediâ Frontis; & poſt horæ quadrantem, cum Auſtrinâ Frontis, ita ut $16^h.$ $54'.$ T. app. æſtimabatur Conjunctio ipſa quoad Longitudinem, quo tempore *Mars* ſat accurate duobus tantum minutis auſtralior erat ſtellâ. Obſervavit etiam D. *Pound* Conjunctionem reſpectu Aſcenſionis Rectæ $17^h.$ $25'.$ T. app. cum diſtantiâ centrorum $2'.$ $07''.$ Jucundum autem erat ſpectaculum, *Martem* videre ſtellam pedetentim aggredientem, motumque ſuum, lentiſſimum licet, manifeſte prodentem.

Conferatur cum hac Obſervatio *Horroxii* noſtri anno 1638. *Februarii* Septimo mane, quam vide in Epiſtolis ejus pag. 304. Tunc enim *Mars* ad eandem ſtellam appulſus, etiam multo propius ad eam acceſſit, ſed ante ortum ejus præterierat Conjunctio.

His adde *Saturni* obſervationem *Januarii* 25to. $12^h.$ $25'.$ T. æq. à D. *Pound* habitam. Cum Planeta diſtabat à ſtella 58va. *Virginis Catal. Brit.* $13'.$ $16''.$ verſus Auſtrum, eamque ſequebatur $2'.$ $30''.$ Aſc. Rect. Stella in ♎ 19^o $21'.$ $52''.$ cum lat. Bor. $2^o.$ $47'.$ $25''.$

II. *An*

Fig. 6

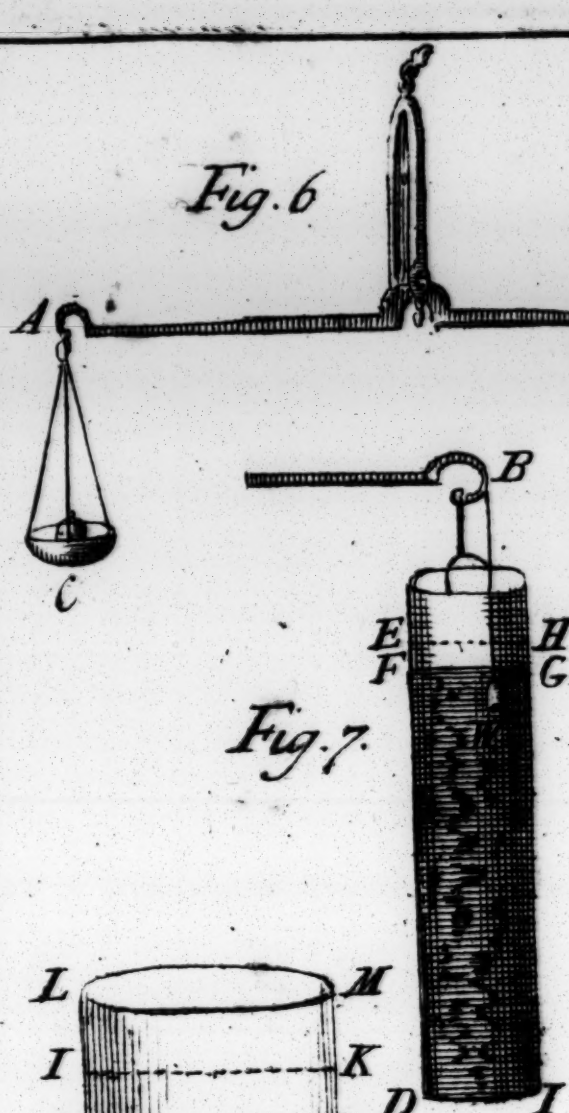

Fig. 7.

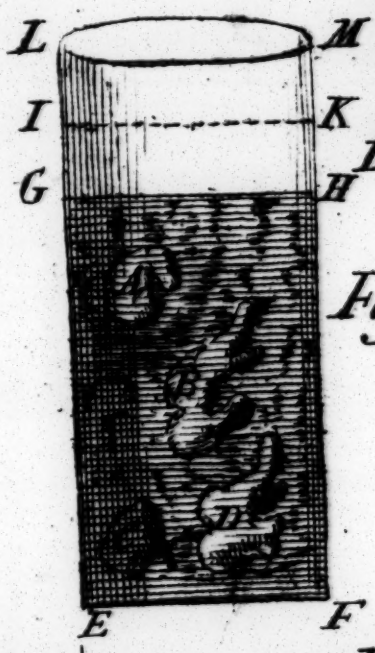

Fig. 5.

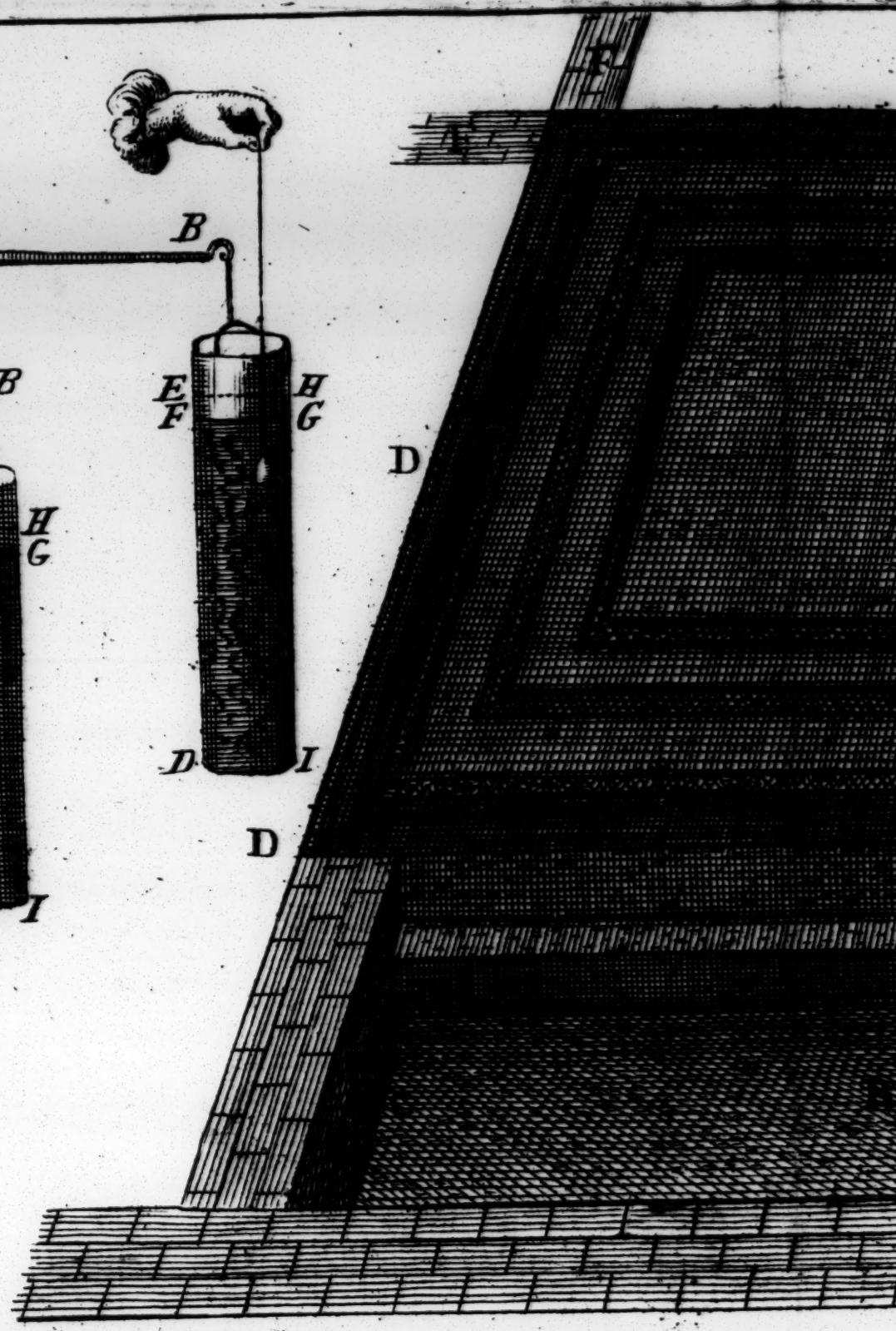

Fig. 8.

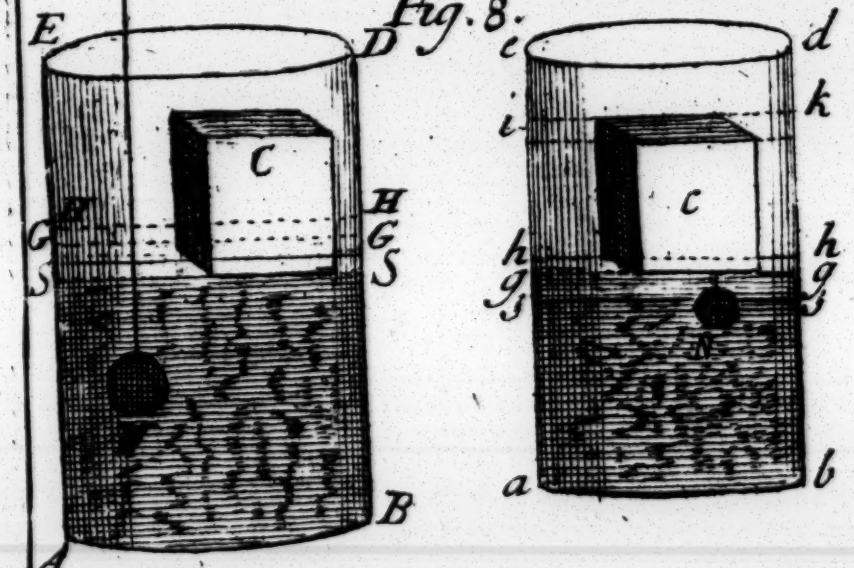

Fig. 4

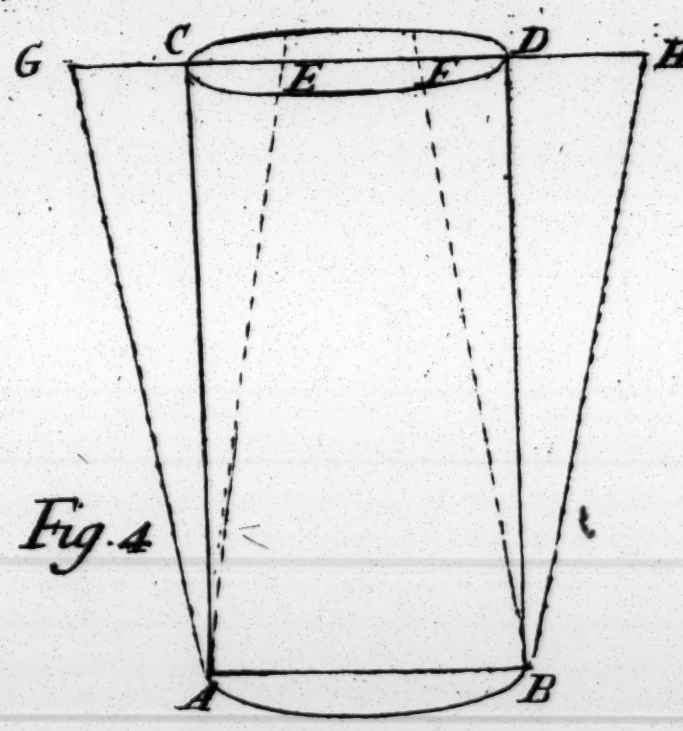

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Fig. 1.

Fig. 2.

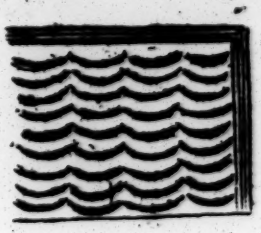
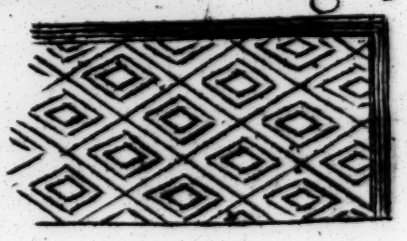


Fig. 3.



A *The Pavement.* B *The Bath.* C *The Sink and the other Passage through the Wall at the East end of the Bath.* D. *The Roman Brick which bounded the Pavement, laid flat and covered with Terrace.* F. *Foundations continued severall ways.* E *The Landing place out of the Bath.* G. *The corner of a large space paved with Roman Brick, and covered with Ashes, Coals. &c.*

II. *An accurate Account of a* teſſellated Pavement, Bath, *and other* Roman *Antiquities, lately diſcover'd near* East Bourne *in* Suſſex. *Being part of a Letter of* January 26. 1717. *from the learned Dr.* John Tabor *of* Lewis, *to Dr.* John Thorpe, R. S. S. *and by him communicated to the* Royal Society.

A Deſcription of the teſſerated Pavement at *East Bourne*, near *Pevenſey*, muſt have been more imperfect than what is now given, had it come to your hands much ſooner. I thought an exact Account could not be taken, unleſs the Ground about it was open'd: and it being part in a Meadow, and part in plough'd Ground, and under a Fence which parts two Perſons Lands; by reaſon alſo the one was ſow'd; I could not procure the Digging in both Places at the ſame time.

It was in *March* laſt when the Meadow was dug; and the laſt Week ſave one in *November*, before we had leave to open the Ground in the Corn Field. The Meadow in which the greateſt part of the Pavement lyes, is near a Mile and half South East of *Bourne*; it contains about four Acres, and is of a triangular Form; the Southern Side is againſt the Sea; only a few Fiſhers Cottages, and a ſmall publick Houſe or two being between that and the Sea. On the Northern Side of the Meadow is a High-Way, which leads from *Bourne* to *Pevenſey*: the Weſt Side is by a Fence of Poſts and Rails ſeparated from a large Corn Field, in Common belonging to the Pariſh.

About the middle of this Fence is the Pavement, distant from High-Water-Mark a Furlong; In former times it might have been somewhat more, because from this Point to the Westward, the Sea is always gaining from the Land.

More than four Years since, *viz.* in the Summer 1712, when the Fence was repair'd; the Workman sinking a Hole to fix a Post in, was hinder'd by something Solid like a Rock; but casting out the Earth clean, found the Obstacle to be Artificial. Mr. *Thomas Willard* of *Bourne*, Owner of the Meadow, being inform'd of the Novelty, gave Order that it should be uncover'd; and sent also to *Herstmonceux*, for one *Purceglove* an ingenious Ingineer (who formerly had been imploy'd in the Mines in the Northern Counties), who with his Instruments bored through the Pavement; and in many places of the Ground about it, which he found to be full of Foundations: but this his Discovery of those Foundations, was only a Confirmation of what the Inhabitants there have always observ'd, as well in Ploughing, as in the Growth of their Corn and Grass: for in the common Corn Field, West to the Meadow, to the distance of near half a Mile, they often raise bits of Foundations with their Ploughs; and in dry Summers, by the different Growth of the Corn, they can plainly perceive all that Tract of Ground to be full of Foundations.

The Pavement was little more than a Foot below the common Surface of the Ground; what lay next it was a small Sea Gravel; the Position of it is very near due East and West (about two Foot of the West end of it reaching into the Corn Field); its length is seventeen Foot and four Inches; its breadth eleven Foot. At first it seem'd to have been bounded with a thin Brick set on Edge, about an Inch above the *Tesseræ*, so exactly strait and even, as if Shot with a Plane; and so well Cemented,

as

as if one entire Brick. But when the outside of the Pavement was broke up, we found, that instead of Bricks set on Edge, as was imagin'd, it was bounded with a Border of Bricks laid flat, and their ends next the *Tessera* turn'd up. The Thickness of these Bricks was an Inch and a Quarter; the Breadth not under Eleven, and not more than twelve Inches; the length full fifteen Inches; which, before they were turn'd up at their Ends, could not have been less than Seventeen. They were very firm, and not in the least Warp'd or Cast in Burning: when broke, their Substance was fine and well mixt, of as uniform and clean a Red Colour, as a piece of fine *Bole*: Except at the ends where turn'd up, they were all over cover'd with a Plaster (the same which *Vitruvius* calls the *Nucleus*, of which more afterwards), half an Inch thick; so hard, entire, and even, that it seem'd as one Stone, quite round the Pavement.

Next within the Bricks, there was a List or Border of white *Tesseræ*, thirteen Inches broad; within that, a List of brown *Tesseræ* (somewhat darker than a Whet-Stone, and somewhat lighter colour'd than the Touch-Stone) four Inches broad; then a List of the White, five Inches broad; next within that, another List of the Brown, four Inches broad: all the rest of the Pavement was set with white *Tesseræ*, without any Ornament or Figure; which though not Gay, lookt very Neat and Clean.

When this was first view'd, none of the Curious doubted, but that the Work was Roman, many were of opinion, that it might have been the Floor of a Temple, or place of Worship. *Pliny* indeed (*a*) informs us, that these sort of Pavements or *Lithostrota*, began to be in use in *Italy*, in the time of *Sylla*; who caus'd one of them

(*a*) Plin. Sec. Hist. Nat. Lib. XXXVI. Cap. XXV.

to be made in the Temple of *Fortune* at *Præneſte*; perhaps the ſame which not long ſince was taken notice of by the Honourable Mr. *Addiſon* (*b*).

I was rather inclin'd to ſuppoſe, it had been that Apartment belonging to the chief Officer where Juſtice was adminiſter'd; and the more, becauſe *Pilat's* final Sentence on our Saviour was pronounced from a Throne on the *Lithoſtroton* (*c*); which Appellation was given to theſe kinds of Pavements by *Varro* (*d*) not leſs than ſixty Years before; and by *Pliny* (*e*) not leſs than forty Years after our Saviours Suffering. That the Roman Generals caus'd ſuch Pavements to be made at their Stations; we may have juſt reaſon to conclude, from that paſſage (*f*) in *Suetonius* cited for this purpoſe by Dr. (*g*) *Plot*.

When the Ground about the Pavement was dug, all theſe Suppoſitions were quaſh'd; for on the North Side of the Pavement, we diſcover'd an entire Bath, ſixteen Foot long, five Foot nine Inches broad, and two Foot nine Inches deep (which the Draught ſent with this repreſents): *Fig.* 1. It was fill'd with Rubbiſh of Buildings, which ſeem'd to have been burnt; *ſc.* hard Mortar adhering to pieces of Roman Brick, ſquar'd Stones, and headed Flint, mingled with Aſhes and Coals of Wood. From the Northweſt Corner of the Pavement, was the Paſſage into the Bath, three Foot three Inches wide; at which place, the Bricks that bounded the Pavement, were not turn'd up at their ends, but lay even with the *Teſſeræ*. At the diſtance of fifteen Inches from the *Teſſeræ*, there was a Fall of two Inches, to the Landing-

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(*b*) Remarks on ſeveral places in *Italy*, *Pag.* 377. (*c*) Εὐαγγ. Ἰωαν. K. XIX. 13. (*d*) Ter. Var. de Re Ruſt. Lib. 3.
(*e*) Plin. Hiſt. Nat. Lib. XXXVI. C. XXV. (*f*) Jul. Cæſ. Sect. 46.
(*g*) *Oxfordſhire* Plots Nat. Hiſtory, Chap. X.

place

place out of the Bath; the Landing place was alſo three Foot three Inches long, and two Foot two Inches broad: Thence by two Stairs, was the Deſcent into the Bath; the length of the Stairs, the ſame as of the Landing-place; the breadth of each Stair was eleven Inches; the height of each Step a little more than ten Inches: the loweſt Stair was twenty Inches from the farther Side of the Bath. The whole Work was very compact, and exactly well made; not in the leaſt injur'd by Time, nor the Violence it underwent when fill'd up; truly anſwering the Precepts of *Vitruvius*; which (*h*) adviſe, that for all Buildings, reſpect ſhould be had to the Strength, Conveniency, and Beauty of the Work deſign'd; and that in order thereto, a careful and judicious Proviſion ſhould be made of Materials, without Parſimony.

Although the Author and Time of theſe Works cannot as yet be diſcover'd; yet 'tis evident the Artificer near enough follow'd the directions *Vitruvius* (*i*) gave for framing ſuch like Structures.

(*h*) M. *Pollio Vitruv.* de Architectur. Lib. II. Cap. III. Hæc autem ita fieri debent, ut habeatur ratio firmitatis, utilitatis, venuſtatis. Firmitatis erit habita ratio, cum fuerit fundamentorum ad ſolidum depreſſio, & ex quaque materia copiarum ſine avaritia diligens electio.

(*i*) M. *Vitruv. Pol.* Lib. VII. Cap. I. Primumque incipiam de Ruderatione, quæ principia tenet Expolitionum, uti curioſius ſummaque providentia ſolidationis ratio habeatur. Et ſi plano pede erit ruderandum, quæratur ſolum ſi ſit perpetuo ſolidum.—Si aut omnis aut ex parte congeſtitius locus fuerit, fiſtucationibus cum magna cura ſolidetur.—Tunc inſuper ſtatuminetur ne minore ſaxo quam quod poſſit manum implere: ſtatuminibus inductis ruderetur. Rudus ſi novum erit, ad tres partes una calcis miſceatur, ſi redivivum fuerit, quinque ad duum mixtiones habeant reſponſum. Deinde Rudus inducatur, & vectibus ligneis Decuriis inductis crebriter pinſatione ſolidetur; & id non minus *poſt* pinſum abſolutum craſſitudine ſit dodrantis. Inſuper ex Teſta Nucleus inducatur, mixtionem habens ad tres partes unam Calcis; uti ne minore ſit craſſitudine pavimentum digitorum ſenum. Supra Nucleum, ad Regulam & Libellam exacta Pavimenta ſtruantur, ſive Sectilibus, ſeu Teſſeris. Cum ea extructa fuerint, & faſtigia extructiones habuerint, ita fricentur, uti, ſi Sectilia ſint, nulli gradus in ſcutulis, aut trigonis, aut quadratis, ſeu favis

First, as to the Pavement, it was ſecur'd on every Side, and the Edges of it reſted on a very firm and neat built Wall, made of Roman Brick, ſquar'd Stone and headed Flint; between five and ſix Foot deep below the Surface of the Pavement, and full twenty three Inches thick; which we may ſuppoſe to have been two Foot by the Roman Meaſure. The Bricks were not in regular courſes, as they are to be ſeen in thoſe Roman Buildings, which are in view above Ground; but without order diſpers'd about in the Wall. The Top of the Wall indeed was but fifteen Inches thick; and that was cover'd with the Bricks firſt mention'd, which bounded the Pavement: but about fourteen Inches below the Top, there was a Set-off (as our Maſons term it) in the inſide of the Wall, eight Inches broad. We did not dig up the Foundation of the Pavement to the Bottom, but opened it at one Corner only, that we might diſcover how it was Fram'd: for when it was bor'd through, they obſerv'd, next under the *Teſſeræ*, a Bed of very ſtrong Mortar, more than a Foot thick; under the Mortar a Bed of Clay two Foot thick; and under the Clay a firm Foundation of Brick. We obſerv'd the Clay (which the Ground thereabouts do not afford) to be very fine and red, and alſo cloſe; no doubt but carefully Ramm'd. The Surface of the Clay was neatly pitch'd with ſmall Flint and Stones, Pointed at their lower ends, and Headed at their upper ends. This Pitching or Paving is by *Vitruvius* call'd *Statuminatio*; and the Stones 'tis done with, he

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extent. Sed coagmentorum compoſitio planam habeat inter ſe directionem. Si Teſſeris ſtructum erit, ut eæ omnes angulos habeant æquales, nullibique à fricatura extantes. Cum enim anguli non fuerint omnes æqualiter plani, non erit exacta ut oportet fricatura.

calls

calls *Statumina*. He directs them to be set, when the Underwork is made Sound and Firm, by well Ramming. Becauſe the firſt Chapter in his Seventh Book, treats only of the Method of making theſe kinds of Pavements, which in his time, and as may be obſerv'd from his words, were had in no ſmall eſteem by the Grandees of *Rome*; I have tranſcribed what may ſhew the accurate Methods which that great People had in Framing them.

But to return, this pitch'd Work was exactly even with the Set-off in the inſide of the Wall; on it was laid a Bed of coarſe Mortar of about nine Inches thick; the Skirts of this Mortar (which by *Vitruvius* is call'd the *Rudus*) reſted on the Set-off above-mention'd; it was compos'd of Lime, a ſharp courſe Sand, ſmall Pebbles, and bits of Brick. Upon this *Rudus* was a finer Compoſition, made, as near as I could gueſs, with Lime, a fine ſharp Sand, ſome kind of Aſhes, and (which was the greater part) ſtampt Brick and Pot-ſherds, in grains not larger than Cabbage-Seed, and the Flower or fine Powder ſeparated from it. This Bed was about half a Foot thick; and is what *Vitruvius* calls the *Nucleus*. Whether we may call it Terrace, I muſt leave it to thoſe who are better skill'd than my ſelf, in giving proper Appellations to the ſeveral parts of Maſonry. Both this *Nucleus* and the *Rudus* under it, very near equall'd the *Portland*-Stone in hardneſs and compactneſs. Upon this *Nucleus* or Terrace were the *Teſſeræ* ſet: they were ſet an end; but ſo exact was the Workman in ſetting them, that he us'd two ſorts of Cement to fix them withal; their lower ends ſtood in a Cement of Lyme only, well work'd; their upper halves were cemented with a fine gray Mortar, conſiſting of fine Sand (and as it ſeem'd) Aſhes and Lyme. This gray Cement every where fill'd the Intervals at their Heads; and was much harder than the *Teſſeræ* themſelves.

'Twas

'Twas before intimated, that the *Tessera* were but of two Colours, White, and of a dark Brown; they were harder than a glaz'd and well burnt Tobacco-Pipe, and of a Grit somewhat finer; the Brown seem'd to be of the same Substance with the White, but colour'd by Art, (as *Pliny* informs us (*k*) the workers in Clay of old had a Method to do): they seem'd to have been form'd in a Mould, and afterwards Burnt. Hence I am inclin'd to take the meaning of *Vitruvius*; where he makes so plain a distinction between the *Tesseræ* and the *Sectilia*; that, the one was according to the import of the name, form'd by Instruments out of Stone, Brick, and Tyle; the other shaped in a Mould and Burnt. They were not of an equal Size, none exceeding an Inch in length; the shortest were $\frac{6}{10}$ of an Inch: most of them were equally made their whole length; but of some the lower ends terminated almost as sharp as a Wedge, on purpose, as may be suppos'd, to be driven where any Interstices were left: At their Heads likewise they were not all equal and alike, some exactly Square, some oblong Square, some Semi-lunar, but none Triangular: the Diameter of those that were Square was about $\frac{4}{10}$ of an Inch; the longest Side of those that were oblong at the Head little exceeded half an Inch. It may be observ'd, that the preparations for fixing this Pavement here, go beyond those which *Vitruvius* prescribes (in the firm Wall near six Foot below the Surface, in the Bed of Clay within it two Foot thick, and in the Foundation of Brick under the Clay). But when we consider the Scituation of the Ground here is low, not many Feet higher than the Sea might be elevated at Spring Tides; and that it might as well be annoy'd by Land-Springs after great Rains, as by Water owzing through the Earth from the

(*k*) Plin. Secun. Hist. Mund. Lib. XXXVII. Cap. XII.

Sea

Sea ſo near; from which the Work in time might receive damage; we muſt allow the abovemention'd Additions to be the reſult of a very judicious Foreſight.

The Bath alſo was form'd and ſecur'd by a very compact Wall, of the ſame breadth and depth with that on which the Pavement reſted: the Wall, which ſuſtain'd the North Side of the Pavement, made the South Side of the Bath. On the South Side of the Bath, from the Eaſt end, to the ends of the Stairs, there was a ſolid Seat; twelve Foot nine Inches long, very near ten Inches broad, and fourteen Inches high. The Bottom or Floor of the Bath, was made after the ſame manner as the Pavement was made, excepting the *Teſſeræ*, and the thick Bed of Clay: for under all, there was Brick; then a Bed of the *Rudus* or coarſe Mortar ſomewhat more than a Foot thick; above that the *Nucleus* or Terrace only, half a Foot thick. The Sides of the Bath, the Seat, and the Stairs, were plaſter'd over with this Terrace about half an Inch thick; all which were throughout ſo Hard, Compact, and Smooth, that when firſt open'd, the whole ſeem'd as if it had been hew'd out of one intire Rock, and poliſh'd. At the middle of the Eaſt end, at the Bottom, there was a Sink-hole, a little more than three Inches long, and above two Inches deep: about four Inches above it, there was another paſſage through the Wall of the ſame ſize; the firſt we may ſuppoſe to let out the Water which had been us'd; the other to let in freſh. The Stairs and Seat were chiefly made of *Roman* Brick, between fifteen and ſeventeen Inches long, between eleven and twelve broad, and near one and a half thick. At the North Side of the Bath the Ground was not open'd; but at the Eaſt end of the Bath and Pavement, at the South Side of the Pavement, and at the Weſt end of both, there ſeem'd to have been ſeveral Vaults or Cellars: for there were

very firm 23 Inch Walls continued every way (to the farther ends of which we did not trace), whoſe Foundations were as low as that which ſupported the Pavement: ſo that to the depth of ſix Foot the Ground was fill'd with ſuch Rubbiſh as was taken out of the Bath. The Bricks in this Rubbiſh, which were all broke, had ſeveral degrees of thickneſs, from three Inches to a little more than one Inch: ſome had one of their Sides wav'd as in *Fig.* 2; ſome Fretwiſe as in *Fig.* 3. others had Roſes on them well imitated: we found alſo two ſorts of channel'd Bricks; the one like a Trough, the Channel three Inch s broad, and as many deep, the Brick it ſelf an Inch and a half thick: The other ſort, had a Cylindrical Channel; ſo that when two were clapt together, they form'd a hollow Cylinder of three Inches Diameter. Theſe channell'd Bricks being all broken, their Length when whole is uncertain, as is the Uſe they ſerv'd to; whether for Paſſages to conveigh Water; or whether they were placed in the Walls to diſtribute Heat throughout the Building, as was uſual in the ancient Structures at *Rome*.

'Tis farther obſervable, when the Ground was open'd the ſecond time; that off from the South-Weſt corner of the Pavement, which the Letter *G* ſhews; five Foot lower than the Surface of the Pavement, there was diſcover'd a large Space (to the end of which we did not ſearch), paved with Brick, eleven Inches broad, almoſt one and a half thick, and fifteen long; ſubſtantially was it pav'd; for it had two Courſes of this Brick. There was half a Foot of Mortar under the lower Courſe; and about an Inch of Mortar between the two Courſes; theſe Bricks alſo were perfectly well made; but on the under Side of each, were two Knobs, about the ſize of half a Wallnut; fix'd on them as may be gueſs'd,

to

to keep them ſteddy, till the Mortar they were ſet in might dry. This pav'd Place was ſearcht 6 or 8 Foot every way; it was all cover'd with a Coat about two Inches thick, of Aſhes and large Coals of Wood: on that lay confuſedly large pieces of the *Rudus* or coarſe Mortar abovemention'd, and lumps of the *Teſſeræ* in all reſpects like thoſe on the Pavement, and cemented as They were. There were moreover mingled with the Aſhes many large Iron Nails, bigger, but not quite ſo long, as thoſe we call double Tenns; ſome Hooks for Doors to ſwing on; ſeveral ſmall pieces of earthen Ware; ſome like bits of Urns; ſome of a fine yellow Clay; ſome red, thin, neatly wrought and adorn'd with Flowers; and laſtly part of a Human Skull, and pieces of Bones near it; which Bones were not inclos'd in any Veſſel, but lay looſe; they were diſcolour'd like thoſe I have ſeen in Urns; ſo that the Body they belong'd to, might periſh by the ſame Flames, that theſe Buildings were deſtroy'd by. There was no Inſcription found either on Stone or Brick; no Statue, or other Figure, ſave thoſe on the Bricks mention'd; neither were there any Coins met with there. But ſomething more than a Furlong North-Weſt of theſe Works, near three Years ſince, there was a Malt-Houſe, and near two Years ſince a Dwelling-Houſe erected; in digging the Foundations for the firſt, there was a Coin of *Poſthumus*; and in the Ground dug for the laſt, a piece of *Conſtantine*'s found; both which I ſend with this, that the Inſcriptions and Reverſes may be incerted if neceſſary.

From the nearneſs of the Bath, it may reaſonably be concluded that the Pavement was neither a part of a Temple, nor for a place of Juſtice: the continuation of the Foundations every way to be traced from it, and what was laſt diſcover'd, are rather an Argument it was an Apartment of a magnificent Palace.

Pliny ſuppoſed that theſe *Lithoſtrota* (*l*) or teſſerated Pavements had their original in *Greece*; but perhaps the *Grecians* borrow'd their Patterns from *Aſia*: for from the Book of *Eſther* (*m*) we learn, there was a moſt Royal Banquet at *Suza*, on a *Lithoſtroton* (ſo the Septuagint has it) of coſtly Stones, four Hundred Years before the time of *Sylla*, who brought them firſt into *Italy*. *Joſephus* affirms (*n*), that the *Grecian* Laws, Learning and Arts were fetch'd from *Aſia*: and indeed when we reflect on the Antiquity of the *Levitick* Law; the Pyramids of *Egypt*; the Temple of *Solomon*; the Walls and Palaces of *Babylon*; and the ſumptuous remains of *Palmyra* and *Perſepolis*; we have no reaſon to eſteem the *Grecians* Authors, but as good Imitators of thoſe early Examples of Learning and Arts they had to follow.

When *Quinctus Cicero* was here with *Cæſar*, the ſecond time he invaded *Britain*; his Brother the incomparable *Tully*, had the overſight of ſome Buildings he had appointed to be made in the *Villa Manliana* at *Arcano*: and in a Letter ſent into *Britain*, *Tully* informs *Quinctus*, that he was well pleas'd with the Seat, and the more, becauſe the *Pavimented Piazza* was Magnificent: that the Pavement ſeem'd (*o*) to be exactly well made: that he had directed ſome Chambers to be alter'd becauſe he did not approve of them: that in the Bathing Apartment, he had remov'd the Sweating Room into another corner of the *Apodyterium*. And afterwards in the ſame Letter makes mention of ſuch another Work which was in hand for him in the City alſo. Again, about the time *Quinctus* return'd out of *Britain*, and was fixt with the Legion he preſided over, in Winter Quarters

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(*l*) Plin. Sect. Hiſt. Lib. XXXVI. Cap. XXV.
(*m*) Eſth. Chap. I v. 6. (*n*) Joſeph *againſt* Appion. Book II.
(*o*) Tull. Cic. ad Quinct. Frat. Lib. III. Ep. I.

among

among the *Nervii* (of which *Cæſar* in his Commentaries makes mention); *Tully* (*p*) takes notice of a Pavement that was making for himſelf alſo: *Expolitiones utriuſque noſtrum, ſunt in manibus; ſed tua pæne ad tectum jam perducta res eſt ruſtica Arcani & Laterii.* 'Tis hinted by *Varro* that a *Lithoſtroton* was one of the Members of a compleat *Villa* (*q*): *Varro* was eighty Years old when his Books *de Re ruſtica* were compoſed: *Tully* was ſomething more than fifty when the above cited Epiſtles were wrote; *Cæſar* when a General, made the *Teſſera* (*r*) and *Sectilia* for Pavements, to be part of his Baggage; and *Vitruvius*, Cotemporary with theſe three, calls the *Lithoſtrota, Principia Expolitionum* (*s*); which make it evident theſe Floors were held in eſteem, by as great Men as the World has afforded, even in their riper Years. From all this, we may obſerve, that ſometime before, and in the firſt Age of the Empire, the humour of theſe kinds of Floorings much prevail'd among the *Romans*: wherefore 'tis no wonder they are found in ſo many places of this Iſland. But, as unprofitable Inventions and Cuſtoms in time grow Stale, and are laid aſide, ſo fared it with that of Pavements: For in the time of *Pliny* they began to be out of uſe on the Ground; but then he tells us, they were made above Stairs (*t*), or in his own Words in Chambers. Whether the *Lithoſtrota* in Chambers were uſual in *Vitruvius*'s days, we have no Warrant to ſuppoſe, from any hint in his Writings; notwithſtanding he gives Rules for making them, *plano pede*, on the Ground; and *ſub* (*u*) *dio*, (which

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(*p*) Ibid. Ep. III. (*q*) Ter. Varro de Re ruſtic. Lib. III.
(*r*) Suet. Tranq. Jul. Cæſ. Cap. 46. (*s*) M. *Vitruv. Pol.* Lib VII Cap. I. (*t*) Plin. Hiſt. Lib. XXXVI. Cap. XXV. Pulſa deinde ex humo Pavimenta in cameras tranſiere è vitro: novitium & hoc inventum. (*u*) M. *Vitruv.* Lib. VII. Cap. I. Sub dio vero maxime idonea faciunda ſunt pavimenta.

from

from the Method by him preſcrib'd muſt be) aloft: becauſe for ſuſtaining thoſe *ſub dio*, he orders the work underneath to be well ſecur'd, with two lays of Plank that ſhould croſs (*w*) each other, and be nail'd down; then the *Statuminatio* or Pitching, the Mortar, Terrace and *Teſſeræ*, as before on the Ground. But becauſe by *ſub dio Vitruvius* could not deſign Chambers; and although *Pliny* informs us the *Græcians* us'd (*x*) to cover or Flat-roof their Houſes with theſe Pavements; yet ſince neither *Vitruvius* nor *Pliny* mention any ſuch Mode prevailing in their times at *Rome*; it remains, that we may imagine *Sub dio*, or the *Subdialia* of *Vitruvius*, to mean Pavements mounted on Pillars or Arches, which might afford delightful Terraces out of the upper Rooms, and ſhady Piazzas underneath: and in this Senſe perhaps may be underſtood the *Porticus Pavimentata* of *Tully* above-mention'd. By the many Apartments, the Foundations about theſe Works point out, there ſeems to have been nothing wherein the Buildings that once ſtood there, might come ſhort of the magnificent Structures, wherewith the *Romans* delighted to gratify their Luxury. The uſes each were deſign'd for, is not to be determin'd: whether there was a Piazza cover'd with a *Lithoſtroton*, cannot be affirm'd. But be that as it will; 'tis next to Demonſtration, there was ſome upper Floor ſuſtain'd by Wood, and pav'd with the *Teſſeræ*, after the ſame manner as *Vitruvius* directs; and, on the Brick Pavement (laſt diſcover'd), the Coat of Aſhes and Wood Coals with Nails, cover'd with large pieces of the *Rudus*, and great lumps of the *Teſſeræ* well cemented

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(*w*) Ibid. itaque ſi neceſſitas coegerit, ut minime vitioſa fiant ſic erit faciundum: cum coaxatum fuerit, ſuper altera coaxatio tranſverſa ſternatur, claviſque fixa, &c.---Statuminatione facta rudus inducatur, &c.

(*x*) Plin. Hiſt. Lib. XXXV. Cap. XXV. Subdialia Græci invenere talibus domus contegentes.

together

together, and the *Nucleus* adhering to them; ſhew there was an upper Pavement broke by its fall, when Fire had conſum'd its ſupport.

I have been thus prolix, in giving you the moſt exact account I could of this piece of Antiquity; becauſe we cannot have a leſs Senſe of the admirable Rules and Methods, the *Roman* People made uſe of, in framing their Buildings, and ordering other Conveniences for Enjoyment and Magnificence; than of the incomparable Management they had in their Military Preparations and Diſcipline; which are ſo to the Life repreſented by (*y*) *Joſephus*, and ſo punctually deſcribed by (*z*) *Vegetius*.

As to the *Roman* Architecture, it may not be amiſs here to note; that when they deſign'd a Building, they could not immediately begin it: their Preparations requir'd time: By their well ſhap'd durable Bricks, and by their Stone-like Mortar, we may plainly perceive, they built not with ſuch haſty Materials as are now us'd. *Vitruvius* (*a*) and *Pliny* both direct, that Brick ſhould be form'd in the Spring, and be two Years drying: And where *Pliny* ſpeaks of their Mortar, he ſays, 'twas ordain'd by the old Laws (*b*) of *Rome*, that no Undertaker ſhould Build a Houſe with Mortar which had not been made three Years before. We find indeed, their Walls ſeem to bid fair for Eternity; whereas ours, from Parcimony and ill Management, are ſcarce able to endure one Age.

The reſt of this learned Diſcourſe, by which 'tis made more than probable that here once ſtood the Roman *City* Anderidæ, *deſtroyed by the* Saxons *about the Year* 500; *though very curious, yet being chiefly Hiſtorical, ſeems not ſo properly the Subject of theſe Tranſactions.*

(*y*) *Joſephus*'s Wars of the *Jews* Book III. Chap. III.
(*z*) Veget. de Re Militari. (*a*) M. *Vitruv. Pol.* Lib. II. Cap. III. Plin. Hiſt. Lib. XXXV. Cap. XIV. (*b*) Plin. Hiſt. Lib. XXXVI. Cap. XXIII.

III. *A ſhort account of the Nature and Vertues of the* Pyrmont *Waters; with ſome Obſervations upon their Chalybeat Quality. Communicated by Dr.* Frederick Slare, R. S. Soc.

HAving procur'd about a dozen Quarts of *Pyrmont* Waters this laſt Summer, I made ſome Tryals with them. I found by the Taſte that they contain'd a rich Chalybeat Vertue, and alſo made a very brisk and lively impreſſion on the Palate, more grateful and ſpirituous, than the beſt *Spaw* Waters I ever taſted. The *Spaw* Waters are look'd upon as moſt excellent, if they Sparkle a little in the Glaſs; but theſe in Summer time, when pour'd into the Glaſs, nay ſometimes even in the Bottle, as ſoon as the Cork was open'd and the Air was admitted, would make a notable Ebullition, ſomewhat like bottled Cyder, tho' this was ſoon over; but they did yet continue their ſmart and brisk Taſte, and high Chalybeat Reliſh to the laſt Drop, tho' we were ſome Hours in Drinking them off. In the Winter time, theſe Waters do not Sparkle, nor Ferment, at leaſt mine did not; but they were not carefully preſerv'd, being expos'd in cold Cellars, where our Beer or Wine ſtood in the Winter; and yet notwithſtanding, they loſt not the Chalybeat Taſte, and alſo retain'd a very pleaſant brisk Guſt. Theſe Waters have been reckon'd in the Number of the *German* Saur Brunnen or *Acidulæ*, and ſome of my Friends to whom I gave a Glaſs of the Water, have aſcrib'd to it a ſharp Taſte, and have been ready to run away with a poſſeſs'd Opinion of its being Sour: but when I requir'd them to call back that haſty Aſſertion, and to conſider it better, whether that Taſte was really

Sour or Acid, they have been forc'd to recant and confeſs, that the ſmart and brisk Taſte miſled them to call it Acid or truly Sour. Thus Cyder and ſoft Ale when Bottl'd, will give ſuch an acute Affection to the Palate, when it is far from being Sour: And even *Volatile Alkalies* of *Sal Armoniac* or of Hartshorn, may be made to give the like pungency to the Tongue.

In order to a more nice Enquiry, whether any Acidity were diſcoverable in theſe *Pyrmont* Waters, we dropt in conſiderable Quantities both of Spirit of Hartshorn, and of Spirit of *Sal Armoniac*, both juſtly prepar'd; but could not diſcover the leaſt Luctation or Motion to appear upon this Conjunction, as it uſually does with an Acid.

I made a yet more nice and certain Examen of theſe Waters, by mixing Milk with them, ſometimes in equal, ſometimes in double proportion; and in various degrees of Warmth, both in Lukewarm degrees, and alſo with a boyling Heat, but I could not perceive any Curdling. But rather on the contrary, the Water preſerv'd the Milk from Coagulation, for four or five Days, even in *September*, it being hot Weather.

Take a very little Gall in Powder, about half a Grain to a Glaſs of a quarter of a Pint; this does in a Moment render it turbid, and make a dark Purple, eſpecially if you ſtir it: but if you drop the Powder on the Surface of the ſame Water, it then cauſes a fine blew Tincture. If you will make a very fine Tincture pleaſant to the Spectator, Take five Leaves of ſtrong Green Tea, put them into the bottom of a Glaſs holding a quarter of a Pint, and you will ſee thoſe Leaves unfold themſelves, and in a quarter of an Hour, tinge the Water with ſuch a Cerulous azure Blue, that few Vegetables do afford the like. We obſerve, that the longer theſe Leaves, or any other Stipticks, (which are the

Precipitators) do ſtay together, the more they degenerate into a deep Purple, or even to an Atramentarious Colour.

In reference to the internal Uſe of theſe Waters, I drank about a Quart at a time, after this manner. I firſt began with the *Spaw* Waters, which I procur'd very good, and drank them for a Week, and they agreed very well. I then drank the *Pyrmont* Waters for three or four Days, and continu'd the uſe of theſe Waters alternately, until I had drank about twenty Days. By the reſult of my Experiment it ſeem'd to me very plain, that the *Pyrmont* Water was more agreeable, gave more Strength and Spirit, and was as much or more preferable for its internal Vertue, as for its excelling the other in a brisker and more ſprightly Taſte.

There is another Excellency in theſe Waters which will make them more uſeful to us, than any Foreign *Chalybeat* Waters we yet know, becauſe theſe will keep better; they are not ſo ſoon ſpoil'd by any accidental Inſinuations of Air, as the *Spaw* are ſubject to be. The *Chalybeat* Mineral is here throughly diſſolved and well united, and mix'd in this Water, ſo that it does not eaſily precipitate: for which Reaſon it may alſo the better paſs the *vaſa lactea*, and even enter into the Maſs of Blood it ſelf, and work the more conſiderable Effects. That this is not a bare Hypotheſis may be prov'd by this Experiment.

Having ſuffer'd the *Spaw* Water to be expoſed in a Bottle which was half full, and unſtopt 12 Hours, I examin'd it, and found it taſt juſt like common Water; but the *Pyrmont* Waters that were open'd to the Air after the ſame Manner, taſted ſtrong of the Mineral, and gave their Tincture as at firſt; nay, they continued thus for full two Days, and perhaps might have done ſo longer, but I thought that Time ſuffic'd.

There

There remain ſeveral other Experiments to be made, in order to a further Search into the Excellencies of this noble Water, but this I cannot do at preſent for want of a Quantity, which I hope to obtain the next Summer; for they can with more Eaſe be brought into *England* than the *Spaw*. I may alſo fairly conclude, that ſince the *Spaw* has been very beneficial to our Patients in Chronical Diſeaſes, theſe Waters of a much ſuperior Virtue will ſurpaſs them in conquering many of our obſtinate Diſtempers.

Some Additions to the aforeſaid Account of the Pyrmont *Waters.*

HAVING had lately ſome Diſcourſe about a Purging Quality contained in theſe Waters, I am now inquiring into the Truth of this Queſtion, whether they in Reality do contain any Purging Ingredients or Properties.

I evaporated about a Quart of this Water *ad ſiccitatem*; I then poured on the *reliquiæ* ſome Rain-Water, enough to diſſolve and take up the Salts, and exhal'd that Water, and had a Grane or two of the Salts, that taſted *muriatic*, ſuch as moſt River and Pump Waters give. It is well known that the Purging Waters have a very bitter Taſte, and by the moſt learned Doctor *Grew piæ Memoriæ*, and an illuſtrious Fellow of this Society, that Salt was called *Sal Catharticum amarum*, which diſtinguiſh'd it from all other Species of natural Salts: that of the *Pyrmont* Water abovementioned has no Relation to this, but to the Sea-Salt, not being in the leaſt bitter.

It is alſo well known, that unleſs our Waters be impregnated with a conſiderable Quantity of this birter Salt, it will not purge at all: Two or three Granes ſignifie nothing, nor have the leaſt Cathartic Power. For Example, Put two Drams of the purging Salts to a Quart of common Water; and this Quantity will give but a Stool or two to one who is naturally very eaſie to work upon. I have examin'd ſeveral other *Chalybeat* Waters, and found much the like Ingredients, and never any that I could ſuſpect to carry any purging Properties.

I think we can much better demonſtrate that the *Chalybeat* Waters do contain Stiptic and Reſtringent Virtues, becauſe they owe their Birth to the *Iron Mineral*, and more particularly to the *Pyrites*, which Doctor *Liſter* ſuggeſts, (not without ſome Reaſon) to be the Parent even of all *Iron Oars*, as it is doubtleſs the Cauſe of all *Chalybeat* Waters: Thus I have often examined the Solution of the *Pyrites* by the Rain-Water at *Debtford*, and at other Places where Copperas is made, and found it a very ſtrong *Chalybeat* Water. It is from this *Minera* we have our ſtrong Stiptic and conſtringent Medicines, for external and internal uſe; we have our Powders and Salts of *Steel*, or *Vitriol* of *Mars*, from hence; nay, even thoſe obſtinate and inveterate *Diarrhæas* which have baffled the Force of all Medicines, have, by a judicious Uſe of *Tunbridge* and other Iron Waters, received a Cure.

But notwithſtanding all we can ſay, it will be retorted, that there is Matter of Fact and Experience againſt us, that the Waters really do purge at *Pyrmont*, where they are drank.

This we do allow to be true, that *Tunbridge* Waters do not only purge but ſometimes vomit, when drank haſtily and in great Quantity; but our Phyſicians have cor-

corrected this Irregularity, and we hear of no ſuch Complaints, where they obſerve a juſt *Regimen*: And we do all agree, that thoſe Waters are, in their own Nature, binding, and do oft require ſome opening Medicine. The Quantities of Water drank at *Pyrmont* are very large, often two or three *Engliſh* Quarts. It is no Wonder that their Weight forces them thorow the Bowels; for any common Water, drank haſtily, and in ſuch Quantity, will do the ſame. Whereas, if you take this Method, and will drink *Pyrmont*, or any other *Chalybeat* Waters leiſurely, *viz.* a Pint-Glaſs in an Hour, or rather two Half Pint Glaſſes, you may drink three Pints in ſo many Hours without Danger of loſing them by Dejection. But if any one will be careful, and take this Caution with him, he will ſcarce fail of Succeſs; that is, let him be very quiet and ſtill, both in Body and Mind; the leſs he ſtirs or walks, the better he will paſs off his Waters by Urine. And tho' this will appear a Paradox, eſpecially to thoſe Phyſicians who practiſe abroad, and commend to their Patients much Action in walking, yet I know I have both Reaſon and Experience on my Side. To avoid Prolixity I ſhall not declare them at this Time, and ſhall only ask leave to mention one Obſervation I have made, that none of our *Engliſh* Steel Waters do ſtrike ſuch a Purple as the Foreign celebrated *Chalybeat* Waters do: for ours do give a more turbid and dark Colour, and the worſe the Waters are, the blacker Sediment they make: Thoſe of *Iſlington* abound with a coarſe *Oker*, the Mineral is not well diſſolved, but gives an *atramentarious* Colour; but the *Pyrmont* Waters excell all I have happened to examine, in its bright *Cærulious* Luſtre.

N. B. *Moſt of the Experiments alledg'd by Dr.* Slare, *in the foregoing Diſcourſe, were likewiſe by him ſhewn before the* Royal

Royal Society, Feb. 28. *laſt: and it was found that the* Pyrmont *Waters gave a much brighter Tincture with Galls and Tea, and had a much more exalted* Chalybeat *Taſte than the* Spaw; *and a ſmall Quantity of each being kept for ſome time in Bottles, to compare them, the* Pyrmont *was found to have retained its Virtues much better than the* Spaw. *The Preſident, and ſeveral of the Members preſent, having drunk a Glaſs of it, found it of a very agreeable Reliſh, and to ſit eaſie on the Stomach.*

IV. *Remarks on the ſecond Paper in the Hiſtory of the Royal Academy of Sciences, for the Year* 1711. *concerning the Cauſe of the Variation of the Barometer: to ſhew that the Way of accounting for it in that Paper is inſufficient, and that the Experiment made uſe of to prove what is there aſſerted, does no way prove it.* By J. T. Deſaguliers, M. A. F. R. S.

The Paper is as follows.

'IT appears by the Barometer, that when it rains, or a little before Rain, the Air commonly becomes lighter.

'That it muſt rain when the Air becomes lighter it is eaſie to imagine; for the imperceivable Particles of Water, that ſwim about in the Air in prodigious Quantity, not being ſufficiently ſuſtain'd when the Air has loſt a certain Degree of its Weight, begin to fall, and ſeveral of them joining together in the Fall, make Drops of Rain. So when about half of the Air is drawn out of the Recipient of the Air-Pump,

'(and

'(and conſequently the remaining Air is as weak again
'as at firſt) ſomething like a ſmall Rain falls. But
'why ſhou'd the Air become lighter? One might ima-
'gine that in the Place where it rains, it may have loſt
'ſome of its Weight and Bulk, by means of the Winds
'carrying away ſome Part of it: but Monſieur *Leibnitz*,
'in a Letter to the Abbot *Bignon*, gives a more inge-
'nious and more new Reaſon for it.

'He pretends that a Body, which is in a Liquid,
'weighs with that Liquid, and makes up part of its
'whole Weight, ſo long as it is ſuſtained in it; but if
'it ceaſes to be ſuſtain'd, and conſequently falls, its
'Weight no longer makes a Part of the Weight of the
'Liquid, which thereby comes to weigh leſs. This
'may naturally be applied to the abovementioned Par-
'ticles of Water; they encreaſe the Weight of the Air
'when it ſuſtains them, which is diminiſhed when it lets
'them fall: and as it may often happen that the Parti-
'cles of Water that are higheſt, fall a conſiderable time
'before they join with thoſe that are low, the Gravity
'of the Air diminiſhes before it rains, and the Barome-
'ter ſhews it.

'This new Principle of Monſieur *Leibnitz* is ſurpri-
'zing. For muſt not a ſtrange Body, whether ſuſtain-
'ed in a Liquid or not, always weigh? Can it gravi-
'tate upon any other bottom than that which ſuſtains
'the whole Liquor? Does that Bottom ceaſe to car-
'ry a ſtrange Body, becauſe it falls? And is not that
'Body all the while it is falling, part of the ſaid Li-
'quid as to the Weight? At that rate, whilſt a Chy-
'mical Precipitation is made, the whole Matter ought
'to weigh leſs, which has never been obſerved, and
'ſcarce appears credible.

'Notwithſtanding theſe Objections the Principle
'holds good, when more cloſely examin'd. What ſu-

'ſtains

'ſtains a heavy Body is preſs'd by it. A Table, for 'Example, which ſuſtains a Pound Weight of Iron, is 'preſſed by it, and is ſo only becauſe it ſuſtains the 'whole Action and Effect of the Cauſe of Gravity, '(whatever it be) to puſh that Lump of Iron lower. 'If the Table ſhou'd yield to the Action of that Cauſe 'of the Weight (or Gravity) it would not be preſs'd, 'and therefore would carry nothing. After the ſame 'manner the Bottom of a Veſſel, which contains a 'Liquid, oppoſes it ſelf to all the Action of the Cauſe of 'Gravity againſt the ſaid Liquid: If a ſtrange Body ſwims 'in it, the bottom oppoſes it ſelf alſo to the ſaid Action 'againſt that Body, which, being in *Æquilibrio* with the 'Liquid, is in that reſpect really a Part of it. Thus the 'Bottom is preſs'd both by the Liquid and the ſtrange 'Body, and ſuſtains them both. But if the Body falls, 'it yields to the Action of Gravity, and conſequently 'the Bottom does no longer ſuſtain it; neither will it 'ſuſtain it, till the ſaid Body is come down to the 'Bottom. Therefore during the whole Time of the 'Fall, the Bottom is caſed of the Weight of that Bo-'dy, which is no longer ſuſtain'd by any thing, but 'puſh'd down by the Cauſe of Gravity, to which no-'thing hinders it from yielding.

'Monſieur *Leibnitz*, to confirm his Notion, propoſed 'an Experiment. He ſays, that two Bodies muſt be 'tied to the two Ends of a Thread, the one heavier, 'and the other lighter than Water, yet ſuch as both 'together may ſwim in Water: Put them into a Tube 'full of Water, the Tube being tied to one End of 'the Beam of a Ballance whoſe other End has a con-'trepoiſing Weight: Then if we cut the Thread which 'ties the Bodies together (that are of unequal Weight) 'ſo that the heavieſt may preſently deſcend, He ſays, that 'in ſuch a Caſe the Tube would be no longer in *Æqui-'brio*, but its counterpoiſing Weight wou'd preponde-

'rate

' rate, becauſe the Bottom of the Tube wou'd be leſs ' preſs'd. It is plain, that the Tube muſt be ſufficiently ' long, that the falling Body may not reach the bot- ' tom before the Tube has time to riſe. In Chymical ' Precipitations, the Veſſels are either too ſhort, or ' what is precipitated falls ſometimes too faſt and ' ſometimes too ſlow; for then the little Bodies are ' always (as to Senſe) *in Æquilibrio* with the Liquor ' that contains them.

' Monſieur *Ramazzini*, the famous Profeſſor at *Padua*, ' to whom Monſieur *Leibnitz* had propoſed his Expe- ' riment, has made it with Succeſs, after ſome fruitleſs ' Trials. Monſieur *Reaumur* (to whom the Academy ' had recommended it) has alſo made it with Succeſs: ' This is a new View in Natural Philoſophy, which, ' tho' it depends upon a well known Principle, is very ' ſubtle and far-fetch'd; and gives us juſt Reaſon to fear ' that in Subjects that ſeem to be exhauſted, ſeveral ' things may yet eſcape us.

Remarks upon Monſieur Leibnitz's *New Principle.*

Figure 4.

LET *AB* be the Bottom of a Veſſel full of any Fluid, whoſe Top is either wider than the Bottom as *GH*, narrower as *EF*, or equal to it as *CD*. The Preſſure of the Fluid upon the Baſe *AB* will be equal to the Weight of *CB*, or of a Cylinder or Priſm of the ſame Fluid, made up of the Area of the Baſe multiplied into the perpendicular Height above it.

If the Fluid be equally denſe every way as Water, or of a Denſity uniformly diminiſh'd as you go upwards, this Propoſition (call'd by Mr. *Boyle* the Hydroſtatical

Paradox) will hold good. This is demonſtrated by all Hydroſtatical Writers.

Figure 5.

Let *EF* repreſent part of the Surface of the Earth, and *GEFH* a Pillar of the Atmoſphere, whoſe Height is *GE* the whole Height of the Air. Let us imagine the Vapours riſing out of the Earth to form themſelves into two Clouds *A* and *B*, and to ſettle in that Place where the Air is of the ſame ſpecifick Gravity with themſelves. It is evident that they will cauſe the Air to riſe ſo much higher as their Bulk amounts to, and will therefore make the Surface which was at *GH* to riſe up to *IK*, ſo that the bottom *EF* which was preſs'd by a Pillar of Air as *GEFH*, is now preſs'd by an higher Pillar as *IEFK*. Now if the Clouds *A*, *B*, by any Cauſe ſoever, change their Place, ſo as to come downwards, (for *Exemple* to *C*, *D*) the Height of the Pillar *IEFK* will remain the ſame as it was, and therefore the Bottom *EF* will be preſs'd as before: by the foregoing Propoſition.

Corollary I.

If the Clouds *A*, *B* deſcend, and in their Deſcent keep the ſame Bulk as they had before, the Surface *IK* will remain the ſame, and therefore *EF* will be preſs'd as before.

Corollary II.

Whether a Body be ſpecifically lighter or ſpecifically heavier than a Fluid; ſo long as it is detain'd in it, it will add to the Fluid as much Weight as the Weight of an equal Bulk of that Fluid: wherefore a Body does not loſe all that Weight which it added to the whole

Weight

Weight of the Fluid, when it ceaſes to be ſuſtain'd in the ſaid Fluid: contrary to Monſieur *Leibnitz*'s Principle.

Scholium.

If a Cloud (by any Cauſe whatſoever) becomes ſpecifically heavier than that Part of the Air in which it ſwims, the Exceſs of its Gravity above an equal Bulk of Air will make it deſcend, and accelerate its Motion downwards; and then indeed it will loſe of its Weight by the Reſiſtance of the *Medium*, till it comes to an uniform (or ſenſibly uniform) Motion: but all the Weight that it will loſe will only be the Exceſs of its Gravity above that of the Air; for with the reſt of its Weight it will ſtill make up part of the Weight of the Air.

Experiment I. *Figure* 6.

Having with a Weight in the Scale *C* of the Balance *AB* counterpois'd the long Glaſs of Water *EI*, with a Horſe-Hair I let down the leaden Weight *W* into the Water, which from *FG* aroſe up to *EH*; and therefore the Water became heavier by the Weight of a Bulk of Water equal to the Lead. Having with another Weight in *C* made up the Counterpoiſe to the whole, with fine Sciſſars I cut the Thread of the Plummet; and all the while the Plummet was falling, the Water deſcended rather than roſe; and when the Lead was at the bottom, the Water overpois'd, becauſe it had then added to it all the Exceſs of Weight of the Lead above an equal Bulk of Water, which by Experiment is about $\frac{10}{11}$ of its Weight. Had Meſſieurs *Reaumur* and *Ramazzini* tri'd the Experiment thus, the Succeſs had been the ſame; but Mr. *Ramazzini* (as I underſtood from a Gentleman who was preſent) tried it in the following Manner, as I have ſince done.

Experiment II. *Figure* 7.

Making uſe of the abovemention'd Machine, after I had balanc'd the Water and Lead in it, I fix'd to the End of the Beam *B* the Thread of the Plummet, which in the former Experiment I held in my Hand. This added to the Weight hanging at *B*, and oblig'd me to put into the other Scale a Weight equal to $\frac{10}{11}$ of the Lead, to recover the *Æquilibrium*. Then cutting the Thread or Hair, the Scale with the Weights overpois'd whilſt the Lead was falling; but the *Æquilibrium* was reſtor'd when it came to the Bottom. So that the Lead even then muſt have loſt only its Exceſs of Weight above Water.

Experiment III. *Figure* 8.

I tried the Way propoſed by Monſieur *Leibnitz* in the following Manner.

I took a Cork *C* weighing an Ounce, and ſomething more than four times lighter than an equal Bulk of Water, and a Ball of Antimony *W* about four times ſpecifically heavier than Water, and of four Ounces Weight. The Cork laid upon the Water in the Veſſel *EABD* rais'd the Water from *SS* to *GG*, and added an Ounce to the Weight of the whole Water: then ſuſpending the Ball of Antimony by a String, and letting it hang in the Water at *N*, it rais'd the Water from *GG* to *HH*, and ſo added another Ounce to the Weight of the Water. Then tying the Antimony to the Cork (See the Figure of the Veſſel mark'd with little Letters) the Cork had added to it three Quarters of the Weight of the Antimony which the Hand before had ſuſtain'd, and made it ſink ſo as to be almoſt cover'd, and raiſed the Water to *ik*, adding three Ounces to its Weight. Hanging this Veſſel of Water upon the Balance, and a Coun-

Counterpoiſe at the other End, upon cutting the String the Veſſel of Water was rais'd up, and the *Æquilibrium* was not reſtor'd till the Antimony came to the Bottom.

By obſerving that as the Cork (being freed from the Weight of the Antimony) aroſe, and that during the Fall of the Body, the Water ſunk to *h h*, it appears that this is, in effect, the ſame Experiment as the former, and concludes no more. As to the real Cauſe of the Variation of the Barometer, namely, the Accumulation of the Air by Winds over the Place where the Barometer riſes; and part of the Air being blown away where the Mercury in the Barometer ſinks, ſee Doctor *Halley*'s Account of it in the *Phil. Tranſactions*. Numb. 181.

POSTSCRIPT.

IN making the firſt Experiment before the R. Society, of a Piece of Lead ſuſpended by a Thread, whilſt it was wholly cover'd with Water in the large Tube in which it hung (whoſe Length was 4 Feet) it was obſervable, not only that the End of the Balance (to which the Tube of Water with the Lead in it was fixed) did not riſe when the Thread was cut, (to let the Lead fall from the Top to the Bottom of the Tube) as it muſt have done according to Mr. *Leibnitz*'s Principle; but that the ſaid End of the Balance began to deſcend from the Time that the Lead began to fall. Therefore to be ſure that it was not the Plummets rubbing againſt the Sides of the Tube in its Fall, which cauſed that *Phænomenon*, I hung to the Balance a long Glaſs of three Inches diameter inſtead of the Tube, and making the Experiment as before, it ſucceeded in the

the ſame manner: the End of the Balance which carried the Veſſel of Water ſunk as ſoon as the Thread of the Plummet was cut; tho' this Glaſs was not above half ſo long as the Tube.

When by holding the String I drew the Lead upwards and downwards in the Water, there was no ſenſible Alteration of the *Æquilibrium*. Neither was it alter'd by cutting the String of a Stone-Plummet, becauſe of the Shortneſs of the Glaſs, and the little Exceſs of ſpecifick Gravity in the Stone: for the greater the Difference is betwixt the Body made uſe of in this Experiment and Water, as well as the bigger the Body it ſelf is, the better the Experiment will ſucceed.

Hence it appears, that when a Body, ſpecifically heavier than a Fluid, is (by what cauſe ſoever) detain'd in any Place of the ſaid Fluid, it adds as much to the Weight of the whole Fluid as an equal Bulk of the ſaid Fluid amounts to: And when the ſaid Body, by the Action of its Exceſs of ſpecifick Gravity above the Fluid, deſcends with an accelerated Motion; ſo long as that Motion is accelerated, the Reſiſtance of the Fluid (which is as the Square of the Velocity) takes off ſomething of the whole Weight of the Body; but as much as the Body loſes, ſo much the Water gains, over and above what was given it by its riſing on Account of the immers'd Body.

A Body therefore that falls in a Fluid is ſo far from making the Fluid lighter as it falls, that it makes it preſs more upon the Bottom that ſuſtains it, when it is falling, than when it was at reſt in the Fluid.

If the Veſſel of Water be long enough for the falling Body to come to an uniform Motion before it reaches the bottom, the Force impreſs'd on the Water under the Body will make it preſs the Bottom, as much as if the Body were actually at bottom; the Body in that Caſe loſing

ſing all its Exceſs of Gravity above that of the Water, and the Water gaining it.

Hence it follows, that a falling Cloud, when it comes to an uniform Motion, will not only add to the Weight of the Air as much as the Weight of an equal Bulk of Air; but even as much as its whole Weight amounts to, tho' it be ſpecifically heavier than the Air about it.

All the Diminution of Weight that can be allow'd in this Caſe is this. If we imagine the Air to have a ſmooth, regular Surface, as we have at firſt ſuppos'd, (or if that be not allow'd, we may take any imaginary Surface of it above the Clouds) when a falling Cloud is diminiſh'd in Bulk, (as when it is chang'd into Rain) the Surface of the Air will ſubſide in proportion to that diminution, and therefore will weigh leſs, by ſo much as is the Weight of a Quantity of Air equal to the Bulk that Cloud has loſt: But when the Drops of Rain after their Acceleration (occaſion'd by their Exceſs of Gravity above that of the Air) are come to an uniform Motion by the Reſiſtance of the Air, they reſtore to the Air the Weight that it had loſt. Now this uniform Motion being acquir'd in about two Seconds of Time, and the Diminution of Gravity in the Air being inſenſible, when compared to near three Inches of Mercury (for ſuch is the Variation of the Barometer with us) can no way be the Occaſion of thoſe ſo ſenſible Alterations in it, which happen ſome time before Rain or Fair Weather.

Add to this that the whole Quantity of Rain that falls in England *and* France, *in the Space of one Year, ſcarce ever equals two Inches of* Mercury: *And in moſt Places between the* Tropicks, *the Rains fall, at certain Seaſons, in very great Quantities, and yet the* Barometer *ſhews there very little or no Alteration.*

V. An-

V. *An Account of an extraordinary Effect of the* Cholick: *communicated to the* Royal Society, *by that curious Anatomiſt* Mr. St. Andre, *and read* March 21. 1717.

THE *Periſtaltick* Motion of the Inteſtins is by all Anatomiſts ſuppos'd the proper Motion of thoſe Cylindrical Tubes.

The uſe of this Motion is to propel the Chyle into the *vaſa lactea*, and to accelerate the groſſer Parts of the Aliment downwards, in order to expel them, when all their nutritive Contents are extracted.

This Motion thus eſtabliſh'd, it naturally ſeems to follow that an Inverſion of it (call'd for that Reaſon an *Antiperiſtaltick* Motion) ſhou'd force the *Aliments*, *Bile*, *pancreatick Juice*, and laſtly the *Faces* to aſcend towards the Mouth.

The Cauſe of this imaginary *Antivermicular* Motion, is aſſigned to a Stoppage of the Inteſtin, or to a great length of it being ingaged in the ſame manner as the Fingers of a Glove are choak'd by inverting the Glove in drawing it off: Or like a sa Silk-Stocking, which when 'tis not gartered, falls upon the Foot, and is in a manner ſtrangled, ſo that ſome Force is required to bring it up again.

This ſuppos'd, the *Antiperiſtaltick Hypotheſis* ſeems at firſt Sight very natural, and anſwers moſt Difficulties. For if the *Vermicular* Motion accelerates the Contents of the Inteſtins downwards; the *Antivermicular*, by the Law of Contraries, ſhould force them upwards towards the Mouth.

Was

Was this Suppoſition as certain as 'tis generally receiv'd, I ſhou'd not preſume to advance that there is no ſuch thing as an *Antiperiſtaltick* Motion of the Inteſtins; nor that the *Miſerere mei* is oftner a violent Contraction of the Abdominal Muſcles, than a Stoppage or Inverſion of the Inteſtins, as 'tis ſuppos'd.

So laying aſide all Prevention, let it be granted that this Diſeaſe is a violent Contraction of the Abdominal Muſcles, as I have already ſuppos'd it, caus'd by the Redundancy of the Inteſtins or their Contents. Then comparing the Symptoms of this Diſeaſe, with thoſe of the different Kinds of *Hernias*, we ſhall find by the Analogy of the Parts, Reaſon and repeated Experience, that the *Chordapſus*, ſo call'd by *Celſus*, is a Diſeaſe in which the Inteſtins and *Omentum*; at other Times the *Pancreas* or *Spleen*; nay, even the *Meſentery* it ſelf are forc'd through the *Diaphragma* into the *Thorax*.

All theſe tender Parts being ſtrongly compreſs'd, by the continual Motion of this Muſcle, muſt by conſequence cauſe the ſame Accidents as in the *Bubonocele* or compleat *Hernia*, there being no difference in theſe two Caſes; but that the firſt is a ſtrangling of the *Inteſtin* by the *Diaphragm*, and the latter a choaking of the *Inteſtins* by the *Abdominal Muſcles*.

One Example of the many of the like Nature, that I can produce, will much confirm this Aſſertion, and may ſerve to convince any Perſon that is impartial.

The Caſe is this: A Gentleman that came to Town yeſterday was Sevennight in good Health, meeting with ſome Friends, drank a great deal of new bottled Oat-Ale, after ſome Pints of Wine. Theſe Liquors fermented ſo violently in his Stomach and Inteſtins; that he was taken with a violent *Cholick* the ſame Night.

In the morning an Apothecary was ſent for, who adminiſtred a Clyſter, and took ſome Ounces of Blood

to relieve the Patient, who complain'd of a great Pain in his left Side.

The Clyſters being repeated the Night following, as alſo the next Morning, and the Patient growing worſe; the Apothecary, without Order of any Phyſician, gave him a violent Vomit; which operated Eight or Nine Times: This added Fewel to the Fire; and the Patient having from that Time been in a deſperate Condition, two eminent Phyſicians were call'd, who order'd that the Clyſters ſhou'd be repeated: But they not prevailing, I was ſent for about ſix Hours before the Patient died: I found him complaining of a violent Pain in all the Region of the *Abdomen*; a frequent Inclination to vomit; having a great Difficulty of breathing, together with a very ſlow Pulſe; his Belly being as hard as a Stone, tho' not ſwell'd.

This laſt Indication made me conclude, that the Diſeaſe was a violent Contraction of the Abdominal Muſcles, which had overcome the *Diaphragm*, and that probably the *Inteſtins* might be forc'd into the *Thorax*.

I was the more confirm'd in this Opinion from the Examples of the like Caſe, which I ſhall ſhortly lay before the Society; upon which I order'd a Fomentation of hot Milk, adding to every Quart a Drachm of *Liquid Laudanum*, which in theſe Maladies gives great Relief: But before it cou'd be got ready, the Patient expir'd in a violent Convulſion.

My Opinion having been highly cenſur'd by the two Phyſicians; I open'd this Gentleman, to juſtifie my ſelf, or to own my Fault openly, if I had been miſtaken: But as the thing happen'd as I conjectured, thoſe Gentlemen will forgive me for taking the Liberty of juſtifying my ſelf.

In opening this Body, I found the Abdominal Muſcles ſo much contracted, that it was almoſt impoſſible to penetrate them with a very ſharp Scalpel. Upon

Upon Examination, I found the Stomach empty, and ſome Parts of the *Duodenum*, but the *Jejunum* and *Ilium* ſo much diſtended with the fermented Oat-Ale, that the *Ilium* had four Inches of Diameter, and the *Colon* above eight.

The *Ilium* was alſo pretty much inflam'd in its inferior Part; and all the Valves of the *Colon* were obliterated, by the great Diſtention of that *Inteſtin.*

But the greateſt Diſaſter was, the Dilatation made in the *Diaphragm*, as I ſuppos'd; made juſt upon the Chink which remits the intercoſtal Nerve to the *Viſcera* of the *Abdomen*, through which a Portion of the *Colon* was forc'd, and the greateſt Part of the *Omentum* and *Pancreas.*

Theſe tender Parts being choak'd, ſoon inflamed, a Mortification of them following; and a Rupture of the *Pancreatick Vein* caus'd an internal *Hæmorrhage*, which fill'd all the left Cavity of the *Thorax*, inſomuch that the whole left lobe of the Lungs was compreſs'd almoſt under the *Muſculus Scalenus.*

The Quantity of extravas'd Blood was very great, and it was not in the leaſt coagulated.

I have brought the diſeas'd Parts with me, to ſhew the Society the Certainty of this Account, and I ſhou'd have been more particular in proving the Impoſſibility of the *Antiperiſtaltick* Motion; if Doctor *Hagumot* had not prevented me by his *Memoir.*

This Gentleman is not far from Truth, and what he ſays is certain: but I am ſurpriz'd that the like Caſe has not occurr'd in his Practice.

VI. *An*

VI. *An Account of Two late* Northern Aurora's, *as they were obſerv'd at the Vicarage of* Sutton at Hone *in* Kent. *By the Reverend* Edmund Barrell, *Prebend of* Rocheſter:

ON *February* the 5th 171$\frac{6}{7}$. at Eight at Night an *Aurora Borealis* appeared. It occupied at leaſt $\frac{1}{6}$ or near $\frac{1}{5}$ of the Horizon; it was low, and ſhot out bright Rays, and, I believe, would have appeared very light, had it not been that the Moon ſhone at the ſame time, being about Five Days old, and that the *Aurora* diſappear'd before the Moon ſet.

Again, on the 30th of *March* following, there was another *Aurora Borealis*. I ſaw it not till paſt Nine: 'twas dim then, and its higheſt Part cover'd the loweſt Star in *Caſſiopea's Chair*. It did not ſeem due *North*, but one Point to the *Weſt*. About Ten it ſhot out very bright Rays, high, and tending ſomewhat towards one another. Near Eleven a Clock, there was (beſides the *Northern* Brightneſs) a long Streak, not very broad, extended *Eaſt* and *Weſt*: Which beginning in the *Serpent's-Head*, near *Hercules-Club*, and covering *Arcturus*, proceeded near *Berenices Hair*, and ſo went over *Cor Leonis*, and thence to the *Canicula*, and ended a little beyond that Star. It ſhone very bright at firſt, but faded away in about Eight or Nine Minutes. If it had Motion (which I am not ſure of) it was Southward. I waited for the next Fit of Brightneſs of the *Aurora*; and in about Seven Minutes, the *Eaſtern* Part of the Streak, *viz.* from the *Serpent's-Head* to near *Berenices Hair*, became viſible again tho' dim, and was quite effaced in Four or Five Minutes more: And I did not yet perceive any Change of its Place.

F I N I S.

Errata. Page 556. in the Note, lege *Lib.* xxxv. pag. 563. line antepenult. lege *Anderida.*

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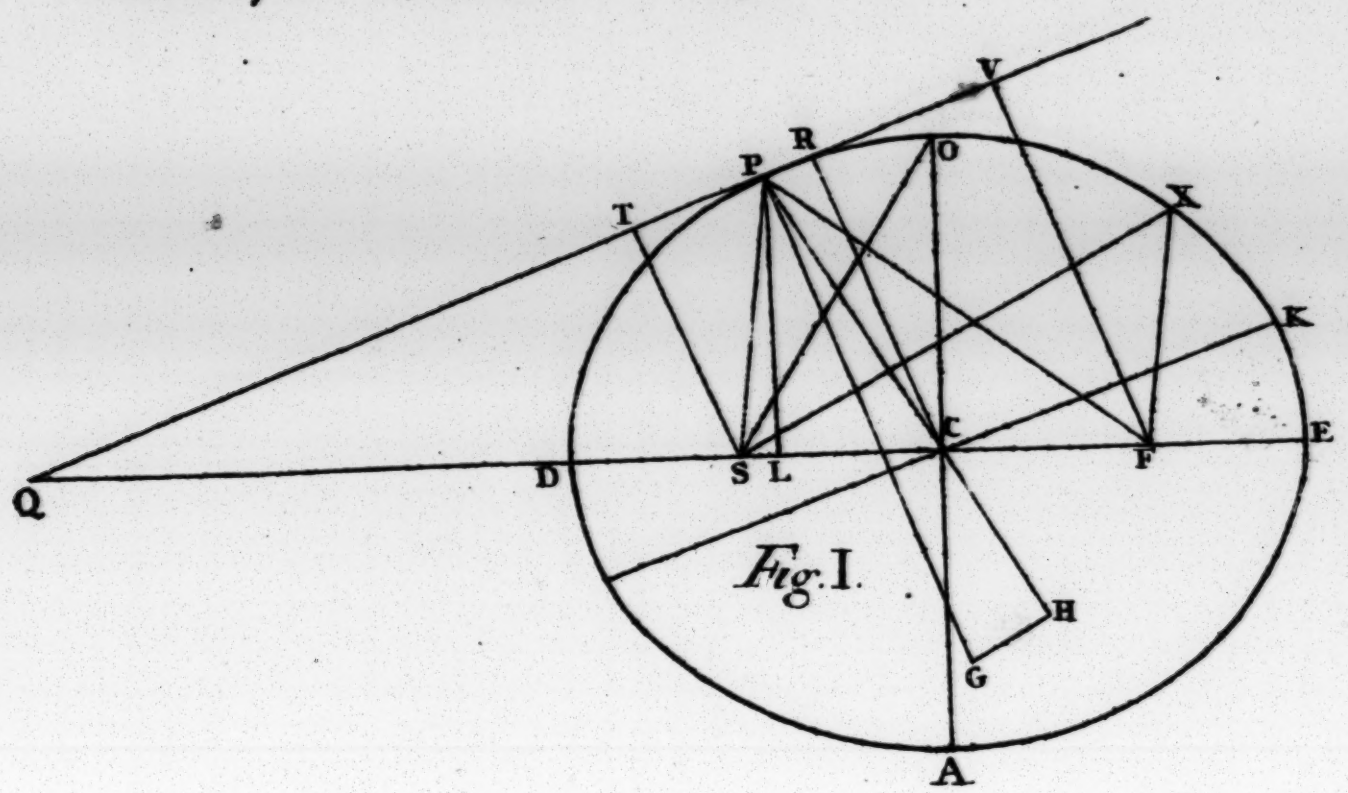


Fig. I.

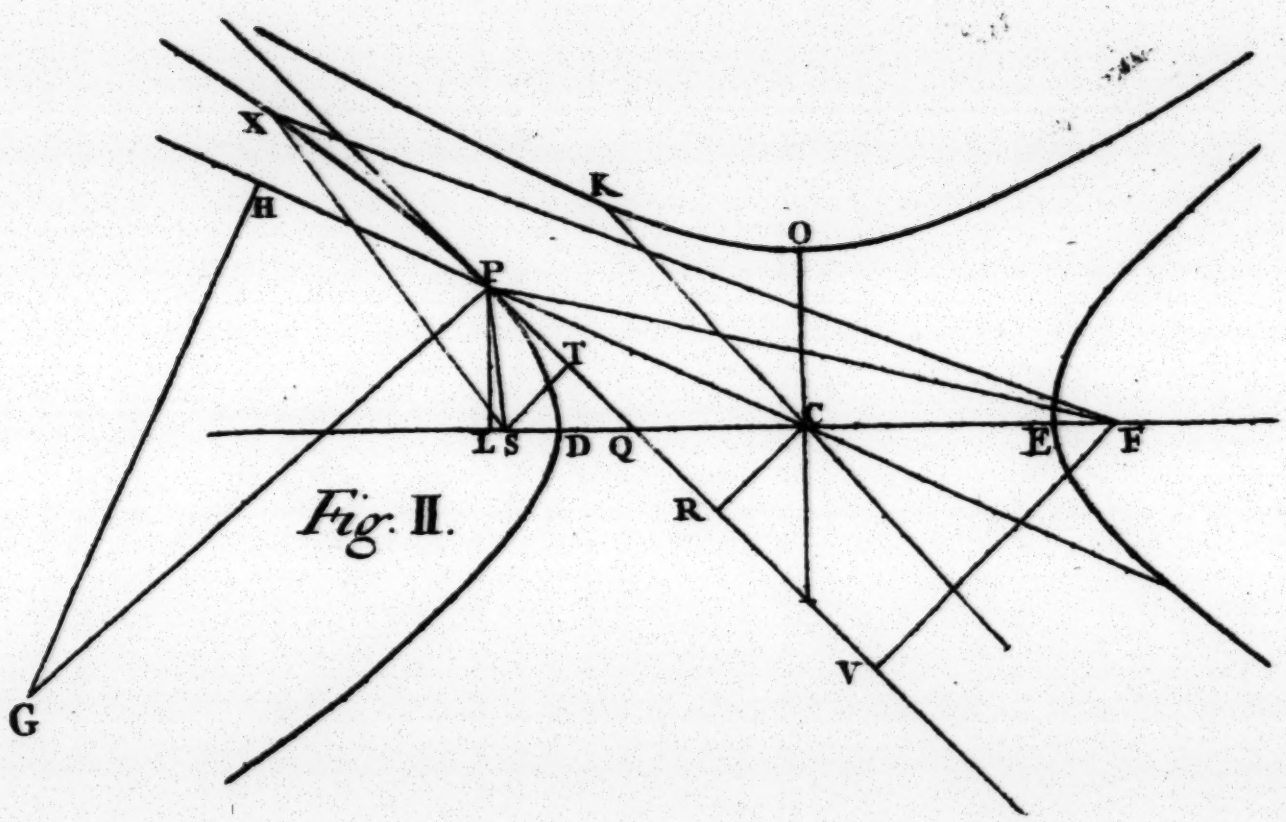
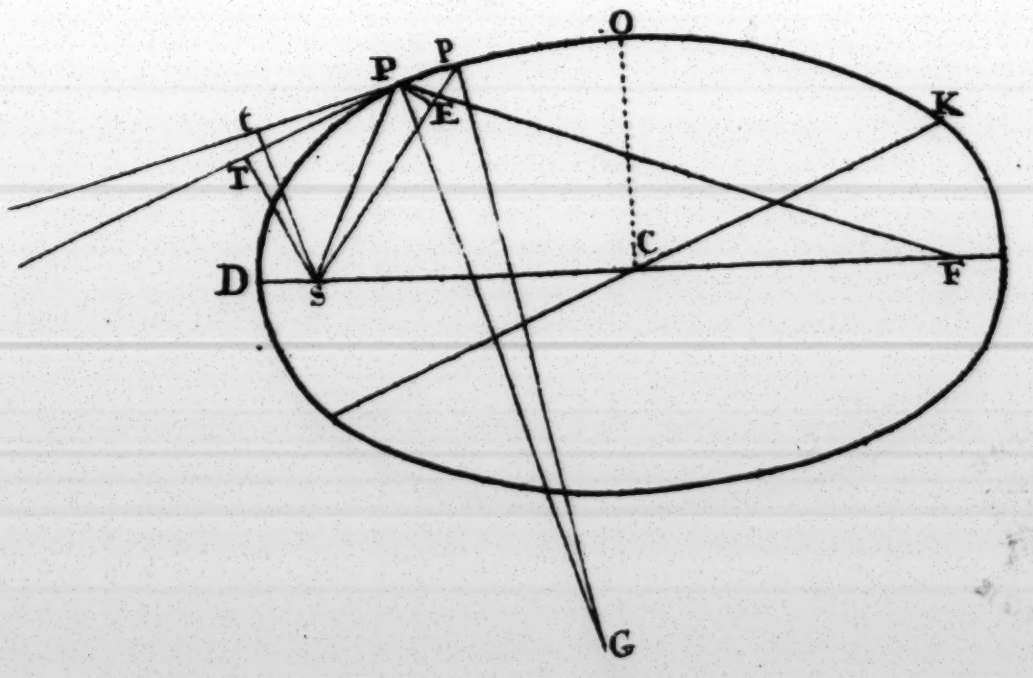


Fig. II.

Fig. III



Jos: Banks

PHILOSOPHICAL TRANSACTIONS.

For the Months of *April*, *May*, and *June*, 1717.

I. *An Account of the* Aurora Borealis, *ſeen at* London, *on the* 30th *of* March *laſt, as it was curiouſly obſerv'd by* Martin Folkes, *Eſq*; R. S. Soc.

II. Guilhelmi Muſgrave *Regiæ Societatis Socii, de* Britannia *quondam Pœne-Inſula,* DISSERTATIO.

III. *Extracts from Mr.* Gaſcoigne's *and Mr.* Crabtrie's Letters, *proving Mr.* Gaſcoigne *to have been the Inventor of the* Teleſcopick Sights *of Mathematical Inſtruments, and not the* French. *By* W. Derham, *Prebend of* Windſor, *and* R. Soc. Soc.

IV. *An Attempt towards the Improvement of the Method of approximating, in the Extraction of the Roots of Equations in Numbers. By* Brook Taylor, *Secretary to the* Royal Society.

V. *Proprietates quædam ſimplices* Sectionum Conicarum *ex natura* Focorum *deductæ; cum Theoremate generali de Viribus Centripetis; quorum ope Lex Virium Centripetarum ad Focos Sectionum tendentium, Velocitates Corporum in illis revolventium, & Deſcriptio Orbium facillime determinantur. Per* Abr. *de* Moivre. R. S. Soc.

I. *An Account of the* Aurora Borealis, *ſeen at* London, *on the* 30th *of* March *laſt, as it was curiouſly Obſerv'd by* Martin Folkes, *Eſq;* R. S. Soc.

BEing in the Street, between 8 and 9 a Clock on *Saturday* laſt, (30 *Martii)* I perceiv'd a Light over the Houſes to the Northwards, little inferiour to that the Full Moon gives when ſhe firſt riſes. Upon this, ſuſpecting ſome ſuch Meteor as we ſaw the laſt Year, I made all the haſt I could into the Fields, where I immediately found my Conjecture verified; and was for ſome time agreeably entertain'd with the ſight of an *Aurora Borealis*, attended with moſt of the *Phænomena* that have been deſcrib'd in that very remarkable one of the 6th of *March*, 1715-6.

The whole Northern Part of the Horizon was in the ſame manner cover'd with ſomewhat reſembling a very conſiderable Light, whoſe lower part was pretty well defin'd by the common Edge of the Cloud, but the upper dy'd away more gradually. This upper Limb of the Light reſembling the Arch of a Circle, whoſe higheſt Point between 9 and 10 of the Clock (when the Meteor was moſt conſiderable) was elevated about 12 Degrees, and bore, as I imagin'd, about 20 deg. Weſtward of the due North. It touch'd the Horizon in the Weſt at the diſtance of about 65 or 70 Degrees from the North, whence the whole intercepted Arc of the Horizon would have been of near 100 Deg. had not ſome few Degrees in the Eaſt been hid by Clouds which lay between us and the Meteor.

The ſeeming black Cloud, when I firſt ſaw it, ran nearly parallel to the Horizon, and at the diſtance of 6 or 7 Degrees: but in about half an Hour it changed its Figure very

very much, ſinking down in the North to about half its height, and riſing in the Weſt near as much. What I principally took notice of this for, was that the Light iſſuing from behind it did not change with it, but remain'd of the ſame Figure, however the Cloud approached or receded from differing Parts of its Limb.

There aroſe at firſt ſome Streams in the *N. N. W.* but of no conſiderable Length, few of them paſſing 5 Degrees above the Arch; but beginning from behind the ſeeming Cloud, ſo as to be about 12 Degrees high in all. They were Pointed at the Ends, and nearly vertical to the Horizon. Between times there was nothing but the Arch to be ſeen, and that only reſembling a common *Aurora*; and again in an inſtant, by a ſort of tremulous Motion, ſeveral Parts of it would appear converted into a vaſt number of parallel Streams, for the moſt part very little higher than the Arch it ſelf. About 20 Minutes before Ten, a ſmall part of the Arch, almoſt due North, grew remarkably lighter than the reſt, and continued to encreaſe for about half a Minute; when there ſuddenly broke out ſome very tall Streams of at leaſt 60 Degrees high, as I found by one in particular which aroſe full North, and paſſing over the Pole Star itſelf, reach'd ſome Degrees beyond it. This was the moſt remarkable time of the Appearance, ſome ſuch Lances, though not ſo high, immediately ſhooting out of the Place that firſt of all radiated, as did ſome more a good way to the Eaſt. They were all nearly Perpendicular to the Horizon, and moſt of them did ariſe quite from the black Subſtance at Bottom, tho' I ſaw ſome few that did not reach ſo low, appearing as if their lower Parts had been broken off Some of them were full as bright as any I ſaw the laſt Year, the Axes (if I may ſo call them) of ſome of the talleſt Streams coming up very

near

near to the Colour of that pale Fire we ſee in ſome ſorts of Lightning.

About this time the Ground Weſtward was all cover'd with an odd ſort of Miſt, the ſame from which I remember laſt Year a great many People ſaid there came an ill ſmell, which I did not at all perceive; however as I remember it to be the very ſame Appearance, I thought it might not be improper juſt to take notice of it.

About 10 the *Phænomenon* very much decreas'd, and ſo continued till after 11, only ſending up now and then 2 or 3 Streams. At half an Hour after 11 it was again pretty much encreas'd, and I ſaw it again ſend out ſome Streams almoſt as conſiderable as any I had before ſeen this Evening; the Arch yet continued, but not ſo entire; and from what I could judge, its middle was ſome Degrees nearer the North than when I firſt took notice of it. Till a quarter of an Hour before 12 the light continually abated, and then I left it; but a Watchman, I order'd to bring me an Account of it next Morning, tells me it continued till towards Daybreak, but never ſtream'd remarkably after I went away.

Tho' I could not this time ſee any Stars through the black Matter at Bottom, I am ſenſible it was not a Cloud, tho' it bore the reſemblance of one: for when a real Cloud (as ſeveral ſmall ones did) came over any part of it, their difference was very conſpicuous.

I have ſince receiv'd two Letters, one from *Wisbich* in the Iſle of *Ely*, the other from within 14 Miles of the *Bath*, both which take notice of it, tho' with no further Particulars, than that on *Saturday* Night, they had ſeen the ſame Light, tho' not ſo conſiderable, as in the beginning of *March* the laſt Year.

II. Guil-

II. Guilhelmi Muſgrave *Regiæ Societatis Socii, de* Britannia *quondam pœne Inſula,* DISSERTATIO.

CUM *Belgium* noſtrum a *Britannico* adluitur Oceano, illo latere Inſulæ hujus triquetræ, quod eſt ex adverſo *Galliæ*; viſum mihi fuit, priuſquam id deſcribere conarer, antiquam & diu agitatam movere quæſtionem, de *Britanniæ* cum *Gallia* conjunctione, &, an revera unquam eſſet, exquirere.

PRIMUM igitur, poſita Cherſoneſo *Britannica*, utrum exedi potuerit: Deinde, utrum exeſa fuerit, ediſſeram.

DE priori propterea dicendum, quod a docto & magni nominis Viro, *Voſſiorum* altero, ſtrenue negatum ſit, unquam, ubi hodie Fretum eſt, fuiſſe Cherſoneſum: & quidem ideo negatum, quoniam, illo ſentiente, nihil ei deterendæ dividendæque par inveniatur. Ut *Taprobanam* (Inſulam *Ceylon*) a vicina continente non avelli probet Vir Clariſſ. [*Otio* (a), inquit, *abundant, qui iſtiuſmodi* Ægyptiorum *fabellis, jam millies productis, totieſq; recoctis, aurem commodant. Quam conſtans & tenoris ſui obſervans ſit rerum natura, patet e Boſporis, omnibuſque omnium terrarum Fretis. Iis cum præcipue Marium & ipſius Oceani vis ſemper incubuerit, eadem tamen ubique a tot annorum millibus & ab ipſo, ut veriſimile eſt, rerum exordio, ſervaret intervalla. Currant licet, ac recurrant Undæ, allatrent undequaque Fluctus, fortius eſt Elementum quod re-*

(a) In Notis ad Melam.

ſiſtit

fiſtit, quam quod oppugnat. Exeſi Scopuli, ac vaſta maris antra, ſatis ubique oſtendunt, quantum Oceani impetus lapſu ſeculorum poſſit efficere: verum hæc ipſa quoque, quid non poſſit efficere Oceanus, multo clarius oſtendunt.]

Hæc *Iſaacus.*

UT diſputationem ea de re ingrediar, Vir hic Clariſſ. Naturæ, nec qua agit illa nec qua patitur, ſtatum ac conditionem, ex omni parte, ſic, ut eſt revera, animo ſatis advertiſſe videtur. Cum non de *Taprobanæ* ſolum Freto commentatur, ſed de Fretis in univerſum, & quidem Argumento *a conſtante & tenoris ſui obſervante rerum natura* accepto, videamus quam hæc cum Oceanis & Freto noſtris conveniant, & in iis quam conſtanter agat Natura.

OCEANI *Britannici*, prout nunc dierum eſt, cum latitudo tum profunditas inveſtiganda, ut ex iis de priſco ſeu Freto, ſive Sinu, poſſimus ſententiam ferre. Ut autem eas comperiamus, adeunda eſt Tabula *Halleiana*, ſui generis omnium accuratiſſima, ex juſſu Regis *Guilhelmi* ejus nominis Tertii conſtructa. Ea docemur, in Oceano *Britannico*, ubi Terrarum hiatus, hac illac ampliſſimus eſt, a veterum *Ocrino* (*Lizard-Point*) ad Inſulam ei oppoſitam *Uſhant*, unum eſſe gradum cum ſemiſſe, id eſt Leucas quaſi triginta, ſive milliaria 90. Hinc Oceanus ſe in oriente parum adducit, at multo magis ubi Promontorium in eum procurrit *Normannicum*: ibi enim eſt dimidio adductior; cum inter *Peverel-Point*, & *Cap. de Hague* e regione ſita, Leucarum *Anglicarum* quaſi 16 diſtantia ſit. Tunc ſe iterum effundit, ubi *Sequanam* recipit: at brevi in arctum agitur, inter *Beachy-Head* & *Cape St. Vallery*. Dein paulatim anguſtior, faſtigiat ſe molliter, uſque dum in Fretum contrahitur, inter *Neſs Anglorum*, & *Gallorum Blackneſs*, non amplius octo Leucis, id eſt 24 milliaribus patens. Terræ

tunc

runc aperiuntur longe lateque vaſtiſſime, & ſpatium Mari faciunt *Germanico*.

HÆ ſunt Oceani Fretique *Britannici* diverſæ latitudines; quibus apparet, eas, ſi non continuo, tamen adeo rara tamque exigua cum ampliatione minui, ut argumento noſtro nihil inde queat derogari. Ita enim Oceanus contrahitur, ut qui initio, ſeu Faucibus ejus *Britannicis*, Leucas triginta, ſive Milliaria præterpropter nonaginta latus ſit, poſt Leucas 153 circiter, ſive Milliaria 460, (quæ hujus Oceani longitudo eſt) ad 24 milliaria contrahatur; id eſt ad primæ latitudinis partem quaſi quartam.

PROFUNDA hujus Oceani altero jam loco ſunt expiſcanda, & quidem optime beneficio ejuſdem Tabulæ. In ea dividitur Oceanus *Britannicus* una cum Freto, in Columellas numero decem, oblongas. Harum ſingulæ latera ſunt ex circulis Meridianis accepta; quæ cum in plano ducta ſint, videntur eſſe recta. Columellæ terminantur adverſis *Galliæ Britanniæque* litoribus: hoc eſt, Lineis huc illuc curvatis in litorum morem.

INCIPIAMÚS a prima in occidente, quæ & longiſſima Columella eſt: & (præmiſſo, quod *Hiberniam* & *Galliam* inter, Oceanus orgyias altus ſit in locis compluribus octoginta; uti paulo ulterius in aperto Mari, 100, 120, 140) notandæ ſunt in prima Columella *profunditates* omnium altiſſimæ; quæ decies exploratæ ſe habent, ut 58, 66, 63, 65, 58, 65, 68, 60, 60, 60. quæ *profunditatum* Summæ faciunt 623 orgyias. Eæ per decem, i. e. *profunditatum* numerum, diviſæ, mediam earum *profunditatem* oſtendunt eſſe 62.

IN Columella altera, decem altiſſimarum media *profunditas*, ſimili modo inveſtigata, eſt orgyiæ 51. In tertia 51. In quarta 40. In quinta 43. In ſexta 40. In ſeptima 36. In octava 37. In nona 33. Poſt nonam Columellam, cum Oceanus in lævam flectitur, &

obliquus in Fretum desinit, accipiam illud ut Terris interjacet, in *Britannia* locis appellatis *South-Foreland* & *Hastings*, in *Gallia St. Valery* & *Estaples* inclusum. Hic *profunditatum* decem media est 30. In Freto angustissimo 16: quæ ad profunditatem mediam in prima Columella, est, ut 16 ad 62; id est, ut 1 ad 4 fere; & ad altissimam profunditatem *Galliam* inter *Hiberniamque*, ut 16 ad 80, i. e. ut 1 ad 5: ad altissimam in aperto Mari, ut 16 ad 140; i. e. ut 1 ad 9 fere.

QUA proportione minuitur altitudo Maris, ea crescit Terræ Mari subjectæ acclivitas; & est illi in *ratione inversa*: quæ utique propositio, si non ex omni parte vera, (propter orbis figuram minime rotundam) tamen adeo veræ proxima est, ut argumentationi nostræ sufficiat. Est ergo Terra, in Freto nostro angustissimo undis subjecta, quam in Oceani *Britannici* faucibus, orgyias 46, id est pedes 276 altior; & quam Terra, *Galliam Hiberniamque* inter, Oceano subjecta, orgyias 64, sive pedes 384 altior; & quam Terra, aperto Mari subjecta, 124 orgyias, sive pedes 744 altior. Vide quanta sit Terræ ab alto Mari ad Fretum acclivitas; eaque ut ex calculo prædicto patet, fere continua. Hæc est Oceani *Britannici*, tam in illius Latitudine, quam Profunditate contractio.

AGE, nunc tendamus ultra, velis expansis, in Oceanum *Germanicum*: Hic Mare subito patentius, sic, ut etiam profundius: quod inter Promontoria *North-foreland*, *Orfordness*, Oppida *Caletum* & *Ostendam* interfluit, decem maximas habet mensuras, quarum media 24 Orgyias cum $\frac{4}{5}$ continet: quod inter *Orfordness*, & *Yarmouth*, *Texellam* & *Ostendam* est, maximas decem mensuras habet, quarum media 25 Orgyias cum $\frac{1}{5}$. Quicquid ultra est, Terris, hinc ad occidentem, illinc ad orientem se retrahentibus, vastissimus est Oceanus, in quo mensuræ sunt ab orgyiis 45 ad 50 numero quamplurimæ.

HÆC

HÆC a Freto *Britannico* tam in oriente, quam occidente Terræ declivitas (quæ a Maris altitudine utrobique aucta patefit) omnino probat, *in ipſo Freto jugum eſſe Terræ excelſum, acutum*; quod cum hodie non multum infra Maris ſuperficiem eſſe reperiatur, olim ſe emergere, hoc eſt *Cherroneſum olim hic fuiſſe, monſtrat.*

ALIA ſunt duo, quæ cum in hac re momenti ſint immenſi, tenore tum minus certo, & natura minus ſunt conſtanti; quandoquidem a Maris motu & ventis accepta. In refluxu Maris Aquæ nunc quieſcentes incubant arenæ; nunc eam molliter præterlabuntur. In æſtu mitiori, Litus & ima Rupium blandiſſime lambunt, tenerrime oſculantur. Fervente vero æſtu, res omnino alia eſt: Aquarum Fremitus auditur, Fluctus cernuntur, & ſe non parum attollunt. Nihilominus ſine multa ſtrage, terriſve aliquot annorum millibus exeſis, hæc omnia poſſe fieri, cum *Voſſio*, cogitandum eſt.

SIN æſtuanti Mari, quod altero loco dicendum eſt, Ventus ſuperveniat (*Flat* autem *Ventus ſic, ut vult*, & quis ei tempus ſtatuerit, aut modum impoſuerit?) papæ! quoti, quantique Fluctus advolvuntur! *Alpes* exiſtimare licet Cryſtallinos, niſi quod cito diffluant. Montem enim Mons invadit, detrudit, diſcutit; tantiſper ejus ſpoliis adauctus, dum in cœlum altiſſimus exurgit Aquæ Mons. Tanta vis Aquarum ex Oceano occidentali in *Britannicum* immittetur, & tanto impetu, quantus in univerſo Terrarum orbe rarus inveniatur; imo quantus ab ipſo rerum initio rariſſime. Oceani *Britannici* tum brevia, tum anguſtiæ, continuo pœne (quod oſtenſum eſt) creſcentes, faciunt, ut Aquæ ſic impulſæ mirum in modum eleventur, & in Iſthmum (quem argumenti gratia fuiſſe damus) arietent, ita ut ab iis Iſthmum exundari, deteri, ablui; ſicque Inſulam fieri *Britanniam* non videatur ἀδυνάτων ἕν; verum e contrario factu probabile. Quantæ Ventorum, at præcipue

Ze-

Zephyri Cauriq; virtutes ſint, in cogendo impellendoq; hoc Oceano *Britannico*, paucis expendam, a Doctiſſimo viro *Rad. Bohun*, (*Novi* olim *Collegii* Socio, qui *de Ventis* omnium eruditiſſime ſcripſit) hac in re adjutus.

ETIAMSI Zephyrus a Poetis *vitam rebus ferre* dicatur.

——— *ejuſque tepentibus auris*
Laxent Arva Sinus; ———

interdum ita fit, ut

—— *Euriq; Zephyriq; tonet domus* ; & ut
— *Zephyro multæ turbentur arenæ.*

Adeo ſævus, horribilis, iracundus ſæpe Zephyrus, Vires in Oceano, qui *Europam* & *Americam* vaſtus interjacet, acquirens, & in ampliſſimo hocce campo recenſens, vix concipi poteſt, quanto *Britanniæ Galliæque* oras impetu invadat. Exploratiſſimum enim eſt, in haſce oras eum communiter anni plus dimidio flare, (quod jam olim a *Julio Cæſare* notatum) & flatu eas ſæviſſime verberare: maxime autumno, a quo ſumunt originem Tempeſtates Idiomate noſtro dictæ [*Michaelmaſs-Storms*] eumq; adeo interdum ſævire, ut ſi cum æſtu fervente jungi Ventus hic acciderit, tam Oceanus *Britannicus*, quam Fretum *Sabrinianum* immane quantum augeantur. *Sabrina* vaſtiſſime turget. *Uzella* longe lateque *Somerſettenſem* Agrum exundat: Mirum ab hiſce Cataclyſmis quantum mea patria perpeſſa eſt. Continuatur æſtus ad uſque *Tewkesbury*, id eſt milliaria magis ducenta. Apud *Chepſtow* Aqua pedes interdum octoginta aſſurgit. Idem fere dicendum de Oceano *Britannico*, Venti ejuſdem viribus elato: niſi quod hic, Cherſoneſo jam effracta, liberius Aquæ moveantur, non adeo ſiſtantur, non tantum eleventur; quæ utique Aquarum libertas ante Cherſoneſum abruptam, nequaquam adeo magna eſſe potuit.

HAC igitur de cauſa [*Zephyro* nempe, *Cauro*, ſive alio Ventorum aliquo, Maris æſtui ſuperveniente] Oceani *Britannici* Undam in Iſthmum validiſſime impingi, & ab illis primum ejus ſuperficiem, quæ ex Silice & Calce (prout hodie Terræ e regione oppoſitæ) conſtabant, ablui; deinde Iſthmi quod reliquum erat, ſpatio bis mille annorum & eo amplius, Aquæ fluxu refluxuq; ad 16 orgyias, quæ hodierna Freti hujus (quod dixi-mus) altitudo eſt, atteri credibile, veriſimile eſt,

TANTUM abeſt, ut *Veſſio*, Fretorum perpetuitatem a naturæ in operibus ſuis *conſtantia* tenoreque eodem arguenti, fidem habeamus, ut e contrario Fretum hocce noſtrum illius *inconſtantiæ* deberi, lubens agnoſcerem. Vir ille clariſſimus, naturæ uſitatum agendi modum unice reſpiciens, *extraordinarium prætermiſit*; qui tamen, in raris hujuſmodi effectis, potiſſime videtur reſpiciendus. In Freto *Siculo* conſiderando, ejuſque diducendi modo inveſtigando, quis Ignis ſubterranei ſupra modum erumpentis, tamquam Cauſæ hac in re probabilis, non meminerit? niſi iſtam *Catanenſium ἀτοπίαν* conſecutus, qui (tradente *Alphonſo* † *Borello*) poſt Eruptionem *Ætnæ* diu intermiſſam, Ignem ejus immodicum ne ſemel quidem unquam fuiſſe, ſatis inſulſe putavere. Ut Vento nihil inconſtantius, ſic ad Fretum hoc aperiendum (poſito cauſarum apparatu cætero) nihil conducibilius: & cum eo res deducta eſt, *fortius elementum eſſe quod oppugnat, quam quod reſiſtit*, (aliter quam *Voſſius* ſtatuit) omnino probabile mihi videtur. *Salmaſium* ille, Virum & candidum & doctum convellit, quod de Navibus abſque Coſtis & Interamentis, ita ſcripſit, ac ſi in *Burgundia* manſiſſet, nec quid rei Navis aut Mare foret, intellexiſſet. Nequeo ſatis *Iſaacum* mirari, quod *de Mari & Ventis* ſcriptor luculentus, horum vires in Mari

† In Libro de *Incendiis Ætnæ* p. 117.

tur-

turbando agitandoque, in terris obruendis abrumpendiſq; prorſus omitteret; atque adeo quod *Batavus* omitteret: ea ſcil. Regione oriundus, quæ Mari & Vento tam obnoxia.

NON alienum erit hic Inundationum aliquot exempla, uti revera fuere, in medium proferre; quibus abunde patet, Terræ faciem frequenter obrui, & ab iis non parum mutari. Hic autem nihil neceſſe eſt, ut *Helicen* & *Burin*, *Achaiæ* Urbes adeamus: de quibus tamquam magnarum Inundationum argumentis, (1) *Ovidius*, & diu ante illum (2) *Ariſtoteles*. Graviſſimas fuiſſe Oceani noſtri, tam *Germanici* quam *Britannici*, ſatis oſtendunt Hiſtorici Geographique.

IN *Zeelandia* (3) Inſulæ undecem, & in iis Oppida & Pagi (quorum hodie ſummitates aliquæ, refluxu Maris in conſpectum veniunt) numero ter centum (4) obruebantur.

ANNO 1014. [*Mare Litus egreditur III. Cal. Octob. & in* Anglia *Villas quamplurimas innumerabilemque populi multitudinem ſummerſit.*] (5) *Simeonis Dunelmenſis* Hiſtoria de Geſtis Regum *Anglorum*. De hac, ut opinor, Inundatione videatur etiam Chronicon *Joh. Brompton* (6).

ANNO 1099. [*Tertio Non. Novemb. mare Litus egreditur, & villas & homines quamplures, Boves & Oves innumeras demerſit*] *Sim. Dunelmenſis* (7) Hiſtoria.

A D. 1176 [*Mare extra fines in* Anglia *erumpens multos in* Hollandia *homines & pecora abſorbuit, & quaſi*

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(1) Invenies ſub Aquis, & adhuc oſtendere nautæ
Inclinata ſolent cum mœnibus oppida verſis.
1 *Metam. Lib.* 15.

(2) Τὰ δὲ ἀνάλογον συμπίπτει τούτοις καὶ ἐν θαλάσσῃ. Χάσματα γάρ, γίνεσθαι θαλάσσης, καὶ ἀναχωρήματα πολλάκις, καὶ κυμάτων ἐπιδρομαὶ, ποτὲ μὲν ἀνάπαυσιν ἔχουσαι ποτὲ δὲ πνεύσιν μόνιμον, ὥσπερ ἱστορεῖται περὶ Ἑλίκην τε καὶ Βοῦριν. *Ariſt.* de Mundo. (3) *Heylin's* Geogr. L. 2. In Belgio. (4) *Laet.* Deſcriptione Belgii, pag. 124. (5) Apud Hiſtoriæ *Anglicanæ* Scriptores X, pag. 171. (6) Apud eoſdem, pag. 892. (7) Pag. 224.

poſt

poſt biduum furore ſedato in ſemet ipſum rediit.] Chronicon *Johannis Brompton.* (8)

INSOLITAM maris inflationem & commotionem Anno D. 1250. factam, tradit *Matthæus* (9) *Pariſienſis.* [*Unde Mare perturbatum fines ſolitos pertranſiens, tam horribilem mugitum cum fremitu edidit, ut per remota Terræ ſpatia, non ſine ſtupore audientium, reboaret. Viſum eſt etiam ſub opaca nocte ipſum Fretum quaſi accenſum ardere, & Fluctus Fluctibus conglomeratos dimicare. Apud* Winchelese *pluſquam 300 domus cum quibuſdam Eccleſiis per Maris violentum aſcenſum ſunt ſubmerſæ.*]

ANNO 1251. inquit idem (10) *Matthæus* [*In* Frigia (*quæ* Friſelandia *appellatur*) *Aqua Diluvium fecit particulare, occupans Terræ illius ſpatium itineris circiter ſeptem dierum. Poſt 40 dies ille damnoſus Fluctus in locum ſuum remeavit.*]

ANNO 1286. [*Ingruente fortiſſimo Vento, flante de partibus Orientis, qui & Eurus dicitur, & fluxu Maris ſuper provinciam* Hollandiæ *terribiliter invaleſcente, prævaluerunt Aquæ Maris, adeo ut Foſſata, quæ Terram ipſam & Mare diſterminant, inopinatius quam credi poterat, tranſgrederentur; poſuitque Terram fructiferam in ſalſuginem tam repentinus Maris impetus; qui per indigenas nullatenus poterat obviari: & maxima pars S.* Botolfi *ſubmerſa, hominumque & pecudum inæſtimabilis periit multitudo.*] Ita Chronicon *Tho.* (11) *Wikes.*

DE *Hollandiæ* Inundatione ſic *Hadrianus Junius* (12) in *Bataviæ* Hiſtoria [*Quadringentis abhinc annis inaudita* illa *Inundatione quæ univerſam* Hollandiæ *faciem longe lateque operuit, obſtructo fluminis* (Rheni) *curſu, ſteriles arenarum colles Litus occuparunt, Mare, Terras, ipſamque Litoris oram attrivit.*]

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(8) pag. 1117. (9) Pag. 535. Ed. *Watſiana.* (10) Pag. 549. (11) Pag. 114. (12) Pag. 196.

ANNO 1404. quo beatiſſimus noſter *Wiccamus* obiit, [*Tanta repente ruptis limitibus irrupit Aquarum influentia in Cantio, quanta nunquam fuerat illic ante viſa, qua ſubmerſa ſunt animalia numero & pretio exceſſivo: nec ſolummodo d flevit* Anglia *damna talia, ſed ut fertur,* Selandia, Flandria & Hollandia, *per Undarum excrementa, innumerabilia ſenſit diſpendia eo anno*] Hypodigmate *Neuſtriæ* per *Tho.* (13) *Walſingham.*

[REGNANTE Edwardo I. *cum Oceanus ventorum violentia exaſperatus, hunc* (Cantii) *tractum operuiſſet, lateq; hominum, pecorum ædificiorumque ſtragem dediſſet, &* Bromhill *viculo frequente peſſundato, etiam* Rother, *qui hic prius ſe in Oceanum exoneravit, alveo emovit, oſtiumque obſtruxit, novo in Mare aditu compendio per* Rhiam *aperto*] *Camdenus* in *Britanniæ Cantio.*

QUID quod *Tungros*, oppidulum *Leodienſe*, a Mari pene centum milliaria jam remotum, Mare quondam adluere opinati ſunt Viri doctiſſimi, argumento non uno perſuaſi (14).

NEQUE noſtra ætas caruit hujuſmodi Exundationibus: Narrant Novellæ *Feb.* 27. 1713. in *Eſſexia*, plura Terræ jugerum millia, per milliaria aliquot, inter *Barking* & *Purfleet*, everſis obſtaculis, Maris influxu obrui.

HÆC de Oceani *Germanici* exundationibus: *Britannici* noſtri, & *Sabrinæ*, neque pauciores, neque minores ſunt. Enimvero vidimus ætate noſtra Iſthmum, uno eodemque *Pedredi* fluxu & refluxu (Terræ ſuperficie prius ab Agricolis ſemota) dilui, fluviumque veteri curſu relicto novum acquirere: hoc, inquam, unico fluvii iſtius æſtu factum vidimus, nullo auxilium præbente vel Undas adigente Vento.

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(13) Pag. 564. Ed. *Francofurt.* MDCIII. (14) *Verſtegan*'s Antiquities, pag. 102. *Ray*'s Phyſico-Theological Diſcourſes, 1693. pag. 169.

SABRINÆ fluminis impulſu fieri probabile eſt Exundationem illam, qua in Agro *Monumetenſi*, Parœciæ No. 26. A. D. 1607. menſe *Januario* Aquâ obruerentur: Cujus eodem Anno publicata fuit (15) Hiſtoriola.

JOHANNE rerum *Anglicarum* potito [*Subita & improviſa Aquarum Inundatio pluribus in locis per* Angliam *facta eſt, unde plures homines ſubmerſi ſunt, & domus everſæ, maxime apud* Exceſtre *& Sanctum* Ivonem] Imagines Hiſtoriarum Autore (16) *Radulfo* de *Diceto*, [Poſt diutinam malaciam, mare *Vergivium*, adeo per totam hyemem, regnante tunc *Henrico* ſecundo, tempeſtatibus agitabatur, ut toto illo temporis ſpatio, Navicula nulla ad *Hiberniam* adpulſa, de reliquo terrarum orbe nihil apud eam inauditum. Terror hinc univerſus, tamquam malo impendente quodam gravi, de cælis miſſo. Arenarum aggeres, in Auſtrali *Cambria*, quaſi Cataclyſmo, abluebantur, Litora ſubvertebantur] *Giraldi Cambrenſis* (17) *Hibernia* expugnata.

AT ô quam terribilis illa tempeſtas, qua. Maris Undam in Oceanum *Britannicum* impellente *Africo*, ita ille turgebat, ut Pharos illa celeberrima, *Eddiſtone* appellata, quæ fuit e regione *Plimutha*, tamquam in contemptum *Æoli Neptunique* fabricata, quaſi ludibrio habita ſimul cum ædificatore dirueretur. Cujus utique tempeſtatis in hoc noſtro Oceano ſi non eadem vis & poteſtas eſſe videatur, atque illarum in prædictis Oceani *Germanici* Exundationibus, propter majorem ab hiſce ſtragem & damnum in *Hollandia Zeelandiaq*; factam; Litoribus hoc noſtris rupibus munitis, quæ & duriores & altiores quam apud *Batavos* ſunt, deberi judico.

TEMPESTATES (ut argumentum hoc conficiam) quæ in Oceanis *Germanico* & *Britannico*, noſtra

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(15) *Lamentable News from* Monmouthſhire. (16) Pag. 710.
(17) Cap. XXXV.

& patrum memoria ſæviere, & de quibus omnino conſtat, adeo fuere turbulentæ, ut ſi earum aliquæ in Cherſoneſum, ad hoſce dies uſque manentem, recta fuiſſent collineatæ, nullus (ut opinor) eſſet dubitandi locus, quin a tanta vi auferretur Iſthmus: & tot annorum ſæculis, quot illico dicentur, revera hoc accidiſſe nequaquam improbabile videtur.

SIN ex *Americâ* turbo maris æſtui ſuperveniat, mare, cœlum, omnia miſcens, omnia confundens, ac ſi naturæ inſtaret diſſolutio, (poſſe vero hæc concurrere nemo ſanæ mentis ibit inficias) En cauſam huic negotio parem!

ALTERUM hujus Diſſertationis membrum jam aggrediamur, & ſpeciatim inquiramus, utrum exeſa fuerit revera hæc Cherſoneſus; annon. Si de ejus dividendæ modo, qui fieri poſſet, conveniat, magna inde lux emanabit, unde argumenta, quæ a Viris doctis paſſim afferuntur, ad diviſionis hujus probabilitatem arguendam, egregie confirmabuntur: præmiſſo nempe (quod hactenus fuit deſideratum) Vento, ejuſque in elidendo hoc Iſthmo virtute omnium cauſarum maxima, tantoque negotio (cum cæteris) pari. Horum ego argumentorum nonnulla perſequar; ſed leviter tangam, utpote ab aliis fuſius antehac tractata.

PRIMO, Terræ jugum illud notabile, quod Freto ſubjicitur, & de quo ſupra; quid aliud ſibi vult, quam quod eo loci Terra olim multo altior eſſet; at Maris per aliquot annorum millia reciprocationibus, ad eum in quo nunc eſt ſtatum, abluta & detrita? Præcipue, ſi advertamus Regulam hanc conſtantem & perpetuæ veritatis eſſe, *Maris* ſcil. *imum, quo magis Oceano præterlabente tritum, (quantum patitur ejus durities) eo magis planum & æquale reddi.*

QUID deinde Rupes in Freti Litoribus oppoſitis, ſive Montes prærupti, albi, ex eadem materia, Calce nimirum & ſilice compoſiti, ad ſex utrobique milliaria, ſibi

ſibi invicem, tamquam Teſſeræ reſpondentes; quid, inquam, volunt, niſi olim interfringi ſe, & ablutione Terræ interpoſitæ diſrumpi?

TERTIUM, apprime convenit cum iſthac Cherſoneſi *Britannicæ* opinione, tractus illius, qui hodie *Rumney-Marſh* appellatur, ratio & ingenium. Durante enim Iſthmo, cum Oceani fluxus eo tamquam obice ſiſteretur, æſtuare eum neceſſe erat, atque adeo Terram illam *Rumneienſem*, utpote planam & humilem, in propinquo exundare. Hoc oſtendunt Oceani *Britannici* Fluxus, hodieque hac planitie altiores, aggere fortiſſimo & magnis ſumtibus aſpulſi: Dentes item oſtendunt, atque Oſſa, ſive Hippopotami ſive alius cujuſdam marini animalis (18), anno 1668, *Charthami*, altitudine pedum 17, dum puteus aperiretur, eruta: at luce clarius oſtendit Anchora, non ita pridem ex alto loca hæc circiter effoſſa. Perrupto autem Iſthmo obiceque jam remoto, Oceani unda ſubſidit, a Terra illa receſſit, in alveum ſe contraxit; unde quæ olim Æſtuarium, hodie Planities, longa viginti milliaria, lata octo, eaque fertiliſſima bobus ſaginandis aptiſſima reperitur.

NOVISSIME, fac Cherſoneſum olim fuiſſe, Lupos, aliaque animalia, generi humano inimica, poſſe huc migrare, conceptu facillimum eſt: at ſi illa non fuit, navigiis ea, tamquam ad tuendas & conſervandas eorum Species, advehi, ſtulte cogitabimus?

NEQUE me moratur, quod nulla ſive *Latinorum*, ſive *Græcorum*, ſive alius cujuſvis populi Hiſtoria Cherſoneſi hujus abruptæ mentionem feciſſe perhibeatur, (quamvis hoc nequaquam ex omni parte verum:) Dic ſodes, Hiſtoriæ quam brevis ſit ætas, ſi ad ætatem mundi comparetur. A rerum initio ad primam, quæ

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(18) Vide Clariſſ. *Somneri* Diatribam *Chartham-News* appellatum: Actorum Philoſoph. *No.* 272. Et Clariſſ. *Walliſii* de hac Cherſoneſo Diſſertationem, Act. Phil. *No.* 275.

nunc

nunc exſtat (i. e. *Herodoti)* Hiſtoriam, 3500 circiter anni ſunt; & a *Noæ* Diluvio, 1800. At tam immenſo temporis ſpatio (quod ſupra innuimus) quæ Cauſarum accidere poſſint *συζυγίαι*; quæque ex iis in orbe noſtro fieri mutationes, nemo tam cito ſtatuere debebit.

DIXI *hoc nequaquam ex omni parte verum:* quid enim planius illo *Virgilii*,

— *Penitus toto diviſos orbe* Britannos.

[*Nonne putatis,* (inquit eruditiſſimus & Antiquitatum *Britannicarum* ſcientiſſimus (19) *Joh. Twinus*) *vocabulum* [diviſos] *habere eam vim ut ſignificat abſciſſionem alicujus ab aliquo? Et Auctorem mire gnarum ſignificationis fuiſſe, & rerum antiquarum maxime peritum, & bene memorem ſui? Ad hæc verba* Servius [*Quia olim juncta fuit Orbi* Britannia.] Nihil clarius eſſe poteſt ad demonſtrandum Iſthmi hujus diviſionem veteribus fuiſſe notam. Ut omnino fruſtra eſſe *Voſſius*, & nimio plus ὑποθέσει δουλεύειν exiſtimetur, cum in Fabellis *Ægyptiacis* (quo nempe ſuæ opinioni habeatur honos) eam poni voluerit.

CONCLUDIMUS ergo e prædictis ſimul acceptis, *Britanniam non jam inde ab initio fuiſſe Inſulam, ſed ex Pæne-Inſula factam: idque ut videtur, a Vento e ſævioribus aliquo, cum Maris æſtu concurrente & Iſthmum perrumpente.*

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(19) De rebus *Albionicis*, pag. 22.

III. *Extracts from* Mr. Gaſcoigne's *and* Mr. Crabtrie's Letters, *proving* Mr. Gaſcoigne *to have been the Inventor of the* Teleſcopick Sights *of Mathematical Inſtruments, and not the* French. *By* W. Derham, *Prebend of* Windſor, *and* R. Soc. Soc.

IN *Monſieur de la Hire*'s firſt Part of his *Tabulæ Aſtron.* publiſhed in 1687. I find an Invention, which was undoubtedly our Countryman Mr. *Gaſcoigne*'s, aſcribed to *Monſieur Picard*, and that is, the *Application of Teleſcopick Sights to Aſtronomical Inſtruments.* Mr. *de la Hire*'s Words are, *Paucis abhinc annis D. Picard inſignis Aſtronomus, atque in eadem Academiâ* [*Regiâ Scientiarum*] *Socius, Dioptrarum crenas ab inſtrumentis ſuſtulit, eorumque loco ſubſtituit Teleſcopia; quæ res Presbytis & Myopibus*, &c. In which Words it is not indeed expreſly ſaid that Mr. *Picard* was the Inventor of this way, but only that he applied Teleſcopes. But by reaſon it implies that it was that curious and ingenious Gentleman Mr. *Picard*'s Invention, and it is in effect claimed as ſuch, in *Monſieur Auzout*'s Account of the *Teleſcopick Micrometer*, in the *Philoſ. Tranſ. No.* 21. therefore I think my ſelf in Duty bound, to do that young but ingenious Gentleman, Mr. *Gaſcoigne*, the Juſtice, to aſſert his Invention to him; by reaſon all his Papers, that by the late ingenious Mr. *Towneley*'s Diligence could be picked up, are now (together with Mr. *Towneley*'s own Papers) in my Hands.

As

As for the Invention of the Micrometer, which Mr. *Auzout* claims as his and *Monſieur Picard*'s, I ſhall ſay little to it, Mr. *Towneley* having ſufficiently prov'd it to be Mr. *Gaſcoigne*'s, in the *Philoſ. Tranſact.* No. 25. And the Deſcriptions and Draughts of that, and ſome other Inſtruments of that kind, are now by me, in Mr. *Gaſcoigne*'s own Hand, to confirm Mr. *Towneley*'s Account, if occaſion were.

And as Mr. *Gaſcoigne* was the firſt that meaſured the Diameters of the Planets, *&c.* by a Micrometer; ſo I ſhall prove that he was the firſt that applied *Teleſcopick Sights* to Aſtronomical Inſtruments. In a long Letter to his ſagacious Friend Mr. *Crabtrie*, of *Jan.* 25. 164⅞. (wherein he deſcribes his Micrometer, and ſhews his way of finding the Refractions, the Moon's Parallax, and how he meaſured the Diameters of the Planets) Mr. *Gaſcoigne* tells him how the *meaſuring Glaſſes*, which he had been ſpeaking of, might be applied to a *Quadrant*. *If*, ſaith he, *here* (that is in the Diſtinct-Baſe) *you place the Scale that meaſures —, or if here an Hair be ſet, that it appear perfectly through the Glaſs —, you may uſe it in a Quadrant, for the finding of the Altitude of the leaſt Star viſible by the Perſpective wherein it is. If the Night be ſo dark, that the Hair or the Pointers of the Scale be not to be ſeen, I place a Candle in a Lanthorn, ſo as it caſt Light ſufficient into the Glaſs; which I find very helpful when the Moon appeareth not, or it is not otherwiſe light enough.*

In another Letter, dated on *Chriſtmas-Eve* 1641. (wherein he deſcribes the Wheel Work of his Micrometer, and ſhews how he could apply it to the taking of three Points; and ſpecifies his Obſervations of the Diameters of the Sun and Moon; and mentions a Theory he had contriv'd of the Sun; *&c.* and ſaith what pains he had taken in the Anatomy of the Eye) he tells Mr. *Crab-*

Crabtrie how he had applied his Teleſcopick Sights to a Sextant. Saith he, *Mr.* Horrox *his Theory of the Moon I ſhall be ſhortly furniſhed to try. For I am fitting my Sextant for all manner of Obſervations, by two Perſpicills with Threads. And alſo I am conſulting my Workman about the making of Wheels like* β, γ, δ, ε, *of* † *Diagr.* 3, *to uſe two Glaſſes like a Sector. If I once have my Tools in readineſs to my Deſire, I ſhall uſe them every Night. I have fitted my Sextant by the Help of the Cane, two Glaſſes in it, and a Thread, ſo as to be a pleaſant Inſtrument, could Wood and a Country-Joiner or Workman pleaſe me.*

In another Letter (the Date of which is worn out, but is, in Mr. *Crabtrie*'s Hand, called his 10th Letter to him) he ſaith, *I have given order for an Iron Quadrant of Five Foot, which will give me the* 1000*th Part of One Degree, which ſhall be furniſhed like my firſt Scale; only my Workman is ſo* **throng for my Father, that I fear it will not be finiſhed before the Eclipſe. I have cauſed a very ſtrong Ruler to be exactly made, and intend to fit it with Curſors of Iron, with Glaſſes in them and a Thread, for my Sextant.*

To theſe I could have added many other Paſſages of the like Nature: but theſe may be ſufficient, to ſhew that *Mr. Gaſcoigne*, as early as 1640, made uſe of Teleſcopes on Quadrants and Sextants, as well as in his Invention of the Micrometer.

What Commendations theſe Contrivances got him, and what Expectations they raiſed in ſome of the Aſtronomers of that Time, particularly in two of the moſt acute of that Age, Mr. *Horrox*, and Mr. *Crabtrie*, may be ſeen in the ſame Mr. *Crabtrie*'s Letters to Mr. *Gaſcoigne*, which are alſo in my Hands. Some Paſſages of which I ſhall recite, and at the ſame time give the Society a Taſte of what thoſe curious Letters do contain.

† This Diagram is wanting in the Letter. * A *Yorkſhire Phraſe* for *fully employed.*

In Mr. *Crabtrie's* ſecond Letter, which is of *October* 30. 1640; after a very clear Demonſtration that the Solar Spots are not Planets at a Diſtance from the Sun, but ſomething adhering to, or very near the Sun's Body: and alſo after a no leſs clear Demonſtration of the Errors of *Lansberges Hipparchian Diagram*, his *Lunar Parallax*, his Doctrine of *Eclipſes*, and indeed his whole *Lunar Aſtronomy*, together with divers other curious Matters, too many to be ſpecified: after this, I ſay, Mr. *Crabtrie* ſaith thus, *Something I am ſure you were telling me concerning a way of obſerving the Places of the Planets by your Glaſſes. But I have not a little lamented that my Time cut me ſo ſhort, when I was with you, that I could not more fully ruminate and digeſt thoſe ſtrange Inventions which you ſhewed me, and told me of. My Laſſitude after an unexpected and unacquainted Journey; my unpreparedneſs for thoſe Cogitations (not intending that Journey the Day before) and the Multiplicity and Variety of the Novelties you ſhewed me, ſo wholly diſtracted my Thoughts into Admiration, that I cannot now give my Meditations any reaſonable Account of what I ſaw: but muſt intreat you, in a few Lines, to rub up my Memory, and tell me again what you ſhewed me, and the Extent of thoſe your Inventions. Which I deſire, that I might conſider, and rejoice to conſider, how much and wherein* Urania's *Structure will grow to Perfection by your Aſſiſtance: and that (what in me lies) I may help you to remember when and wherein your Inventions and Obſervations will be of moſt uſe. I ſhould alſo deſire you to inform me what Bigneſs of a Quadrant you conceive to be large enough for Obſervation with your Devices. For I am e're long going to* Wigan, 12 *Miles from hence, where much Braſs is caſt; and then I could ſee whether I could procure ſuch an one caſt. You told me (as I remember) you doubted not in time to be able to make Obſervations to Seconds. I cannot but admire it and yet, by what I ſaw, believe it: but long*

to have ſome farther Hints of your Conceit for that Purpoſe. One Means, I think, you told me was, by a ſingle Glaſs in a Cane, upon the Index of your Sextant, by which (as I remember) you find the exact Point of the Sun's Rays. But the way how, I have quite forgotten, and much deſire: Your Device for the exact Diviſion of a Quadrant, by dividing 11 *Degrees into* 10 *Parts, I did then underſtand, but do not now fully remember. If it might not be too much Trouble to you, I ſhould intreat you to give me ſuch a Paper-Demonſtration thereof as you ſhewed me, and two or three Lines plainly of the Uſe thereof, how to find thoſe ſmall Parts. I loſt the little Paper, wherein I noted the Moon's Diameter, which we obſerved when I was with you: I pray you ſend it me, if,* &c.

I cannot conceal how much I am tranſported beyond my ſelf with the Remembrance (of that little I do remember) of thoſe admirable Inventions which you ſhewed me when I was with you. I ſhould not have believed the World could have afforded ſuch exquiſite Rarities, and I know not how to ſtint my longing Deſires, without ſome further Taſte of theſe ſelected Dainties. Happier had I been, had I never known there had been ſuch Secrets, than to know no more, but only that there are ſuch. Of all Deſires the Deſire of Knowledge is moſt vehement, moſt impatient: and of all kinds of Knowledge, this of the Mathematicks affects the Mind with moſt intenſe Agitations. I doubt not but you can experimentally witneſs the Truth hereof, and one time or other have been no Stranger to ſuch Thoughts as mine. And therefore although Modeſty would forbid me to requeſt any thing (until you give me leave) but what you pleaſe voluntarily to impart, yet the Vehemence of my Deſire forceth me to let you know how much I deſire, and how highly I ſhould prize any thing that you ſhould be pleaſed to communicate to me in thoſe Optick Practices. Could I purchaſe it with Travel, or procure it for Gold, I would not long be without a Teleſcope for ob-

ſerving ſmall Angles in the Heavens; nor want the Uſe of your other Device of a Glaſs in a Cane upon the moveable Ruler of your Sextant (as I remember) for helping to the exact Point of the Sun's Rays. But ſeeing Urania *is*, &c.

Thus was the moſt ingenious Mr. *Crabtrie* tranſported with Mr. *Gaſcoigne*'s Devices, although at that time far leſs perfect than they were in a ſhort Time after. And no leſs affected was the incomparable *Horrox*, as Mr. *Crabtrie* ſets forth, in his third long Letter of *Dec.* 28. 1640. which hath theſe Words, *My Friend Mr.* Horrox *profeſſeth, that little Touch which I gave him of your Inventions, hath raviſhed his Mind quite from it ſelf, and left him in an Extaſie between Admiration and Amazement. I beſeech you, Sir, ſlack not your Intentions for the perfecting of your begun Wonders. We travel with Deſire till we hear of your full Delivery. You have our Votes, our Hearts, and our Hands ſhould not be wanting, if we could further you.* And then after many curious Matters (which would take up too much of the Societies time to relate) he thus proceeds, *Your Diagrams for Perſpectives I have viewed again and again, and cannot ſufficiently admire your indefatigable Induſtry, and profound Ingenuity therein. I am much affected with the Symbolical Expreſſions of your Demonſtrations. I never uſed them before (but I will do) yet I underſtand them all at the firſt Sight, and ſee well the Truth of your Demonſtrations.*

To theſe I ſhall only add one Paſſage more, and this becauſe it ſhews ſome other of Mr. *Gaſcoigne*'s exquiſite Contrivances, or at leaſt the Accuracy of what are mentioned; and that is in Mr. *Crabtrie*'s Letter of *Dec.* 6. 1641. at the Beginning of which he ſaith, *That which you give me a full Projection of was above my Hope: and if the Screws keep an exact Equality of Motion forward in each Revolve, it is a moſt admirable Invention; and with the other Accommodations, I had almoſt ſaid without*

Com-

Compare. But that the Diviſions of a Circle ſhould be meaſured to Seconds, without the Limb of an Inſtrument, or that Diſtances, Altitudes, Inclinations, and Azimuths ſhould be taken all at one Moment, without the Limb of an Inſtrument likewiſe, and each to any required Number of Parts; or that the Diameter of Jupiter ſhould be projected in ſuch prodigious Meaſures as you ſpeak of, &c. were enough to amuſe and amaze all the Mathematicians in Europe, and may indeed be rather a Subject of Admiration than Belief, to any that hath not known your former Inventions to exceed Vulgar (I had almoſt ſaid Humane) Abilities. And for my Part, I muſt confeſs Modeſty ſo checks my ambitious Deſires, that I dare ſcarce hope ſuch Miracles ſhould ever be produced in real Practice to ſuch Exactneſs. Then (to give the Society a further Taſte of thoſe Letters) follows an Account of the Agreement of Mr. *Horrox*'s Theory of the Moon with Mr. *Gaſcoigne*'s Obſervations; and alſo very curious Ratiocinations, and a Diſquiſition about finding the Parallax of the Sun and Moon, and their Diſtance from the Earth. In which he cenſures *Morinus*'s Braggs, *&c.* and then ſaith, that *no Man that hath written of the Diagram* [of Hipparchus] *underſtood it fully, or deſcribed it rightly, but only* Kepler *and our* Horrox; *for whoſe immature Death* [which was ſuddenly, and about the Age of 25.] *there is yet ſcarce a Day which I paſs without ſome Pang of Sorrow.*

Thus, among many, I have related ſome of the Paſſages of Mr. *Gaſcoigne*'s and Mr. *Crabtrie*'s Letters relating to *Teleſcopick Sights.* From whence it is very manifeſt, that long before the *French* Gentleman's Claims, our Countryman Mr. *Gaſcoigne* had made uſe of thoſe Sights in his Aſtronomical Inſtruments; particularly in two or more Sorts of *Micrometers* (as I plainly find) and in his *Quadrant* and *Sextant.* And had it pleaſed God to have given him a longer Life, we might have expected

pected greater things from his pregnant and ſagacious Wit. For he was ſcarce 20 Years of Age when he held theſe Correſpondencies with Mr. *Crabtrie*. And at the Age of 23. he was killed at *Marſton-Moor-Battle*, on *July* 2. 1644. fighting for King *Charles* I. His Father was *Henry Gaſcoigne* Eſq; of *Middleton*, between *Leeds* and *Wakefield*.

IV. *An Attempt towards the Improvement of the Method of approximating, in the Extraction of the Roots of Equations in Numbers.* By Brook Taylor, *Secretary to the Royal Society.*

IN *Phil. Tran.* No. 210. Dr. *Halley*, now Secretary of the Royal Society, has publiſh'd a very compendious and uſeful Method of extracting the Roots of affected Equations of the common Form, in Numbers. This Method proceeds by aſſuming the Root deſired nearly true to one or two Places in Decimals (which is done by a Geometrical Conſtruction, or by ſome other convenient way) and correcting the Aſſumption by comparing the Difference between the true Root and the aſſumed, by means of a new Equation whoſe Root is that Difference, and which he ſhews how to form from the Equation propoſed, by Subſtitution of the Value of the Root ſought, partly in known and partly in unknown Terms.

In doing this he makes uſe of a Table of Products (which he calls *Speculum Analyticum*,) by which he computes the Coefficients in the new Equation for finding the Difference mentioned. This Table, I obſerved, was formed in the ſame Manner from the Equation pro-

propos'd, as the Fluxions are, taking the Root ſought for the only flowing Quantity, its Fluxion for Unity, and after every Operation dividing the Product ſucceſſively by the Numbers 1, 2, 3, 4, *&c.* Hence I ſoon found that this Method might eaſily and naturally be drawn from *Cor.* 2. *Prop.* 7. of my *Methodus Incrementorum*, and that it was capable of a further degree of Generality; it being Applicable, not only to Equations of the common Form, (*viz.* ſuch as conſiſt of Terms wherein the Powers of the Root ſought are poſitive and integral, without any Radical Sign) but alſo to all Expreſſions in general, wherein any thing is propoſed as given which by any known Method might be computed; if *vice versâ*, the Root were conſider'd as given: ſuch as are all Radical Expreſſions of Binomials, Trinomials, or of any other Nomial, which may be computed by the Root given, at leaſt by Logarithms, whatever be the Index of the Power of that Nomial; as likewiſe Expreſſions of Logarithms, of Arches by the Sines or Tangents, of Areas of Curves by the *Abſciſſa's* or any other Fluents, or Roots of Fluxional Equations, *&c.*

For the ſake of this great Generality, it may not be improper to ſhew how this Method is derived from the foreſaid *Corollary*. Therefore z and x being two flowing Quantities (whoſe Relation to one another may be expreſt by any Equation whatſoever) by this *Corollary*, while z by flowing uniformly becomes $z + v$, x will become $x + \frac{\dot{x}}{1\cdot\dot{z}}v + \frac{\ddot{x}}{1.2\dot{z}^2}v \mp \frac{\dddot{x}}{1.2.3\dot{z}^3}v^3 + \&c.$

or $x + \frac{\dot{x}v}{1} + \frac{\ddot{x}v^2}{1\times 2} + \frac{\dddot{x}v^3}{1.2.3} \mp \&c.$ for $\dot{z}$ putting 1.

Hence if y be the Root of any Expreſſion formed of y and known Quantities, and ſuppoſed equal to nothing,

and

and z be a part of y, and x be formed of z and the known Quantities, in the ſame manner as the Expreſſion made equal to nothing is formed of y; and let y be equal to $z + v$: the difference v will be found by Extracting the Root of this expreſſion $x + \frac{\dot{x}v}{1} + \frac{\ddot{x}v^2}{1.2} + \frac{\dddot{x}v^3}{1.2.3} + \&c. = 0$. For in this Caſe z being become $z + v = y$, x, which is now become $x + \dot{x}v + \frac{\ddot{x}v^2}{2} + \&c.$ muſt become equal to nothing.

The Root v in the Equation $x + \frac{\dot{x}v}{1} + \frac{\ddot{x}v^2}{1.2} + \frac{\dddot{x}v^3}{1.2.3} + \&c. = 0$, is to be found upon the Suppoſition of its being very ſmall with reſpect to z, (as it muſt be, if z be taken tolerably exact) by which means the Terms $\frac{\dddot{x}v^3}{1.2.3} + \frac{\ddddot{x}v^4}{1.2.3.4} + \&c.$ may be neglected, upon account of their ſmallneſs with reſpect to the other Terms, ſo as to leave the Equation $x + \frac{\dot{x}v}{1} + \frac{\ddot{x}v^2}{1.2} = 0$, for finding the firſt approximation of v.

By extracting the Root of this Equation, we have $v = \sqrt{\frac{\dot{x}^2}{\ddot{x}^2} - \frac{2x}{\ddot{x}}} - \frac{\dot{x}}{\ddot{x}}$. That is,

Firſt, $\sqrt{\frac{\dot{x}^2}{\ddot{x}^2} - \frac{2x}{\ddot{x}}} - \frac{\dot{x}}{\ddot{x}}$, if $x + \dot{x}v + \frac{\ddot{x}v^2}{2} = 0$.

Sec. $\sqrt{\frac{\dot{x}^2}{\ddot{x}^2} + \frac{2x}{\ddot{x}}} - \frac{\dot{x}}{\ddot{x}}$, if $-x + \dot{x}v + \frac{\ddot{x}v^2}{2} = 0$.

Thirdly

3. $\frac{\dot{x}}{\ddot{x}} - \sqrt{\frac{\dot{x}^2}{\ddot{x}^2} - \frac{2x}{\ddot{x}}}$, if $x - \dot{x}v + \frac{\ddot{x}v^2}{2}$, &c. $= 0$.

4. $\frac{\dot{x}}{\ddot{x}} - \sqrt{\frac{\dot{x}^2}{\ddot{x}^2} + \frac{2x}{\ddot{x}}}$, if $-x - \dot{x}v + \frac{\ddot{x}v^2}{2}$, &c. $= 0$.

This approximation gives v exact to twice as many places as there are true Figures in z, and therefore trebles the number of true Figures in the Expreſſion of y by $z + v$, which may be taken for a new Value of z, for computing a ſecond v, ſeeking other Values of x, $\dot{x}$, $\ddot{x}$, &c. Tho' when z is tolerably exact (which it may be eſteem'd when it contains two or three or more true Figures in the Value of y, according to the Number of Figures the Root is propoſed to be computed to,) the Calculation may be reſtor'd without ſo much trouble, only by taking $\sqrt{\frac{\dot{x}^2}{\ddot{x}^2} \pm \frac{2x}{\ddot{x}} - \frac{2\dot{x}}{2 \cdot 3\ddot{x}}v^3 - \frac{2\dddot{x}}{1 \cdot 2 \cdot 3 \cdot 4\ddot{x}}v^4}$ &c. inſtead of $\sqrt{\frac{\dot{x}^2}{\ddot{x}^2} \pm \frac{2x}{\ddot{x}}}$ taking every time for v its Value laſt computed.

4. From the ſame Equation $x + \dot{x}v + \frac{\ddot{x}v^2}{2} + \frac{\dddot{x}v^3}{1 \cdot 2 \cdot 3} + \&c. = 0$, may be gather'd alſo a rational Form, *viz.* $v = \frac{-x}{\dot{x} - \frac{x\ddot{x}}{2\dot{x}}}$. For neglecting the Terms $\frac{\dddot{x}v^3}{1 \cdot 2 \cdot 3}$, &c. we have $v = \frac{-x}{\dot{x} + \frac{\ddot{x}}{2}v}$ which is nearly $= \frac{-x}{\dot{x}}$. Therefore in the Diviſor inſtead of v writing $\frac{-x}{\dot{x}}$ we have

more exactly $v = \frac{-x}{\dot{x} - \frac{\ddot{x}\,x}{2\,\dot{x}}}$, that is

1. $\frac{-x}{\dot{x} - \frac{\ddot{x}\,x}{2\,\dot{x}}}$, when $x + \dot{x}v + \frac{\ddot{x}\,v^2}{2}$ &c. $= 0$.

2. $\frac{x}{\dot{x} + \frac{\ddot{x}\,x}{2\,\dot{x}}}$, when $-x + \dot{x}v + \frac{\ddot{x}\,v^2}{2}$ &c. $= 0$.

3. $\frac{x}{\ddot{x} - \frac{\dddot{x}\,x}{2\,\ddot{x}}}$, when $x - \dot{x}v + \frac{\ddot{x}\,v^2}{2}$ &c. $= 0$.

4. $\frac{-x}{\ddot{x} + \frac{\dddot{x}\,x}{2\,\ddot{x}}}$, when $-x - \dot{x}v + \frac{\ddot{x}\,v^2}{2}$ &c. $= 0$.

This *Formula* will alſo triplicate the number of true Figures in z. And the Calculation may be repeated, after every Operation, taking for a Diviſor $\ddot{x} \pm \frac{\dddot{x}}{2}v + \frac{\ddot{x}\,v^2}{1.2.3} + \frac{\ddddot{x}\,v^3}{1.2.3.4}$ &c. inſtead of $\dot{x} + \frac{\ddot{x}\,x}{2\,\dot{x}}$.

Dr. *Halley* has fully explain'd the manner of uſing both theſe *Formula*'s in Æquations of the common Form; wherefore I ſhall be the ſhorter in explaining two or three Examples of another ſort.

Ex. I. Let it be propoſed to find the Root of this Equation $\overline{y^2 + 1}|^{\sqrt{2}} + y - 16 = 0$. In this Caſe, for y writing z, and for 0 writing x, we have $\overline{z^2 + 1}|^{\sqrt{2}} + z$

$+z-16=x$. Whence by taking the Fluxions, we have $\dot{x}=2\sqrt{2}\times z\times \overline{z^2+1}|^{\sqrt{2}-1}+1$, and $\ddot{x}=2\sqrt{2}\times 8-4\sqrt{2}\,z^2\times \overline{z^2+1}|^{\sqrt{2}-2}$. For finding the first Figures of the Root y, for $\sqrt{2}$ take $\frac{3}{2}$, and we have the Equation $\overline{y^2+1}|^{\frac{3}{2}}+y-16=0$, which being expanded gives $y^6+3y^4+2y^2+32y-255=0$.

By this Equation I find that for the first ſuppoſition we may take $z=2$. Therefore in order to find v, let us now make $\sqrt{2}=\frac{7}{5}$, (which is nearer than before) and we have $x=\overline{z^2+1}|^{\frac{7}{5}}+z-16=\overline{2^2+1}|^{\frac{7}{5}}-14$ $=5^{\frac{7}{5}}-14=-4,48$; $\dot{x}=10,66$; $\ddot{x}=4,72$. Whence by the ſecond rational Form $v=\dfrac{4,48}{10,66+\frac{4,72\times 4,48}{2\times 10,66}}$ $=0,38$; which muſt be too big, becauſe $\frac{7}{5}<\sqrt{2}$, and therefore will require a larger Value of y to exhauſt the Equation, than where $\sqrt{2}$ is exact. For the ſecond ſuppoſition therefore, let us take $z=2,3$, and make $\sqrt{2}=1,4142136$, and by help of the Logarithms we ſhall have $\overline{z^2+1}|^{\sqrt{2}}=13,47294$, whence $x=-0,22706$; $\dot{x}=14,93429$, and $\ddot{x}=5,18419$. Hence by the 2d. irrational *Formula* $v=\sqrt{\frac{14,93429^2}{5,18419^2}+\frac{0,45412}{5,18419}}-\frac{14,93429}{5,18419}=0,01516$, which gives $y=z+v=2,31516$, which is true to ſix Places. If you deſire it more exact than to the extent of the Tables of Logarithms, taking $z=2,31516$ for the next ſuppoſition, the Calculation muſt be repeated by computing of $\overline{zz+1}|^{\sqrt{2}}$ to a ſufficient number of Places; which muſt be done by the Binomial Series, or by making a Loga-

rithm on purpoſe, true to as many places as are neceſſary.

Ex. II. For another Example, let it be required to find the Number whoſe Logarithm is 0, 29, ſuppoſing we had no other Table of Logarithms but Mr. *Sharps* of 200 Logarithms to a great many places. This amounts to the reſolving this Equation $ly = 0,29$, or $ly - 0,29 = 0$.

Hence therefore we have $x = lx - 0,29$, $x = \frac{1}{z}$ (1

being the *Modulus* belonging to the Table we uſe, *viz.* 0, 4342944819, *&c.*) $\ddot{x} = \frac{-a}{z^2}$, $\dddot{x} = \frac{2a}{z^3}$, $\ddddot{x} = \frac{-6a}{z^4}$

&c. In this Caſe becauſe $\dot{x}$ has a negative Sign, changing the Signs of all the Coefficients, the Canon for v will be found in the fourth Caſe, which in the irrational Form gives $v = \frac{\dot{x}}{\ddot{x}} - \sqrt{\frac{\dot{x}^2}{\ddot{x}^2} + \frac{2x}{\ddot{x}} - \frac{2\dddot{x}}{2.3\ddot{x}}v^3 - \frac{2\ddddot{x}}{2.3.4\ddot{x}}v^4}$

$\&c. = z - \sqrt{z^2 + \frac{2lz - 0,58}{a} \times z^2 + \frac{2v^3}{3z} - \frac{2v^4}{4z^2}}$

$+ \frac{2v^5}{5z^3}$ *&c.* In this Caſe to avoid often dividing by z, it will be moſt convenient to compute $\frac{v}{z}$, which is got from this Equation $\frac{v}{z} = 1 - \sqrt{1 + \frac{2lz - 0,58}{a} + \frac{2v^3}{3z^3} - \frac{2v^4}{4z^4} + \frac{2v^5}{5z^5}}$, *&c.* The neareſt Logarithm, in the Tables propoſed, to the propoſed Logarithm 0, 29 is 0, 2900346114, its Number being 1, 95. Therefore for the firſt ſuppoſition taking $z = 1,95$, we have x $(= lz - 0,29 = 0,2900346114 - 0,29) =$ 0, 0000

0,00003461 14, and $\frac{2lz - 0,58}{d} = \frac{0,0000692228}{0,4342944819} =$ 0,0001593913 9, and $1 + \frac{2lz - 0,58}{d} = 1,0001593913 9$. Whence for the first approximation we have $\frac{v}{z} = 1 - \sqrt{1,0001593913 9} = -0,0000796924 7$, and $v = -0,0001554003^2$, and $y = z + v = 1,9498445996 8$. Which is true to eleven places, and may easily be corrected by the Terms $\frac{12v^3}{3}$ &c. which I leave to the Readers curiosity.

Being upon the Subject of Approximations, it may not be amiss to set down here two Approximations I have formerly hit upon. The one is a Series of Terms for expressing the Root of any Quadratick Equation: and the other is a particular Method of Approximating in the invention of Logarithms, which has no occasion for any of the Transcendental Methods, and is expeditious enough for making the Tables without much trouble.

A general Series for expressing the Root of any Quadratick Equation.

Any Quadratick Equation being reduc'd to this Form $xx - mqx + my = 0$, the Root x will be expresst by this Series of Terms.

$$x = \frac{y}{q} + A \times \frac{1}{\frac{mq^2}{y} - 2} + B \times \frac{1}{a^2 - 2} + C \times \frac{1}{b^2 - 2}$$

$+ D \times \frac{1}{c^2 - 2}$ &c. Which must be thus interpreted.

1. The Capital Letters A, B, C, &c. stand for the whole Terms with their Signs, preceding those wherein

in they are found, as $B = A \times \frac{1}{\frac{mq^2}{y} - 2}$.

2. The little Letters a, b, c, &c. in the Diviſors, are equal to the whole Diviſors of the Fraction in the Terms immediately preceding; thus $b = a^2 - 2$.

For an Example of this, let it be required to find $\sqrt{2}$. Putting $\sqrt{2} = x + 1$, we have $x^2 + 2x - 1 = 0$, which being compared with the general *Formula*, gives $mq = -2$, and $my = -1$: therefore for m taking -1, we have $q = 2$, and $y = 1$, which Values ſubſtituted in the Series give $x = \frac{1}{2} - \frac{1}{2 \times 6} - \frac{1}{2 \times 6 \times 34} - \frac{1}{2 \times 6 \times 34 \times 1154} - \frac{1}{2 \times 6 \times 34 \times 1154 \times 1331714}$, &c. The Fractions here wrote down giving the Root true to twenty three Places.

A new Method of computing Logarithms.

This Method is founded upon theſe Conſiderations.

1. That the Sum of the Logarithms of any two Numbers is the Logarithm of the Product of thoſe two Numbers Multiplied together.

2. That the Logarithm of Unite is nothing; and conſequently that the nearer any Number is to Unite, the nearer will its Logarithm be to 0. 3*dly*. That the Product by Multiplication of two Numbers, whereof one is bigger, and the other leſs than Unite, is nearer to Unite than that of the two Numbers which is on the ſame ſide of Unite with its ſelf; for Example the two Numbers being $\frac{2}{3}$ and $\frac{4}{3}$, the Product $\frac{8}{9}$ is leſs than Unite, but nearer to it than $\frac{2}{3}$, which is alſo leſs than Unite. Upon theſe Conſiderations, I found the preſent Approximation;

proximation; which will be beſt explain'd by an Example. Let it therefore be propoſed to find the Relation of the Logarithms of 2 and of 10. In order to this, I take two Fractions $\frac{128}{100}$ and $\frac{8}{10}$, *viz.* $\frac{\overline{2}|^{7}}{\overline{10}|^{2}}$ and $\frac{\overline{2}|^{3}}{\overline{10}|^{1}}$, whoſe Numerators are Powers of 2, and their Denominators Powers of 10; one of them being bigger, and the other leſs than 1. Having ſet theſe down in Decimal Fractions in the firſt Column of the Table annext, againſt them in the ſecond Column I ſet A and B for their Logarithms, expreſſing by an Equation the manner how they are Compounded of the Logarithms of 2 and 10, for which I write *l* 2 and *l* 10. Then Multiplying the two Numbers in the firſt Column together, I have a third Number 1,024, againſt which I write C for its Logarithm, expreſſing likewiſe by an Equation in what manner C is formed of the foregoing Logarithms A and B. And in the ſame manner the Calculation is continued; only obſerving this *Compendium*, that before I Multiply the two laſt Numbers already got in the Table, I conſider what Power of one of them muſt be uſed to bring the Product the neareſt to Unite that can be. This is found, after we have gone a little way in the Table, only by Dividing the Differences of the Numbers from Unite one by the other, and taking the Quotient with the neareſt, for the Index of the Power wanted. Thus the two laſt Numbers in the Table being 0,8 and 1,024, their Differences from Unit are 0,200 and 0,024; therefore $\frac{0,200}{0,024}$ gives 9 for the Index; wherefore Multiplying the ninth Power of 1,024 by 0,8, I have the next Number 0,990352031429, whoſe Logarithm is $D = 9C + B$. In ſeeking the Index in this manner by Diviſion of the Differences, the Quotient ought generally to be taken with

with the leaſt: but in the preſent caſe it happens to be the moſt, becauſe inſtead of the Difference between 0,8 and 1, we ought ſtrictly to have taken the difference between the reciprocal 1,25 and 1, which would have given the Index 103, and that would be too big, becauſe the Product by that means would have been bigger than 1, as 1,024 is. Whereas this Approximation requires that the Numbers in the firſt Column be alternately greater and leſs than 1, as may be ſeen in the Table.

When I have in this manner continued the Calculation till I have got the Numbers ſmall enough, I ſuppoſe the laſt Logarithm to be equal to nothing. Which gives me an Equation, from which having got away the Letters by means of the foregoing Equations, I have the relation of the Logarithms propoſed. In this manner if I ſuppoſe $G = 0$, I have $2136l3 - 642l10 = 0$. Which gives the Logarithm of 2 true in ſeven Figures, and too big in the Eighth; which happens becauſe the Number correſponding with G is bigger than Unite.

There is another Expedient which renders this Calculation ſtill ſhorter. It is founded upon this Conſideration, that when x is very ſmall $\overline{1+x}|^n$ is very nearly $1+nx$. Hence if $1+x$, and $1-z$ are the two laſt Numbers already got in the firſt Column of the Table, and their Powers $\overline{1+x}|^m$ and $\overline{1-z}|^n$ are ſuch as will make the Product $\overline{1+x}|^m \times \overline{1+z}|^n$ very near to Unite, m and n may be found thus: $\overline{1+x}|^m = 1+mx$, and $\overline{1-z}|^n = 1-nz$, and conſequently $\overline{1+x}|^m \times \overline{1-z}|^n = 1+mx-nz-mnzx$, or (neglecting $mnzx$) $1+mx-nz$. Make this equal to 1, and we have $m:n::z:x::l1-z:l1+x$. Whence $xl1-z+zl1+x=0$. To give an Example of the Application of this, let 1,024 and 0,990352 be the laſt Numbers in the Table, their Logarithms being C and D. Then we have

1,024

1, 024 $=$ 1 $+ x$, and 0, 990352 $=$ 1 $- z$, and consequently $x =$ 0, 024, and $z =$ 0, 009648. Whence the Ratio $\frac{z}{x}$ in the leaſt Numbers is $\frac{201}{500}$. So that for finding the Logarithms propoſed we may have 500 D $+$ 201 C $=$ 48510 l 2 $-$ 14603 l 10 $=$ 0, which gives l 2 $=$ 0, 3010307, which is too big in the laſt Figure; but it is nearer the truth, than what is got from the Logarithm F ſuppoſed equal to nothing. So that by this means we have ſaved four Multiplications, which were neceſſary to find the Number 9989595 *&c.* correſpondent to F, and which muſt have been had if we would make the Logarithm true to the ſame Number of places without this *Compendium*.

1, 280000000000	$A = 7\,l\,2 - 2\,l\,10$	$l\,2 >$ 0, 28
0, 800000000000	$B = 3\,l\,2 - l\,10$	$<$ 0, 33
1, 024000000000	$C = B + A = 10\,l\,2 - 3\,l\,10$	$>$ 0, 300
0, 990352031429	$D = 9\,C + B = 93\,l\,2 - 28\,l\,10$	$<$ 0, 30107
1, 004336277664	$E = 2\,D + C = 196\,l\,2 - 59\,l\,10$	$>$ 0, 301020
0, 998959536107	$F = 2\,E + D = 485\,l\,2 - 146\,l\,10$	$<$ 0, 3010303
1, 000162894165	$G = 4\,F + E = 2136\,l\,2 - 643\,l\,10$	$>$ 0, 30102996
0, 999936281874	$H = 6\,G + F = 13301\,l\,2 - 4004\,l\,10$	$<$ 0, 301029997
1, 000035441215	$I = 2\,H + G = 28738\,l\,2 - 8651\,l\,10$	$>$ 0, 3010299951
0, 999971720830	$K = I + H = 42039\,l\,2 - 12655\,l\,10$	$<$ 0, 3010299959
1, 000007161046	$L = K + I = 70777\,l\,2 - 21306\,l\,10$	$>$ 0, 3010299962
0, 999993203514	$M = 3\,L + K = 254370\,l\,2 - 76573\,l\,10$	$<$ 0, 3010299967
1, 000000364511	$N = M + L = 325147\,l\,2 - 97879\,l\,10$	$>$ 0, 30102999566 35
0, 999999764687	$O = 18\,N + M = 6107016\,l\,2 - 1838395\,l\,10$	$<$ 0, 30102999566 40
Com. Ar. 235313		
$0 = 364511\,O + 235313\,N = 2302585825187\,l\,2 - 693147400972\,l\,10$		$>$ 0, 30102999566 3987

I have computed this Table ſo far, that the Reader may ſee in what manner this Method Approximates; this whole Work, as it appears, coſting a little more than three Hours time.

V. *Proprietates quædam ſimplices* Sectionum Conicarum *ex natura* Focorum *deductæ; cum Theoremate generali de Viribus Centripetis; quorum ope Lex Virium Centripetarum ad Focos Sectionum tendentium, Velocitates Corporum in illis revolventium, & Deſcriptio Orbium facillime determinantur.* Per Abr. *de* Moivre. R. S. Soc.

Sit *DE* Axis Tranſverſus Ellipſeos, *AO* Axis alter, & *C* centrum Sectionis. Sit *P* punctum quodvis in circumferentia ejus; *PQ* Tangens curvæ ad *P*, occurrens Axi Tranſverſo ad *Q*; puncta *S*, *F* Foci; *CP*, *CK* ſemidiametri Conjugatæ; *PH* Semilatus rectum ad diametrum *PC*; *PG* normalis ad Tangentem, cui occurrat *HG*, perpendicularis ipſi *PCH*, in puncto *G*, ut fiat *PG* radius Curvaturæ Ellipſeos in puncto *P*: ſint etiam *ST*, *CR*, *FV* perpendiculares in Tangentem *PQ* demiſſæ: Jungatur *SO*, & demittatur in Axem normalis *PL*. His poſitis, Dico quod,

1. *Rectangulum ſub diſtantiis ab utroque Ellipſeos Foco, ſive SP × FP æquale eſt quadrato Semidiametri CK.*

Demonſtratio.

$PSq = PCq + CSq - 2CS \times CL$ *per* 13. II. Elem.
$PFq = PCq + CSq + 2CS \times CL$ *per* 12. II. Elem.
Unde $PSq + PFq = 2PCq + 2CSq$.
Jam $PS + PF = DE = 2CD$; ac propterea
$PSq + PFq + 2PS \times PF = 4CDq$. Quare

Quare transponendo, $2PS \times PF = 4CDq - 2PCq - 2CSq$.

Ac Dimidiando $PS \times PF = 2CDq - PCq - CSq$.

Est autem CS *quad.* $= CD$ *quad.* $- CO$ *quad*, atque adeo $PS \times PF = CDq + COq - PCq$.

Sed $CDq + COq = PCq + CKq$ *per* 12. VII. *Conic. Apollonii.*

Quare $PS \times PF = CKq$. *Q. E. D.*

II. *Distantia à Foco SP est ad perpendicularem in Tangentem demissam, ut Semidiameter Conjugata CK ad Semiaxem minorem CO.*

Demonstratio.

Ob similia Triangula SPT, FPV, erit $PS : PF :: ST : FV$; ac componendo $PS + PF$ erit ad $ST + FV$, & earundem dimidia CD ad CR, ut PS ad ST. Unde $CD \times CK$ erit ad $CR \times CK$ ut PS ad ST. Sed $CR \times CK$ æquale est rectangulo sub Semiaxibus CD in CO, per 31. VII. *Conic.* Proinde PS est ad ST ut CD in CK ad $CD \times CO$, sive ut CK ad CO. Ac pari argumento demonstrabitur PF esse ad FV in eadem ratione. *Q. E. D.*

III. *In eadem etiam est ratione Semiaxis Transversus CD ad normalem è centro C ad Tangentem demissam, sive ad CR.*

Etenim cum rectangulum $CR \times CK$ æquale sit rectangulo $CD \times CO$, uti jam dictum est, erit ἀνάλογον CD ad CR ut CK ad CO. *Q. E. D.*

IV. *Semidiameter quævis PG est ad distantiam puncti P à foco S, sive ad SP, ut distantia ab altero Foco FP ad dimidium lateris recti ad Verticem P pertinentis, sive ad PH.*

Hoc autem manifestum est ob *Propr.* I. cum nempe quadratum ex CK æquale sit rectangulo sub $SP \times PF$.

V. *Rectangulum Semiaxium $CD \times CO$ est ad quadratum semidiametri conjugatæ CK, ut CK ad Radium Curvaturæ in puncto P, sive ad PG.*

Sunt

Sunt enim Triangula PCR, PGH inter se similia, unde CR est ad PC, ut semilatus rectum PH ad PG: hoc est, per præmissam Proprietatem III, $\frac{CD \times CO}{CK} =$ CR est ad PC ut $\frac{CK^2}{PC} = PH$ ad $\frac{CK^3}{CD \times CO} = PG$. proinde ἀναλογον $CD \times CO : CK^2 :: CK : PG$. Q.E.D.

THEOREMA GENERALE I.

Vis centripeta ad idem punctum S tendens, in Curvis omnibus, est semper proportionalis Quantitati $\frac{SP}{PG \times ST^3}$

Hoc Theorema ante plures annos à me investigatum & cum amicis communicatum, propriis demonstrationibus firmavere Geometræ Clarissimi D. *J. Bernoullius* in *Act. Lipsiæ*; D. *J. Keillius* in harum *Transact.* N. 317. & D. *Jac. Hermannus* in *Phoronomiâ* suâ pag. 70. quos vide.

Scribendo autem CK^3 pro PG, per *Propr.* V; & $\frac{SP}{CK}$ juxta *Propr.* II, pro ST; (ob datas scilicet CD, CO) erit Vis centripeta tendens ad focum Ellipseos S, semper ut $\frac{SP \times CK^3}{CK^3 \times SP^3}$, hoc est ut $\frac{SP}{SP^3}$ vel $\frac{1}{SP^2}$, nempe reciprocè ut quadratum ex SP. Unde patet quod si Sectio fuerit Ellipsis motu corporis descripta, erit Vis Centripeta ut quadratum distantiæ à centro Virium reciprocè. Ex his Proprietatibus consequuntur Corollaria nonnulla notatu non indigna.

Coroll 1. *Velocitas Corporis in Ellipsi revolventis, ad punctum quodlibet P, est ad Velocitatem revolventis in circulo ad eandem distantiam SP à centro Virium, in subdupla ratione distantiæ ab altero foco PF, ad Semiaxem transversum Sectionis, sive ut media proportionalis inter PF & CD ad CD.*

Est enim velocitas revolventis in Ellipsi ad distantiam SP, ad Velocitatem revolventis in Circulo vel Ellipsi ad dist-

distantiam Semiaxis CD vel SO, ut CO ad ST; hoc est per *Propr.* II. ut $\surd PF$ ad $\surd SP$. Velocitas autem revolventis in Circulo ad distantiam CD est ad velocitatem revolventis in Circulo ad distantiam SP, ut $\surd SP$ ad $\surd CD$. Ex æquo igitur, Velocitas revolventis in Ellipsi ad distantiam SP, est ad Velocitatem revolventis in Circulo ad eandem distantiam ut $\surd PF$ ad $\surd CD$.

Coroll. 2. *Ex datis Velocitate in Ellipsi, positione Tangentis, & centro Virium seu Foco, facile est determinare Focum alterum.*

Sit enim Velocitas Data R; ea autem Velocitas quâ describeretur Circulus ad datam à centro distantiam SP sit Q; ac per *Coroll.* præcedens, R est ad Q ut $\surd PF$ ad $\surd CD$, adeoque $2QQ$ est ad RR ut CD ad PF, & $2QQ - RR$ erit ad RR ut SP ad PF: Datur autem SP; data est igitur PF magnitudine. Datur etiam positione, ob angulum VPF angulo SPT æqualem. Datur igitur punctum F alter Focorum: Quo invento pronum est Sectionem describere.

Si vero $\frac{1}{2}RR$ majus fuerit quadrato ex Q, $2QQ - RR$ fit quantitas Negativa, & loco Ellipseos Trajectoria describenda in Hyperbolam transit. Eritque $RR - 2QQ$ ad RR ut SP ad PF distantiam alterius Foci, ad alterum Tangentis latus ponendam, ut habeatur Focus F. Proprietates autem omnes quas in Ellipsi demonstravimus; mutatis mutandis etiam Hyperbolæ competunt. *Fig.* II.

Quod si acciderit $2Q$ æquale esse dimidio quadrati ex R; evanescente quantitate $2QQ - RR = 0$, quarta proportionalis PF fit infinita: proinde Trajectoria describenda Parabolica est, Foco scilicet altero in infinitum abeunte. Axis autem Trajectoriæ positione datur; est enim ipsi PF parallelus, existente scilicet angulo FPV angulo dato SPT æquali.

Coroll. 3. *Velocitas revolventis in data Sectione Conica ad distantiam SP est ad Velocitatem ejusdem ad distantiam aliam SX, ut media proportionalis inter FP & SX ad mediam proportionalem inter SP & FX.* Velo-

Velocitas enim in P eſt ut $\sqrt{\frac{F\,P}{S\,P}}$ (per *propr.* II.) & per eandem, Velocitas in X eſt ut $\sqrt{\frac{F\,X}{S\,X}}$. Unde manifeſta eſt propoſitio.

Coroll. 4. *Ratio etiam Velocitatum duorum Corporum in eadem Syſtemate, ſed in datis Coniſectionibus diverſis, revolventium, datis utriuſque à communi Orbium Foco diſtantiis, ope Corollarii* 1^mi. *ſtatim obtinebitur.*

Cum enim Velocitas corporis in P ſit ad Velocitatem in Circulo ad eandem diſtantiam $S\,P$, ut $\sqrt{P\,F}$ ad $\sqrt{C\,D}$; & in alia ſuppoſita Coniſectione, cujus Semiaxis $c\,d$ & Foci S, f, ad diſtantiam $S\,p$ Velocitates illæ ſint ut $\sqrt{p\,f}$ ad $\sqrt{c\,d}$: Velocitas autem revolventis in circulo ad diſtantiam $S\,P$ ſit ad Velocitatem in Circulo ad diſtantiam $S\,p$ ut $\sqrt{S\,p}$ ad $\sqrt{S\,P}$; Compoſitis rationibus, erit Velocitas in P ad Velocitatem in p, ut $\sqrt{P\,F \times c\,d \times S\,p}$ ad $\sqrt{p\,f \times C\,D \times S\,P}$. Quod ſi Sectio illa altera fuerit Parabola, erunt $c\,d$, $p\,f$ infinitæ, ſed in ratione 1 ad 2; proinde ratio Velocitatum erit ut $\sqrt{P\,F \times S\,p}$ ad $\sqrt{2\,C\,D \times S\,P}$.

Coroll. 5. *Quod ſi in Hyperbola punctum P abeat in infinitum, ex præcedentibus manifeſtum eſt, Velocitatem ultimam ac minimam, qua cum corpus in æternum aſcenderet, æqualem eſſe ei qua, ad diſtantiam C D Semiaxi tranſverſo æqualem, Circulum deſcriberet.*

Coroll. 6. *Ex data diſtantia à Foco, datur quoque Poſitio Tangentis, ſive angulus S P T, ſub diſtantia S P & Tangente P T contentus.*

Eſt enim (per *propr.* II.) $P\,S$ ad $S\,T$ ut $C\,K$ ad $C\,O$ ſive ut $\sqrt{S\,P \times P\,F}$ ad $C\,O$, atque ita Radius ad Sinum anguli $S\,P\,T$. At in Ellipſibus Circulis affinibus præſtaret angulum $P\,S\,T$, ejuſdem complementum ad quadrantem, inquirere: Hujus autem Sinus eſt ad Radium ut $\sqrt{S\,P \times P\,F - C\,O^q}$ ad $\sqrt{S\,P \times P\,F}$.

Coroll.

Coroll. 7. Atque hinc consequuntur Velocitates quibuscum distantiæ S P crescunt vel decrescunt.

Nam cum, ex Corollario præcedente, $\checkmark \overline{SP \times PF}$ sit ad $\checkmark \overline{SP \times PF - CO^q}$ ut Radius ad sinum anguli PST, ac in eadem sit ratione Velocitas Corporis in P ad Velocitatem momenti ipsius SP; Velocitas autem illa in P sit (per *propr.* II.) ut $\checkmark \frac{PF}{SP}$; elisis superfluis, erit $\checkmark \frac{\overline{SP \times PF - CO^q}}{SP}$ Velocitati, qua crescit vel decrescit distantia SP, semper proportionalis.

THEOREMA GENERALE II.

In omni Trajectoria Curvilinea Velocitates angulares circa centrum Virium sunt reciprocè proportionales quadratis distantiarum à centro.

Nam ob Sectorum minimorum Areas æquales, arcus angulis minimis subtensi sive Bases, sunt reciprocè ut Radii: Anguli autem minimi quibus Bases æquales subtenduntur sunt etiam reciproce ut Radii. Proinde anguli Sectorum minimorum Areâ æqualium, sunt inter se reciprocè in dupla ratione Radiorum, sive ut quadrata distantiarum.

Coroll. 8. Hinc Velocitates angulares revolventium in diversis Ellipsibus datis comparantur inter se.

Velocitates enim angulares quibuscum ad distantias Semiaxibus Transversis æquales circuli describerentur, sunt reciproce in ratione sesquialtera Axium, sive ut $\frac{I}{CD\checkmark CD}$. Velocitates autem angulares has medias habent Corpora revolventia, cum quadrata distantiarum æquantur rectangulis sub semiaxibus Ellipseôn. Ideo (per *Theor.* II.) erit SP^q ad $CD \times CO$ ut $\frac{I}{CD\checkmark CD}$ ad $\frac{CO}{SP^q \times \checkmark CD}$: quæ quidem Quantitas est ut Velocitas anguli ad centrum S, motu rectæ SP, tempore quam minimo dato, descripti.

Coroll. 9. Velocitas angularis qua circumgyratur Tangens PT, sive recta in Tangentem perpendicularis ST, est ad Velocitatem

locitatem angularem rectæ SP, ut Semiaxis transversus CD ad distantiam ab altero Foco PF.

Demonstratio.

In *Fig.* III. Sint puncta *P*, *p*, quamproxima inter se; ductisque *SP*. *Sp*, sint *PT*, *pt* duæ Tangentes, ad quas demittantur normales *ST*, *St*; iisque parallelæ ducantur radii Curvaturæ *PG*, *pG* coeuntes in *G*: ac describatur, centro *S* & radio *SP*, arcus minimus *PE* occurrens ipsi *Sp* in *E*. Manifestum est angulum *PGp* æqualem esse angulo *TSt*, sive angulari Velocitati normalis *ST*. Est autem angulus *PSp* angularis velocitas rectæ *SP*; quare angulus *PGp* est ad angulum *PSp* ut angularis Velocitas ipsius *ST* ad angularem velocitatem rectæ *SP*; hoc est, ut $\frac{Pp}{PG}$ ad $\frac{PE}{PS}$. Sed $Pp . PE :: SP . ST :: CK : CO$ (per *propr.* II). Hæ igitur Velocitates sunt ut $\frac{CK}{PG}$ ad $\frac{CO}{PS}$. Pro PG scribe $\frac{CK^2}{CD \times CO}$ (per *propr.* V.) ac $\frac{CK}{PG}$ fiet $\frac{CD \times CO}{CK^q} = \frac{CD \times CO}{PS \times PF}$. Hinc $\frac{CD \times CO}{PS \times PF}$ erit ad $\frac{CO}{PS}$ sive, deletis superfluis, CD ad PF, ut angulus TSt ad angulum PSp, sive Velocitas angularis Tangentis ad angularem Velocitatem distantiæ SP: proinde Velocitas qua circumgyratur Tangens, semper proportionalis est quantitati $\frac{CO \times \sqrt{CD}}{PF \times SP^q}$.

Pleraque horum Corollariorum ex aliis Conicarum Sectionum Proprietatibus deducta, vel facile deducenda, inveniet Lector in Sect. III. Lib. I. Princip. Nat. Philosophiæ.

FINIS.

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PHILOSOPHICAL TRANSACTIONS.

For the Months of *July*, *August*, and *Sep.* 1717.

I. AN *Account of a Diſſection of a Child, Communicated in a Letter to Dr.* Brook Taylor, *R. S. Sec. By Dr.* Patrick Blair, *R. S. S.*

II. *De Seriebus infinitis Tractatus. Pars Prima. Auctore* Petro Remundo *de* Monmort. *R. S. S. Una cum Appendice & Additamento per D.* Brook Taylor, *R. S. Sec.*

Diagnostics

I. *An Account of the Diſſection of a Child. Communicated in a Letter to Dr.* Brook Taylor, *R. S. Secr. By Dr.* Patrick Blair, *R. S. S.*

AS nothing is more apt to lead us to the Knowledge of the ſeveral Diſtempers which affect the Human Body, and to acquaint us with the juſt Prognoſticks of the like Caſes, than the opening of diſtemper'd Perſons. I hop'd it would not be an unacceptable inſtance of my Zeal and Readineſs to ſerve the Moſt Honourable the *Royal Society* upon all occaſions, to deſire you to preſent them with the following account of the Diſſection I lately made of a Child.

This Child was five Months old, and was ſo emaciated, that he appear'd rather to have decreaſed, than to have encreaſed in Bulk, from the time of his Birth; his whole Body not weighing above five Pounds. The Skin and Muſcles of the Abdomen were very thin, but the Peritoneum was preternaturally thick. The Ventriculus was more like to an Inteſtin than to a Stomach, its length being five Inches, and its breadth but one Inch. The Coats of it were thick and fleſhy, and the Cavity very inconſiderable. The Pylorus, and almoſt half of the Duodenum were Cartilaginous, and ſomething inclin'd to an Oſſification, ſo that no Nouriſhment could have paſſed into the Inteſtins, tho' the Stomach had been capable of containing it, which makes it no Wonder that the Body was ſo emaciated. There were ſcarce any foot-ſteps of the Omentum to be ſeen, even at the Bottom of the Stomach, to which it uſually adheres.

The

The right Lobe of the Lungs adhered firmly to the Ribs and had three Exulcerations, which contain'd purulent Matter. It was ſo very thin and compact, that it ſeem'd as if that Lobe had never been of uſe in Reſpiration. The left Lobe was of a more florid Red, ſpongy, and free from any Adheſion.

Upon enquiring after the Symptoms this Child had been affected with, his Mother told me, he ſeem'd to be healthy till he was about a Month old, when he was ſeized with a violent Vomiting, and a Stoppage of Urine and Stool. Some time after, both theſe became more regular, but the Vomiting ſtill continued. He ſeem'd to have a great Appetite, taking what Suck, Drink, or other Food was offer'd him, with a kind of eagerneſs; but he immediately threw it all up again. He had all along breathed freely, and had no Cough, notwithſtanding the Exulcerations above mention'd. This confirm'd me in the Opinion that he had never Breath'd, by the right Lobe of the Lungs.

There could be nothing more emaciated than this Child was; and it ſeems to be worth conſidering, whether his Illneſs might not be owing in a great meaſure to the want of the Omentum, (for he ſeem'd never to have had any); as alſo, whence it is that this Part is generally conſum'd in an Atrophy, and in moſt Hydropical Caſes, except where it ſelf is more eſpecially concerned.

II. De

II. *De Seriebus infinitis Tractatus. Pars Prima.* *Auctore* Petro Remundo *de* Monmort. R. S. S.

Prop. 1. *Prob.*

INvenire ſummam terminorum quot libuerit Seriei hujus $a \times \overline{a+n} \times \overline{a+2n} \times \&c. \times \overline{a+p-1n}$
$+ \overline{a+n} \times \overline{a+2n} \times \overline{a+3n} \times \&c. \times \overline{a+pn}$
$+ \overline{a+2n} \times \overline{a+3n} \times \overline{a+4n} \times \&c. \times \overline{a+p+1n}$
$+ a+3n \times \&c.$ Ubi eſt n differentia data, tam inter Factores continuos, a, $a+n$, $a+2n$, &c. ejuſdem cujuſvis termini, quàm inter Factores homologos terminorum diverſorum in Serie continuatâ; atque deſignat p numerum factorum hujuſmodi in quovis termino.

Solutio Per x deſignetur primus Factorum in ultimo terminorum quorum ſumma requiritur, atque ſumma illa erit

$$\frac{x \times \overline{x+n} \times \&c. \times \overline{x+pn} - \overline{a-n} \times a \times \&c. \times \overline{a+p-1n}}{p+1n}$$

Q. E. I.

Ex. 1. Proponatur Series numerorum naturalium $1+2+3+4+\&c.$ & invenienda ſit ſumma tot terminorum quot ſunt unitates in numero z, qui in hoc caſu eſt etiam ultimus terminorum quorum ſumma requiritur. In hoc itaque caſu ſunt $a=1$, $n=1$, $p=1$, & $x=z$. Unde fit $x \times \overline{x+n} \times \&c. \times \overline{x+pn} = z \times \overline{z+1}$, $a-n \times a \times \&c. \times a+p-1n = 0 \times 1$, atque $p+1n = 2 \times 1$; adeoque ſumma quæſita eſt $\frac{z \times z+1}{2}$.

Ex. 2. Invenienda ſit ſumma tot terminorum, quot ſunt unitates in numero z, Seriei $1+3+6+10+\&c.$ Numerorum Triangularium. Numeri 1, 3, 6, 10, &c. in hac

Serie ſic ſcribi poſſunt $\frac{1\times2}{2}$, $\frac{2\times3}{2}$, $\frac{3\times4}{2}$, $\frac{4\times5}{2}$, &c. Hoc pacto, ſepoſito diviſore dato 2, Series revocatur ad formam Propoſitionis, exiſtentibus $a=1$, $n=1$, & $p=2$. $x=z$. Unde ſumma Seriei duplicata eſt $\frac{x\times x+1\times x+2-0\times1\times2}{3}=\frac{x\times x+1\times x+2}{3}$; adeoque habitâ ratione diviſoris 2, Summa Seriei ipſius eſt $\frac{x\times x+1\times x+2}{2\times3}$, vel $\frac{z\times z+1\times z+2}{2\times3}$, in hoc caſu exiſtente x eodem ac z. Ad eundem modum inveniuntur ſummæ cæterorum numerorum figuratorum, quorum formulæ jam vulgò innoteſcunt.

Ex. 3. Sint $a=1$, $n=2$, $p=3$, ut ſit Series poſita $1\times3\times5+3\times5\times7+5\times7\times9+$ &c. In hoc itaque caſu formula ſummæ fit $\frac{x\times x+2\times x+4\times x+6-1-2\times1\times3\times5}{4\times2}=\frac{x\times x+2\times x+4\times x+6+15}{8}$. Verbi gratiâ, ſi quæratur ſumma decem terminorum, fit $x=19$ (nempe terminus decimus in Serie Arithmeticè proportionalium, 1, 3, 5, 7, &c.) adeoque ſumma eſt $\frac{19\times21\times23\times25+15}{8}$ $=28680$. Propoſitio vero ſic demonſtratur.

Demonſtratio. Sit Series quantitatum $A, B, C, D, E,$ &c. quarum differentiæ conſtituant Seriem $a, b, c, d,$ &c. (nempe ut ſint $a=B-A$, $b=C-B$, $c=D-C$, &c.) Hinc ſtatim colligitur eſſe $a+b=C-A$, $a+b+c=D-A$, $a+b+c+d=E-A$: & in genere aggregatum quotlibet terminorum Seriei $a, b, c, d,$ &c. æquale eſt termino proximè inſequenti Seriei $A, B, C, D, E,$ &c. mulctato termino primo A. Pro $A, B, C,$ &c. ſume terminos

$a-n$

$\frac{a-n \times a \times \&c. \times a+p-1\,n}{p+1\,n}$, $\frac{a \times a+n \times \&c. \times a+p\,n}{p+1\,n}$, $\frac{a+n \times a+2\,n \times \&c. \times a+p+1\,n}{p+1\,n}$, &c. hoc est, valores successivos ipsius $\frac{x \times x+n \times \&c. \times x+p\,n}{p+1\,n}$; & eorum differentiæ, pro a, b, c, d, &c. sumendæ, erunt $a \times a+n \times \&c. \times a+p-1\,n$, $a+n \times a+2\,n \times \&c. \times a+p\,n$, &c. qui sunt ipsissimi termini Seriei propositæ. Sed comparando has Series, si terminus aliquis Seriei posterioris sit $x \times x+n \times \&c. \times x+p-1\,n$, constat terminum uno ulteriorem in Serie priori fore $\frac{x \times x+n \times \&c. \times x+p\,n}{p+1\,n}$. Summa itaque Seriei posterioris usque terminum $x \times x+n \times \&c. \times x+p-1\,n$ inclusivè est $\frac{x \times x+n \times \&c. \times x+pn-a-n \times a \times \&c. \times a+p-1\,n}{p+1\,n}$

Q. E. D.

Scholium 1. In hâc propositione continetur particula quædam Methodi incrementorum, de quâ ante biennium librum edidit D. *Brook Taylor Soc. Reg. Lond. Secr.* mihi amicitiâ conjunctissimus. Librum ipsum adeat qui de eâ methodo plura scire velit: ad institutum nostrum sufficit observare quanta intersit affinitas inter Methodum hanc & Methodum Fluxionum seu differentialem Nam ut in Methodo differentiali, ad inveniendum differentiale ipsius x dignitatis x^m, unum latus x convertendum est in differentiam dx; & ortum ducendum est in dignitatis Indicem m, ut sit $m\,dx\,x^{m-1}$ differentiale quæsitum; sic in Methodo Incrementorum *Ad inveniendum Incrementum facti hujusmodi* $x \times x+n \times x+2\,n$, (*ubi factores* x, $x+n$, $x+2\,n$,

$x + 2n$, *sunt in progressione Arithmeticâ, cujus differentia communis est ipsius* x *Incrmentum datum* n,) *Factorum minimus* x *convertendus est in Incrementum, & ortum ducendum est in numerum Factorum, ut sit* $3n \times x + n \times x + 2n$ *Incrementum quæsitum, numero Factorum in casu exposito existente* 3. Sic etiam ipsius $x \times x + n$ Incrementum fit $2n \times x + n$.

2. Incrementa etiam Reciprocorum hujusmodi Factorum inveniuntur per eandem regulam; hoc nempe observato, quòd cum sit Divisio contrarium Multiplicationis, vice ablationis minimi Factorum, sit jam addendus alius factor adhuc uno Incremento major; item quòd Factorum numerus sit scribendus cum signo negativo. Hoc pacto ipsius $\frac{1}{x}$ Incrementum fit $\frac{-1 \times n}{x \times x + n}$; ipsius $\frac{1}{x \times x + n}$ Incrementum fit $\frac{-2 \times n}{x \times x + n \times x + 2n}$; & sic de aliis hujusmodi. Hoc facile probatur sumendo differentias inter Integralium valores duos continuos.

3. Insistendo vestigiis Methodi directæ, hinc colliguntur præcepta Methodi inversæ, quibus inveniuntur Integralia Incrementorum oblatorum. *Applicetur* enim *Incrementum oblatum ad lateris Incrementum datum; addatur Factor adhuc uno Incremento minor, & applicetur ortum ad numerum Factorum sic auctorum.* Sic *e. g.* oblato Incremento $n \times x \times x + n \times x + 2n$, fit primò $x \times x + n \times x + 2n$; deinde $x - n \times x \times x + n \times x + 2n$, addito Factore $x - n$; denique $\frac{x - n \times x \times x + n \times x + 2n}{4}$, quod est Integrale quæsitum. Hoc quidem ubi Factores sunt Multiplicantes; Ubi vero Factores occupant locum divisoris, mutatis mutandis, regula hæc est, *Applicetur Incrementum oblatum ad lateris incrementum datum; rejiciatur Factorum*

Factorum maximus, & applicetur ortum ad numerum Factorum relictorum cum signo negativo. Exempli gratiâ oblato Incremento $\frac{n}{x \times x + n \times x + 2n}$, fit primò $\frac{1}{x \times x + n \times x + 2n}$, deinde $\frac{1}{x \times x + n}$, denique $\frac{1}{-2 \times x \times x + n}$, seu $\frac{-1}{2x \times x + n}$, quod est Integrale quæsitum.

4. In casu hoc novissimo Integrale inventum, cum signo contrario, æquale est summæ omnium Incrementorum in Serie in infinitum continuatâ; *v. g.* est $\frac{1}{2x \times x + n}$ $= \frac{n}{x \times x + n \times x + 2n} + \frac{n}{x + n \times x + 2n \times x + 3n}$ $+ \frac{n}{x + 2n \times x + 3n \times x + 4n} + \&c.$ Nam in hoc casu, facto x tandem infinito, evanescit $\frac{1}{2x \times x + n}$, hoc est, ultimus terminorum $A, B, C, \&c.$ fit nihil; & ob contrarietatem signorum Integralis & Incrementi, vice $-A$ exprimitur aggregatum per $+A$.

Lemma 1.

Per X designetur terminus quilibet in Serie quâvis numerorum $M, N, O, P, \&c$; per x designetur locus termini istius X in Serie illâ (*v. g.* ut sit $x = 1$, quando designat X terminum primum M, sit $x = 2$, quando designat X terminum secundum N, & sic de cæteris) & sint terminorum M, N, O, P prima differentiarum primarum b, c prima differentiarum secundarum, d prima tertiarum, e prima quartarum, & sic porrò. Tum erit

$$X = M + b \times \frac{x-1}{1} + c \times \frac{x-1}{1} \times \frac{x-2}{2} + d \times \frac{x-1}{1} \times \frac{x-2}{2} \times \frac{x-3}{3} + e \times \frac{x-1}{1} \times \frac{x-2}{2} \times \frac{x-3}{3} \times \frac{x-4}{4} + \&c.$$

Sequitur hoc ex tabulâ æquationum *pag.* 66. tractatûs noſtri *Eſſay d'Analyſe, &c.*

Lemma 2.

Iiſdem poſitis, per z deſignetur terminus quilibet in Serie Arithmeticè proportionalium $a, a+n, a+2n$, *&c.* & ſit jam $X = A + Bz + Cz \times \overline{z+n} + Dz \times \overline{z+n} \times \overline{z+2n} + Ez \times \overline{z+n} \times \overline{z+2n} \times \overline{z+3n} + \&c.$ Tum ipſorum A, B, C, D, E, *&c.* valores erunt.

$$A = M + b \times \frac{-a}{n} + c \times \frac{-a}{n} \times \frac{-a-n}{2n} + d \times \frac{-a}{n} \times \frac{-a-n}{2n} \times \frac{-a-2n}{3n} + e \times \frac{-a}{n} \times \frac{-a-n}{2n} \times \frac{-a-2n}{3n} \times \frac{-a-3n}{4n} + \&c.$$

$$B = \frac{1}{n} \times b + c \times \frac{-a-n}{n} + d \times \frac{-a-n}{n} \times \frac{-a-2n}{2n} + e \times \frac{-a-n}{n} \times \frac{-a-2n}{2n} \times \frac{-a-3n}{3n} \&c.$$

$$C = \frac{1}{n} \times \frac{1}{2n} \times c + d \times \frac{-a-2n}{n} + e \times \frac{-a-2n}{n} \times \frac{-a-3n}{2n} + \&c.$$

$$D = \frac{1}{n} \times \frac{1}{2n} \times \frac{1}{3n} d + e \times \frac{-a-3n}{n} + \&c.$$

$$E = \frac{1}{n} \times \frac{1}{2n} \times \frac{1}{3n} \times \frac{1}{4n} e + \&c.$$

Ordo

Ordo formandi coefficientes ipſorum b, c, d, e, &c. in his valoribus, per ſe eſt ſatis manifeſtus.

Demonſtratio. Quoniam per x & z deſignantur termini correſpondentes progreſſionum Arithmeticarum 1, 2, 3, 4, &c. & a, $a+n$, $a+2n$, $a+3n$, &c. indicabit $x-1$ numerum differentiarum n qui in z continetur, ut ſit $z=a+\overline{x-1}\,n$. Hinc fit $x-1=\frac{z-a}{n}$, $x-2=\frac{z-n-a}{n}$, $x-3=\frac{z-2n-a}{n}$, &c. Subſtituendo itaque hos valores $x-1$, $x-2$, $x-3$, &c. in Serie Lemmatis præcedentis, & terminis in ordinem redactis, prodeunt ipſorum A, B, C, &c. valores exhibiti.

Cor. Ubi $a=n$, prodeunt A, B, C, D, &c. per formulas ſimpliciores, nempe

$$A=M-b+c-d+e\ \&c.$$

$$B=\frac{1}{n}\times\overline{b-2c+3d-4e\ \&c.}$$

$$C=\frac{1}{n}\times\frac{1}{2n}\times\overline{c-3d+6e\ \&c.}$$

$$D=\frac{1}{n}\times\frac{1}{2n}\times\frac{1}{3n}\times\overline{d+4e\ \&c.}$$

Lemma 3.

Symbolis X & x eodem modo interpretatis ac in Lemmate primo, ſint q, r, s, t, u, &c. generatores Trianguli Arithmetici cujus lineam tranſverſam, occupat Series M, N, O, P, Q &c in ordine nempe inverſo, ut ſit $q\,(=M)$ generator ultimus, r penultimus, s antepenultimus, & ſic porrò. Tum erit

$$X=q+r\times\frac{x-1}{1}+s\times\frac{x-1}{1}\times\frac{x}{2}+t\times\frac{x-1}{1}\times\frac{x}{2}\times\frac{x+1}{3}+\&c.$$

Conſtat

Conſtat ex contemplatione ipſius Trianguli Arithmetici, quam exhibuimus *pag.* 63 tractatûs *Eſſay d'Analyſe, &c.* ubi idem fuſius explicatur.

Lemma 4.

Iiſdem poſitis, & Symbolo z eodem modo interpretato ac in *Lem.* 2. ſi ſit $X = A + Bz + Cz \times \overline{z+n} + \&c.$ ut in *Lem.* 2. erunt coefficientium $A, B, C, D, \&c.$ valores.

$$A = q + r \times \frac{-a}{n} + s \times \frac{-a}{n} \times \frac{-a+n}{2n}$$
$$+ t \times \frac{-a}{n} \times \frac{-a+n}{2n} \times \frac{-a+2n}{3n} + \&c.$$

$$B = \frac{1}{n} \times r + s \times \frac{-a}{n} + t \times \frac{-a}{a} \times \frac{-a+n}{2n} + \&c.$$

$$C = \frac{1}{n} \times \frac{1}{2n} \times s + t \times \frac{-a}{n} + \&c.$$

$$D = \frac{1}{n} \times \frac{1}{2n} \times \frac{1}{3n} \times t + \&c.$$

Ordo coefficientium in his valoribus eſt manifeſtus, & demonſtratur Lemma ad modum *Lemmatis* 2.

Cor. 1. Ubi $a = n$, coefficientes, $A, B, C, D, \&c.$ prodeunt per formulas ſimpliciores, nempe

$$A = q - r, \qquad C = \frac{1}{n} \times \frac{1}{2n} \times \overline{s - t}$$

$$B = \frac{1}{n} \times \overline{r - s}, \qquad D = \frac{1}{n} \times \frac{1}{2n} \times \frac{1}{3n} \overline{t - u}$$

&c.

Cor. 2. Unde ſi generatorum $q, r, s, t, u, \&c.$ aliquot ſint inter ſe æquales, exhibebitur X per formulam ſimpliciorem, evaneſcentibus aliquot coefficientium $A, B, C, D, \&c.$

Sic

Sic exempli gratiâ, propositâ Serie numerorum 4, 69, 530, 2676, 10350, &c. qui constituunt lineam decimam transversam in Triangulo Arithmetico cujus generatores tres priores sunt 54, — 18, 5, & septem posteriores sunt æquales 4; existente $a = 1 = z$, Terminus X exhibetur per formulam quatuor tantùm terminorum.

$$-\frac{z}{1}.\frac{z+1}{2}.\frac{z+2}{3}\,\&c.\times\frac{z+6}{7}+23\,\frac{z}{1}.\frac{z+1}{2}.\,\&c.$$

$$\times\frac{z+6}{7}-72\,\frac{z}{1}.\frac{z+1}{2}.\,\&c.\times\frac{z+7}{8}+54\,\frac{z}{1}.\frac{z+1}{2}.\,\&c.$$

$$\times\frac{z+8}{9}.$$ evanescentibus coefficientibus sex primis A, B, C, D, E, F.

Prop. II. *Prob.*

Invenire summam quotlibet terminorum Seriei

$$\frac{M}{a\times a+n\times\&c.\times a+p-1n}+\frac{N}{a+n\times\&c.\times a+pn}$$

$$+\frac{O}{a+2n\times\&c.\times a+p+1n}+\&c.$$ ubi numeratores $M, N, O, \&c.$ constituunt Seriem quamlibet terminorum, quorum differentiæ, vel primæ, vel secundæ, vel aliæ quædam dantur; vel quod perinde est, qui constituunt lineam quamvis transversam in dato quovis triangulo Arithmetico; Denominatores autem constituunt Seriem in *Prop.* I. exhibitam.

Solutio. Per X designetur primus factorum a, $a+n$, $a+2n$, &c. in denominatore ejusdem termini, ut sint X & z iidem ac in *Lemm*: præmissis, adeoque designetur terminus quilibet Seriei per $\frac{X}{z\times z+n\times\&c.\times z+p-n}$

Per *Lem.* 2, vel per *Lem.* 4. (prout magis commodum videatur

videatur vel differentias, vel generatores trianguli Arithmetici adhibere,) resolvatur X in Multinomium $A + Bz \times \overline{z+n} + Cz \times \overline{z+n} \times \overline{z+2n} + \&c.$ Hoc pacto (terminis multinomii ad denominatorem $z \times \overline{z+n} \times \&c. \times \overline{z+p-n}$ applicatis) terminus quilibet Seriei revocabitur ad formulam $\frac{A}{z \times \overline{z+n} \times \&c. \times \overline{z+p-1\,n}}$

$$+\frac{B}{\overline{z+n} \times \&c. \times \overline{z+p-1\,n}} + \frac{C}{\overline{z+2n} \times \&c \times \overline{z+p-1n}}$$

$+ \&c.$

Unde (per Scholium 4 *Prop.* I.) aggregatum totius Seriei, à termino $\frac{X}{z \times \overline{z+n} \times \&c. \times \overline{z+p-1\,n}}$ inclusive in infinitum continuatæ, est

$$\frac{A}{\overline{p-1} \times n \times z \times \overline{z+n} \times \&c. \times \overline{z+p-2\,n}}$$

$$+\frac{B}{\overline{p-2} \times n \times \overline{z+n} \times \&c. \times \overline{z+p-2\,n}}$$

$$+\frac{C}{\overline{p-3} \times n \times \overline{z+2n} \times \&c. \times \overline{z+p-2\,n}} + \&c.$$

re si dematur hoc aggregatum ab ejusdem aggregati valore quando $z = a$, residuum erit summa omnium terminorum ante terminum $\frac{X}{z+\&c.}$, hoc est, tot terminorum quot sunt unitates in $\frac{z-a}{n}$. *Q. E. I.*

Ex. 1. Sit primum exemplum in Serie $\frac{5}{3 \cdot 5 \cdot 7 \cdot 9 \cdot 11 \cdot 13}$ +

$$+\frac{41}{5\cdot7\cdot9\cdot11\cdot13\cdot15}+\frac{131}{7\cdot9\cdot11\cdot13\cdot15\cdot17}$$
$$+\frac{275}{9\cdot11\cdot13\cdot15\cdot17\cdot19}+\frac{473}{11\cdot13\cdot15\cdot17\cdot19\cdot21}$$

$+$ &c. Sunt hic $a=3$, $n=2$, $p=6$, $M=5$. & capiendo differentias numeratorum inveniuntur $b=36$, $c=54$, $d=0=e=$ &c. Hinc in Lemmate secundo sunt $A=5+36\times\frac{-3}{2}+54\times\frac{-3}{2}\times\frac{-5}{4}=\frac{209}{4}$, $B=\frac{1}{2}\times36+54\times\frac{-5}{2}=\frac{-99}{2}$, $C=\frac{1}{2}\times\frac{1}{4}\times54=\frac{27}{4}$, $D=0=E=$ &c. Summa itaque totius Seriei est $\frac{209}{4\times5\times2\times3\cdot5\cdot7\cdot9\cdot11}+\frac{-99}{2\times4\times2\times5\cdot7\cdot9\cdot11}+\frac{27}{4\times3\times2\times7\cdot9\cdot11}=\frac{283}{80\times3\cdot5\cdot7\cdot9\cdot11}$, atque summa terminorum numero $\frac{z-3}{2}$ $(=\frac{z-a}{n})$ est

$$\frac{283}{80\times3\cdot5\cdot7\cdot9\cdot11}-\frac{209}{40\times z.z+2.z+4.z+6.z+8}$$
$$+\frac{99}{16\times z+2\times z+4.z+6.z+8}-\frac{27}{24\times z+4.z+6.z+8}$$

Quærantur *v. g.* octo termini; tum existente $\frac{z-3}{2}=8$ fit $z=19$, quo valore in formulâ adhibito, prodit summa $\frac{155891}{2\cdot3\cdot3\cdot3\cdot3\cdot5\cdot5\cdot5\cdot7\cdot11\cdot19\cdot23}$.

Iidem Numeratores occupant lineam tertiam transversam in Triangulo Arithmetico

54 . 54 . 54 . 54 . 54 . 54 . &c.
$=$ 18 . 36 . 90 . 144 . 198 . &c.
5 . 41 . 131 . 275 . &c.

Unde

Unde in formula *Lem.* 4. sunt generatores $q = 5$, $r = -18$, $s = 54$, $t = 0 = \&c.$ & prodeunt coefficientes $A = 5 - 18 \times \frac{-3}{2} + 54 \times \frac{-3}{2} \times \frac{-3+2}{4} = \frac{209}{4}$, $B = \frac{1}{2} \times -18 + 54 \times \frac{-3}{2} = \frac{-99}{2}$, $C = \frac{1}{4} \times \frac{1}{4} \times 54 = \frac{27}{4}$, $D = 0 = E = \&c.$ iidem ac supra.

Ex. 2. Sit Series $\frac{4}{1.2.3.4.5.6.7.8.9.10.11} + \frac{69}{2.3.\&c.12} + \frac{530}{3.4.\&c.13} + \frac{2676}{4.5.\&c.14} + \frac{10350}{5.6.\&c.15} + \&c.$ Ubi sunt $a = 1$, $n = 1$, $p = 11$, atque Numeratores constituant Seriem in *Corol.* 20. *Lem.* 4. exhibitam. Applicando itaque valorem X in *Corol.* illo ad denominatorem $z \times z+1 \times \&c. \times z+10$, fit Seriei propositæ Terminus

$$\frac{-1}{1.2.3.4.5.6 \times z+6.z+7.z+8.z+9.z+10} + \frac{23}{1.2.3.4.5.6.7 \times z+7.z+8.z+9.z+10} - \frac{72}{1.2.3.4.5.6.7.8 \times z+8.z+9.z+10} + \frac{54}{1.2.3.4.5.6.7.8.9 \times z+9 \times z+10}.$$

Adeoque per hanc *Prop.* summa Seriei à termino illo in infinitum continuatæ est

$$\frac{-1}{4 \times 1.2.3.4.5.6 \times z+6.z+7.z+8.z+9} +$$

$$+\frac{23}{3\times1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7\times z+7\cdot z+8\cdot z+9}$$

$$-\frac{72}{2\times1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7\cdot8\times z+8\cdot z+9}$$

$$+\frac{54}{1\times1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7\cdot8\cdot9\times z+9}.$$

Itaque pro z ſumpto 1, fit ſumma totius Seriei $\frac{305}{12\times1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7\cdot8\cdot9\cdot10}$. Et in genere ſumma terminorum numero $\frac{z-1}{1}$, eſt

$$+\frac{305}{12\times1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7\cdot8\cdot9\cdot10}$$

$$+\frac{1}{4\times1\cdot2\cdot3\cdot4\cdot5\cdot6\times z+6\cdot z+7\cdot z+8\cdot z+9}$$

$$-\frac{23}{3\times1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7\times z+7\cdot z+8\cdot z+9}$$

$$+\frac{72}{2\times1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7\cdot8\times z+8\times z+9}$$

$$-\frac{54}{1\times1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7\cdot8\cdot9\times z+9}.$$

Scholium 1. In computandis ſummis hujuſmodi Serierum, calculus plerumque levior eſt adhibitis generatoribus trianguli Arithmetici, quam ſi adhibeantur differentiæ. Libet itaque hac occaſione oſtendere quomodo ex datis differentiis inveniri poſſunt generatores Trianguli Arithmetici.

Sunto itaque ω primus Seriei terminus, a differentia ultima data, b prima differentiarum penultimarum, c prima antepenultimarum, & ſic porrò d, e, &c. atque ſint t, u, x, y, &c generatores quæſiti Trianguli Arithmetici, cujus lineam tranſverſam ordine p occupet Series

propoſita. Tum (quod ex contemplatione Trianguli Arithmetici facile conſtat) ſunt

$$a = t$$

$$b = \frac{p-1}{1} t + u$$

$$c = \frac{p-1}{1} \times \frac{p-2}{2} t + \frac{p-2}{1} u + x$$

$$d = \frac{p-1}{1} \times \frac{p-2}{2} \times \frac{p-3}{3} t + \frac{p-2}{1} \times \frac{p-3}{2} u + \frac{p-3}{1} x + y \text{ &c.}$$

Unde colliguntur generatorum valores

$$t = a$$

$$u = b - \frac{p-1}{1} t$$

$$x = c - \frac{p-1}{1} \times \frac{p-2}{2} t - \frac{p-2}{1} u$$

$$y = d - \frac{p-1}{1} \times \frac{p-2}{2} \times \frac{p-3}{3} t - \frac{p-2}{1} \times \frac{p-3}{2} u - \frac{p-3}{1} x \text{ &c.}$$

Ultimus autem generator æqualis eſt Seriei termino primo ω.

2. D^nus^ de *Monſoury Abbas Orbacenſis* mihi amiciſſimus, & ruri vicinus, poſtquam cum eo hæc communicaveram, aliam invenit hujus Problematis Solutionem, cujus formulam ob ejus miram ſimplicitatem hic referre juvat. Itaque in Serie numeratorum ſint ω terminus primus, b prima differentiarum primarum, c prima ſecundarum, d prima tertiarum, & ſic porrò; atque ſit termini primi Denominator $\overline{z \times z + n} \times \text{&c.} \times \overline{z + p - 1\, n}$; Tum ſumma

ſumma totius Seriei in infinitum continuatæ exhibebitur per formulam $\frac{\omega}{n \times p-1 \times z \times z + n \times \&c. \times z + p-2n} + \frac{b}{n^2 \times p-1 \times p-2 \times z + n \times \&c. \times z + p-2n} + \frac{c}{n^3 \times p-1 \times p-2 \times p-3 \times z + 2n \times \&c. \times z + p-2n} + \&c.$

Sit exemplum in Serie $\frac{5}{3 \cdot 5 \cdot \&c. 13} + \frac{41}{5 \cdot 7 \cdot \&c. 15} + \frac{131}{7 \cdot 9 \cdot \&c. 17} + \frac{275}{9 \cdot 11 \cdot \&c. 19} + \&c.$ cujus ſummam jam exhibuimus. In hoc caſu ſunt $\omega = 5$, $b = 36$, $c = 54$, $d = 0 = e = \&c.$ Unde per formulam ſumma Seriei integræ fit $\frac{5}{2 \cdot 5 \times 3 \cdot 5 \ldots 11} + \frac{36}{4 \cdot 5 \cdot 4 \times 5 \ldots 11} + \frac{54}{8 \cdot 5 \cdot 4 \cdot 3 \times 7 \ldots 11} = \frac{283}{80 \times 3 \cdot 5 \ldots 11}$, ut per formulam noſtram exhibetur. Si quæratur ſumma ejuſdem Seriei incipientis à termino decimo $\frac{2273}{21 \ldots 31}$, in eo caſu $\omega = 2273$, $b = 522$, $c = 54$, & ſumma eſſet $\frac{2273}{2 \cdot 5 \times 21 \ldots 29} + \frac{522}{4 \cdot 5 \cdot 4 \times 23 \ldots 29} + \frac{54}{8 \cdot 5 \cdot 4 \cdot 3 \times 25 \ldots 29}$.

Hæc formula eſt commodiſſima, & ſummam exhibet nullo ferè negotio, quoties quæritur ſumma Seriei integræ, & differentiæ non ſunt nimis multæ. Sed ubi plures ſunt differentiæ, & quæritur non Series integra, ſed termini tantùm initiales aliquammulti, formulæ noſtræ ſunt commodiores.

3. Quando

3. Quando Serierum termini formantur tantùm per Multiplicationem, nec afficiuntur diviſoribus variabilibus, ſummæ ſemper exhiberi poſſunt per Methodum in *Prop.* I. traditam, ſint licet formulæ quantumlibet compoſitæ. Nam poſſunt ſemper revocari ad terminos in formâ quam poſtulat Propoſitio illa. Sic ſi differentiæ ipſorum z & x ſint m & n, & deſignetur terminus Seriei per $z\,x$; hic terminus revocabitur ad formam $a - nz + \frac{n}{m}z \times \overline{z + m}$; cujus Integrale datur *per Prop.* I; nempe quoniam $dx = n$, & $dz = m$, eſt $dx = dz \times \frac{n}{m}$; unde regrediendo ad integralia fit $x = \frac{n}{m}z + a$ (adjecto invariabili a, ut habeatur ratio relationis inter z & x in Seriei termino primo,) quod ſic ſcribi poteſt $\overline{a - n} + \frac{n}{m} \times \overline{z + m}$, ut deinde in z ductum induat formam requiſitam. Et ad eundem modum procedere licet in aliis caſibus ejuſmodi. Sed ubi formulæ oblatæ diviſoribus afficiuntur, eædem ac in Calculo integrali, ut vocant, difficultates occurrunt, eâdem induſtriâ ſuperandæ. Nec tamen ſemper ſuperari poſſunt. Nam præterquam quod vix certò ſciri poſſit quæ debeat relatio intercedere inter Numeratorem fractionis & Denominatorem, ut formula oblata ad Integrale revocari poſſit; ſæpe etiam difficillimum eſt explorare an adſit jam talis relatio in formulâ iſtâ, aut ſi deſit, an introduci poſſit. Quicquid ego in hâc materiâ potiſſimum inveni, continetur in tribus ſequentibus propoſitionibus.

Prop. III. *Prob.*

Creſcentibus, z, u, y, x, *&c.* per differentias datas n, m, l, o, *&c.* invenire valorem numeratoris integri

tegri N, ut existente Denominatore $z.\overline{z+n}.\&c.\overline{z+pn} \times u.\overline{u+m}.\&c.\overline{u+qm} \times y.\overline{y+l}.\&c.\overline{y+rl} \times x.\overline{x+o}\ \&c.\ \overline{x+so}.\&c.$ Fractio ad Integrale revocari possit.

Solutio. Fiat $N = z+pn \times u+qm \times y+rl \times x+so \times \&c. - zuyx\ \&c.$ atque Integrale erit fractio, cujus Denominator $z.\overline{z+n}.\&c.\ z+p-1\,n \times u.\overline{u+m}.\&c.\ u+q-1\,m \times y.\overline{y+l}.\&c.\ y-x-1\,l \times x+o.\&c.\ x+s-1\,o \times \&c.$ existente 1 Numeratore.

Differentia enim hujus fractionis est fractio cujus numerator est ipsius N valor exhibitus, & denominator idem est ac denominator propositus, ut fieri debuit.

Ex. 1. Sit denominator propositus $z \times \overline{z+2} \times u \times \overline{u+3}$. In hoc casu sunt $n=2$, $m=3$, $p=1$, $q=1$; adeoque est $N = z+2 \times u+3 - zu = 3z+2u+6$, & per $\frac{3z+2u+6}{z.\overline{z+2} \times u.\overline{u+3}}$ repræsentatur terminus Seriei summabilis, cujus nempe in infinitum continuatæ summa exhibetur per $\frac{1}{zu}$. Sint verbi gratiâ, ipsorum z & u primus valor communis 1, atque Series summabilis erit $\frac{11}{1.3 \times 1.4} + \frac{23}{3.5 \times 4.7} + \frac{35}{5.7 \times 7.10} + \&c$, quippe cujus totius summa est 1. Per p designetur ordo termini cujusvis in hâc Serie, erit $p = \frac{z-1+2}{2} = \frac{u-1+3}{3}$, adeoque $z = 2p-1$, & $u = 3p-2$; quibus valoribus pro z & u scriptis, designabitur terminus per formulam $\frac{12p-1}{2p-1 \times 2p+1 \times 3p-2 \times 3p+1}$. Summa autem terminorum omnium ante terminum illum, hoc est terminorum initialium numero $\frac{z-1}{2} = p-1$, est

$1 - \frac{1}{zu}, = \frac{zu-1}{zu}$, hoc eſt $\frac{6pp-7p+1}{2p-1 \times 3p-2}$. Quare pro p ſcripto $p+1$, erit $\frac{p \times 6p+5}{2p+1 \times 3p+1}$ aggregatum tot terminorum initialium quot ſunt unitates in p.

Ex. 2. Iiſdem manentibus z, u, n, m, ſit denominator $z.z+2.z+4 \times u.u+3$. Tum per formulam numerator erit $z+4 \times u+3 - zu = 3z+4u+12$, & ſumma Seriei exhibebitur per formulam $\frac{1}{z.z+2 \times u}$. Sit ipſorum z & u primus valor communis 1, & hinc elicietur Series $\frac{19}{1.3.5 \times 1.4} + \frac{37}{3.5.7 \times 4.7} + \frac{55}{5.7.9 \times 7.10} + \&c. = \frac{1}{3}$.

Scholium. In Seriebus jam expoſitis eadem ubique eſt differentia inter factores continuos ejuſdem cujuſvis termini, ac inter factores homologos terminorum continuorum. In ſequentibus exempla quædam ſunt Serierum, quarum ſummæ in terminis numero finitis exhiberi poſſunt, quamvis ea regula non obſervetur.

Prop. IV. *Prob.*

Creſcente z per differentias datas $q\,n$, invenire numeratorem integrum N, ut ad Integrale revocari poſſit fractio, cujus Denominator fit ex certo numero p terminorum z, $z+n$, $z+2n$, &c. Arithmeticè proportionalium in invicem ductorum. Debet autem eſſe q numerus integer minor quam factorum numerus p.

Solutio. Erit $N = z+p-1\,n \times z+p-2\,n \times \&c. \times z+p-q\,n - z \times z+n \times \&c. \times z+q-1\,n$, Integrale

tegrale exiſtente $\frac{1}{z \times z+n \times \&c. \times z+p-q-1n}$ Demonſtratur ad modum propoſitionis præcedentis.

Sumptis ad libitum n, p, q, & primo valore z, hinc oriuntur infinitæ Series ſummabiles, cujuſmodi ſunt Series tres ſequentes.

$$A = \frac{5}{1.2.3.4} + \frac{9}{3.4.5.6} + \frac{13}{5.6.7.8} + \frac{17}{7.8.9.10} \&c.$$

$$B = \frac{1}{1.2.3.4.5} + \frac{4}{4.5.6.7.8} + \frac{9}{7.8.9.10.11} + \frac{16}{10.11.12.13.14} + \&c.$$

$$C = \frac{1}{1.2.3.4.5} + \frac{14}{5.6.7.8.9} + \frac{55}{9.10.11.12.13} + \frac{140}{13.14.15.16.17} + \&c.$$

Has Series jampridem communicavi cum primariis quibuſdam Geometris, à quibus minimè contemni videntur. Sic ad me ſcribit peritiſſimus Geometra D *Nicolaus Bernoulli* in epiſtolâ datâ 25 *Julii* 1716. " Vous " me ferez un extreme plaiſir, *Monſieur*, de me com- " muniquer la Solution de voſtre probleme, *Etant donnée* " *une ſuitte des Fractions dont les Numerateurs ſoient des* " *nombres figurés quelconque, & dönt les Denominateurs* " *ſoient formés du produit d'un nombre egal de Facteurs* " *qui ſoient en Progreſſion Arithmetique, trouver la ſom-* " *me*; & principalement comment vous avez trouvé " ces deux formules $\frac{p}{24 \times 4p+1}$, $\frac{p.p+1}{12 \times 3p+1 \times 3p+2}$.

Hæ formulæ ſpectant ad Series C & B, deſignante p numerum terminorum, quorum ſumma requiritur. Sic etiam ad me ſcribit D. *Taylor* in epiſtola datâ 22 *Aug.* 1716. " Ut & quâ ratione incidiſti in ſummationem " Serierum à te exhibitarum, præſertim loquor de

Serie

" Serie $\frac{1}{1.2.3.4.5} + \frac{4}{4.5.6.7.8} + \frac{9}{7.8.9.10.11} + \&c.$
" quæ videtur esse altioris indaginis.

Sed ut ad exempla jam redeamus. In Serie *A* sunt $p = 4$, $q = 2$, $n = 1$, primo valore z existente 1. Est itaque $z+3 \times z+2 - z \times z+1 = 2 \times 2z+3$ formula, unde (rejecto dato numero 2) derivantur numeratores 5, 9, 13, 17, *&c.* Formula etiam summæ est $\frac{1}{z \times z+1}$. Quare habitâ ratione numeri 2, quem ex numeratoribus rejecimus, summa totius Seriei, à termino in quo est z in infinitum continuatæ, exhibetur per formulam $\frac{1}{2z \times z+1}$; adeoque summa Seriei integræ est

$$\frac{1}{2 \times 1 \times 2} = \frac{1}{4}$$

In Serie *B* sunt $n = 1$, $p = 5$, $q = 3$, primo valore z existente 1. Est itaque $N = z+4 \times z+3 \times z+2 - z \times z+1 \times z+2 = 6 \times z+2|^2$. Ipsius autem $z+2$ valores continui sunt 3, 6, 9, *&c.* qui quoniam omnes sunt divisibiles per 3, ponendo $z+2 = 3x$, fit $N = 6 \times 3x|^2 = 6 \times 9x^2 = 54x^2$, ipsius x valoribus continuis existentibus 1, 2, 3, *&c.* Rejecto itaque numero dato 54, hinc prodeunt numeratores $1, 2^2, 3^2$, *&c.* hoc est 1, 4, 9, *&c.* Formula etiam Integralis est $\frac{1}{z \times z+1}$; quare habitâ ratione numeri 54 quem ex numeratoribus rejecimus, summa Seriei à termino in quo est z in infinitum continuatæ est $\frac{1}{54z \times z+1}$. Unde summa Seriei integræ est $\frac{1}{108}$.

In Serie denique *C* sunt $n = 1$, $p = 5$, $q = 4$, & primus valor $z = 1$. Unde fit $N = z+4 \times z+3 \times z+2 \times z+1 - z \times z+1 \times z+2 \times z+3 = 4 \times z+1$

$\overline{z+2} \times \overline{z+3}$. Valores autem N per hanc formulam prodeuntes ſemper poſſunt dividi per $4 \times 2 \times 3 \times 4 = 96$. Ergò hoc diviſore rejecto prodeunt numeratores 1, 14, 55, 140, *&c.* Et formula Summæ, habitâ ratione numeri 96, eſt $\frac{1}{96z}$. Adeoque Summa Seriei integræ eſt $\frac{1}{96}$.

Scholium 1. Per Propoſitiones has duas noviſſimas nullo negotio inveniri poſſunt Series quot libuerit ſummabiles. Et viciſſim oblatâ Serie hujus ſpeciei, ſi ſummari poteſt, ejus ſumma plerumque revocatur ad alterutram ex his Propoſitionibus. In examine tamen ſolertiâ eſt opus. Optime autem procedit ſi termini Seriei oblatæ revocentur ad formulam *Prop.* III. Sic *e. gr.* propoſitâ Serie $\frac{7}{3 \cdot 5 \cdot 7 \cdot 9 \cdot 11} + \frac{11}{7 \cdot 9 \cdot 11 \cdot 13 \cdot 15} + \frac{15}{11 \cdot 13 \cdot 15 \cdot 17 \cdot 19} + \&c.$ Denominatores ſic ſcribi poſſunt $3 \cdot 7 \cdot 11 \times 5 \cdot 9, 7 \cdot 11 \cdot 15 \times 9 \cdot 13, 11 \cdot 15 \cdot 19 \times 13 \cdot 17 \cdot \&c.$

Unde juxta *Prop.* III. fit $n = 4$, $m = 4$, $p = 2$, $q = 1$, primus valor $z = 3$, primus valor $u = 5$. Hinc formula Numeratoris invenitur $4 \times z + 2u + 8$, Eſt autem $z + 2u + 8$ ſemper diviſibile per 3; quare rejectis diviſoribus datis 4 & 3, per hanc formulam prodeunt Numeratores 7, 11, 15, *&c.* iidem ac Numeratores in Serie propoſita, quæ proinde ſummabitur per illam propoſitionem.

2. Cùm Series illas *A*, *B*, *C*, communicaveram cum D. *Taylor*, reſcripſit ſe earum ſummas inveniſſe primam quidem *A* & tertiam *C*, eas revocando ad caſus ſimplices Methodi Incrementorum, tertiam *C*, *e g.* revocavit ad hanc formam $\frac{1}{24} \times \frac{1}{1 \cdot 5} + \frac{1}{5 \cdot 9} + \frac{1}{9 \cdot 13} + \frac{1}{13 \cdot 17} \&c.$, ut habeatur ſumma per præcepta tradita in *Scholio Prop* 1.

In Serie autem ſecundâ *B*, cùm hoc non æquè ſucceſſit, ſequenti uſus eſt Analyſi, quam, ipſius venia jam impetratâ, ob ejus eximiam elegantiam huc transferre non piget. " Seriei iſtius terminus [in Stylo ejus] exhibetur per formulam $\frac{\overline{z+2}\times z}{27\,z\times\overline{z+1}\times z\times z+1}$; pro $z+3$ in denominatore ſcripto z, quoniam eſt $z=3$.

" Pone $\frac{B}{27\,C}$ æquale eſſe Integrali quæſito, hoc eſt $\frac{B}{C}$ " eſſe Integrale ipſius $\frac{\overline{z+2}\times z}{z\cdot\overline{z+1}\times z\cdot z+1}$, ſepoſito divi- " ſore dato 27. Ipſius autem $\frac{B}{C}$ incrementum eſt " $\frac{BC-BC}{CC}$. Debet ergo $\frac{BC-BC}{CC}$ idem eſſe ac $\frac{\overline{z+2}\times z}{z\cdot\overline{z+1}\times z\cdot z+1}$. Comparando denominatores inveni- " tur $C=z\times\overline{z+1}$. Hinc itaque ſumendo incremen- " ta fit $C=2\,z\,z+z^2+z$ $(=2\,z\,z+4\,z$, quoniam " eſt $z=3$,) His valoribus in locum C & C ſubſtitu- " tis prodit $B\,C-B\,C=\overline{z\,z+z}\,B-2\,z\times\overline{z+2}\,B$, " quod debet eſſe idem ac $z+2\times z$. Sit $B=a+v$, " exiſtente a ipſius B parte invariabili, & v parte va- " riabili. Tum ſumendo incrementa fit $B=v$. Unde " ad invenienda a & v habetur æquatio $z\,z+z\,v$ " $-2\,z\times\overline{z+2}\times a+v=\overline{z+2}\times z$, quæ ſic ſcribi " poteſt $\overline{z\,z+z}\,v-2\,z\times\overline{z+2}\,v=z\times\overline{z+2}\times\overline{1+2a}$ " vel etiam $C\,v-C\,v=z\times\overline{z+2}\times 1+2a$. Pone " $1+2a=0$ (unde fit $a=\frac{-1}{2}$,) & fit $C\,v-C\,v=0$;

" ubi

" ubi fieri poteſt $v = 0$, (quoniam æquationis termini
" ſinguli afficiuntur vel ab v, vel ab $\dot{v}$) Hinc ergò fit $B =$
" $A = \frac{-1}{2}$, adeoque $\frac{B}{C} = \frac{-1}{2z \times z + 1}$. Unde habitâ ra-
" tione diviſoris 27, Integrale quæſitum fit $\frac{-1}{54 \times z \times z + 1}$.
" Sed & comparando æquationem $C\dot{v} - \dot{C}v = 0$ cum
" formulâ generali $\frac{BC - B\dot{C}}{C\dot{C}} = 0$, inde etiam conclude-
" re licet eſſe $\frac{v}{C} =$ quantitati datæ, (quoniam ipſius
" incrementum eſt 0.) Unde pro n ſumpto quovis
" numero dato, fit $v = nC$, atque $B = -\frac{1}{2} + nC$.
" Quo pacto Integrale quæſitum fit $\frac{B}{C} = \frac{-\frac{1}{2} + nC}{C} = \frac{-1}{2C}$
" $+ n$, quod ab Integrali prius invento differt quan-
" titate datâ n. Hoc inde fit, quòd, ut in quadraturâ
" Curvarum Area inventa augeri poteſt vel minui areâ
" datâ, ſic in Methodo incrementorum Integrale inven-
" tum augeri poteſt vel minui quantitate datâ. Per
" Integrale autem primum, ubi deeſt n, exhibetur
" ſumma Seriei in infinitum continuatæ.

Prop. V.

Creſcente z per unitates, & exiſtentibus a, b, c, *&c.* numeris datis integris, quorum nullæ inter ſe æquantur; invenire Integrale ipſius $\frac{1}{z \times \overline{z+a} \times \overline{z+b} \times \overline{z+c} \times \&c.}$.

Solutio. Ducendo tam numeratorem quam denominatorem fractionis in terminos $z + 1$, $z + 2$, *&c.* $z + a + 1$, $z + a + 2$, *&c.* $z + b + 1$, $z + b + 2$, *&c.* $z + c + 1$, $z + c + 2$, *&c.* in denominatore deficientes, revocetur Denominator ad formulam $z \times z + 1$

$\times z+2\times$ &c. denominatoris in *Prop.* I. *Schol. n.* 3. Deinde revocetur Numerator ad formam $A+Bz+Cz \times \overline{z+1}+Dz\times\overline{z+1}\times\overline{z+2}+$ &c. Tum applicando terminos ad Denominatorem novum $z\times\overline{z+1}\times\overline{z+2}\times$ &c. revocetur fractio ad hanc formam $\frac{A}{z\times\overline{z+1}\times \text{\&c.}}+\frac{B}{\overline{z+1}\times\overline{z+2}\times \text{\&c.}}+\frac{C}{\overline{z+2}\times\overline{z+3}\times \text{\&c.}}+\frac{D}{\overline{z+3}\times\overline{z+4}\times \text{\&c.}}$ &c. Unde denique quæratur Integrale per *Schol. Prop.* I. *n.* 3.

Ratio Solutionis per se satis est manifesta.

Scholium 1. Hujus Solutionis tota difficultas latet in revocatione numeratoris ad formam requisitam, quod tamen quomodo sit faciendum uno exemplo patebit. Proponatur itaque factum $z+2\times z+3\times z+7$, quod ad formam propositam sit revocandum. Terminos itaque evolvo gradatim ut sequitur. Factorem primum $z+2$ sic scribo $2+z$, cujus terminum primum 2 duco in $3+z$, unde fit $6+2z$: Terminum secundum z duco in $2+\overline{z+1}$ $(=z+3)$ unde fit $2z+z\times\overline{z+1}$. Dein facta in unam summam colligendo, fit $z+2\times\overline{z+3}=\begin{matrix}6+2z\\+2z\end{matrix}+z\times\overline{z+1}=6+4z+z\times\overline{z+1}$. Supereſt ut hoc ducatur in $z+7$. Itaque terminum primum 6 duco in $7+z$ $(=z+7)$ unde fit $42+6z$; terminum secundum $4z$ duco in $6+\overline{z+1}$ $(=z+7)$ unde fit $24z+4z\times\overline{z+1}$; terminum tertium $z\times\overline{z+1}$ duco in $5+z+2$ $(=z+7,)$ unde fit $5z\times\overline{z+1}+z\times\overline{z+1}\times\overline{z+2}$. Factis itaque in unum collectis ut prius, fit $z+2\times\overline{z+3}\times\overline{z+4}=42+30z+9z\times\overline{z+1}+z\times\overline{z+1}\times\overline{z+2}$. Et ad eundem modum procedere licet in aliis casibus.

2. Sit

2. Sit autem exemplum Propositionis in fractione $\frac{1}{z \times z+2 \times z+5}$. Restituendo factores $z+1$, $z+3$, $z+4$ in Denominatore deficientes, fractio fit $\frac{z+1 \times z+3 \times z+4}{z \times z+1 \times z+2 \times z+3 \times z+4 \times z+5}$. Revocandus itaque est Numerator $z+1 \times z+3 \times z+4$ ad formam requisitam. Itaque per methodum jam traditam fit primo $z+1 \times z+3 = 1 \times 3 + z + z \times 2 + z + 1$ $= 3 + z + 2z + z \times z + 1 = 3 + 3z + z \times z + 1$. Deinde $z+1 \times z+3 \times z+4 = 3 \times 4 + z + 3z \times 3 + z + 1 + z \times z + 1 \times 2 + z + 2 = 12 + 3z + 9z + 3z \times z + 1 + 2z \times z + 1 + z \times z + 1 \times z + 2$ $= 12 + 12z + 5z \times z + 1 + z \times z + 1 \times z + 2$. Applicando hoc factum ad Denominatorem $z \times z+1 \times$ &c. $\times z+5$ fractio tandem revocatur ad hanc formam $\frac{12}{z \times z+1 \times z+2 \times z+3 \times z+4 \times z+5}$ $+ \frac{12}{z+1 \times z+2 \times z+3 \times z+4 \times z+5}$ $+ \frac{5}{z+2 \times z+3 \times z+4 \times z+5} + \frac{1}{z+3 \times z+4 \times z+5}$ Cujus denique Integrale est $\frac{-12}{5z \times z+1 \times z+2 \times z+3 \times z+4}$ $+ \frac{-12}{4 \cdot z+1 \times z+2 \times z+3 \times z+4} + \frac{-5}{3 \cdot z+2 \times z+3 \times z+4}$ $\frac{-1}{2 \cdot z+3 \times z+4}$.

3. Quando duo tantùm sunt factores z & $z+a$, exhibebitur etiam Integrale per formulam $\frac{1}{2} - \frac{1-a}{2z \times z+1}$ $- \frac{1-a \times 2-a}{3z \times z+1 \times z+2} - \frac{1-a \times 2-a \times 3-a}{4z \times z+1 \times z+2 \times z+3}$ &c. Seriem nempe continuando donec abrumpatur per evanescentiam

neſcentiam terminorum. Si Factores duo ſint z & $z-a$ exhibebitur Integrale per formulam $\frac{-1}{z-1} - \frac{-1+a}{2\cdot z-1\cdot z-2}$ $- \frac{-1+a\times-2+a}{3\cdot z-1\cdot z-2\cdot z-3}$ — &c. Poteſt idem Integrale exprimi utroque modo, prout fractionis oblatæ factor vel minor vel major ſumatur pro z.

4. Si primus valor z ſit $a+1$, migrabit formula poſterior in hanc $\frac{-1}{a}\times\frac{1}{1}\times\frac{1}{2}\times\frac{1}{3}+$ &c. uſque $\frac{1}{a}$ inclusivè, quâ, cum ſigno contrario, exhibetur ſumma Seriei $\frac{1}{1\times1+a}+\frac{1}{2\times2+a}+\frac{1}{3\times3+a}+$ &c. in infinitum continuatæ. Sit *e. gr.* $a=1$, atque Series erit $\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+$ &c. $=\frac{1}{1}\times\frac{1}{1}=1$. Si $a=2$, erit Series $\frac{1}{1\times3}+\frac{1}{2\times4}+\frac{1}{3\times5}+$ &c. $=\frac{1}{2}\times\overline{\frac{1}{1}+\frac{1}{2}}=\frac{3}{4}$; Si $a=3$, Series erit $\frac{1}{1\times4}+\frac{1}{2\times5}+\frac{1}{3\times6}+\frac{1}{4\times7}$ &c. $=\frac{1}{3}\times\overline{\frac{1}{1}+\frac{1}{2}+\frac{1}{3}}=\frac{11}{18}$.

5. Ex eâdem Serie $\frac{1}{1\times1+a}+\frac{1}{2\times2+a}+\frac{1}{3\times3+a}$ + &c. pro diverſo valore a oriuntur Series plures formâ ſatis elegantes, quarum nonnullas Lectori ob oculos ſiſtere, credo, ingratum non erit.

Si pro a ſumantur ſucceſſivè numeri pares, 2, 4, 6, 8, &c. Series erunt

Si $a=2$) $\frac{1}{1\times1+2}+\frac{1}{2\times2+2}+\frac{1}{3\times3+2}+\frac{1}{4\times4+2}+$ &c.

4) $\frac{1}{1\times1+4}+\frac{1}{2\times2+4}+\frac{1}{3\times3+4}+\frac{1}{4\times4+4}+$ &c.

6) $\frac{1}{1\times1+6}+\frac{1}{2\times2+6}+\frac{1}{3\times3+6}+\frac{1}{4\times4+6}+$ &c.

8) $\frac{1}{1\times1+8}+\frac{1}{2\times2+8}+\frac{1}{3\times3+8}+\frac{1}{4\times4+8}+$ &c.

Vel

Vel $\frac{1}{4-1} + \frac{1}{9-1} + \frac{1}{16-1} + \frac{1}{25-1} + \&c.$

$\frac{1}{9-4} + \frac{1}{16-1} + \frac{1}{25-4} + \frac{1}{36-4} + \&c.$

$\frac{1}{16-9} + \frac{1}{25-9} + \frac{1}{36-9} + \frac{1}{49-9} + \&c.$

$\frac{1}{25-16} + \frac{1}{36-16} + \frac{1}{49-16} + \frac{1}{64-16} + \&c.$

Vel $\frac{1}{4-1} + \frac{1}{9-1} + \frac{1}{16-1} + \frac{1}{25-1} + \&c.$

$\frac{1}{4+1} + \frac{1}{9+3} + \frac{1}{16+5} + \frac{1}{25+7} + \&c.$

$\frac{1}{4+3} + \frac{1}{9+7} + \frac{1}{16+11} + \frac{1}{25+15} + \&c.$

$\frac{1}{4+5} + \frac{1}{9+11} + \frac{1}{16+17} + \frac{1}{25+23} + \&c.$

Si pro *a* ſumantur ſucceſſive numeri impares 1, 3, 5, 7, &c. Series erunt

$a=1)\ \frac{1}{1\times1+1} + \frac{1}{2\times2+1} + \frac{1}{3\times3+1} + \frac{1}{4\times4+1} + \&c.$

$3)\ \frac{1}{1\times1+3} + \frac{1}{2\times2+3} + \frac{1}{3\times3+3} + \frac{1}{4\times4+3} + \&c.$

$5)\ \frac{1}{1\times1+5} + \frac{1}{2\times2+5} + \frac{1}{3\times3+5} + \frac{1}{4\times4+5} + \&c.$

$7)\ \frac{1}{1\times1+7} + \frac{1}{2\times2+7} + \frac{1}{3\times3+7} + \frac{1}{4\times4+7} + \&c.$

Vel $\frac{1}{2}\times\frac{1}{1} + \frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \&c.$

$\frac{1}{2}\times\frac{1}{3-1} + \frac{1}{6-1} + \frac{1}{10-1} + \frac{1}{15-1} + \&c.$

$\frac{1}{2}\times\frac{1}{6-3} + \frac{1}{10-3} + \frac{1}{15-3} + \frac{1}{21-3} + \&c.$

$\frac{1}{2}\times\frac{1}{10-6} + \frac{1}{15-6} + \frac{1}{21-6} + \frac{1}{28-6} + \&c.$

Vel $\frac{1}{2}\times\frac{1}{1+0} + \frac{1}{3+0} + \frac{1}{6+0} + \frac{1}{10+0} + \&c.$

$\frac{1}{2}\times\frac{1}{1+1} + \frac{1}{3+2} + \frac{1}{6+3} + \frac{1}{10+4} + \&c.$

$\frac{1}{2}\times\frac{1}{1+2} + \frac{1}{3+4} + \frac{1}{6+6} + \frac{1}{10+8} + \&c.$

$\frac{1}{2}\times\frac{1}{1+3} + \frac{1}{3+6} + \frac{1}{6+9} + \frac{1}{10+12} + \&c.$

$\times z+2\times$ &c. denominatoris in *Prop.* I. *Schol. n.* 3. Deinde revocetur Numerator ad formam $A+Bz+Cz \times \overline{z+1}+Dz\times\overline{z+1}\times\overline{z+2}+$ &c. Tum applicando terminos ad Denominatorem novum $z\times\overline{z+1}\times\overline{z+2}\times$ &c. revocetur fractio ad hanc formam $\frac{A}{z\times\overline{z+1}\times \text{\&c.}}+\frac{B}{\overline{z+1}\times\overline{z+2}\times \text{\&c.}}+\frac{C}{\overline{z+2}\times\overline{z+3}\times \text{\&c.}}+\frac{D}{\overline{z+3}\times\overline{z+4}\times \text{\&c.}}$ &c. Unde denique quæratur Integrale per *Schol. Prop.* I. *n.* 3.

Ratio Solutionis per se satis est manifesta.

Scholium 1. Hujus Solutionis tota difficultas latet in revocatione numeratoris ad formam requisitam, quod tamen quomodo sit faciendum uno exemplo patebit. Proponatur itaque factum $z+2\times\overline{z+3}\times\overline{z+7}$, quod ad formam propositam sit revocandum. Terminos itaque evolvo gradatim ut sequitur. Factorem primum $z+2$ sic scribo $2+z$, cujus terminum primum 2 duco in $3+z$, unde fit $6+2z$: Terminum secundum z duco in $2+\overline{z+1}$ $(=z+3)$ unde fit $2z+z\times\overline{z+1}$. Dein facta in unam summam colligendo, fit $\overline{z+2}\times\overline{z+3}=\begin{matrix}6+2z\\+2z\end{matrix}+z\times\overline{z+1}=6+4z+z\times\overline{z+1}$. Supereſt ut hoc ducatur in $z+7$. Itaque terminum primum 6 duco in $7+z$ $(=z+7)$ unde fit $42+6z$; terminum secundum $4z$ duco in $6+\overline{z+1}$ $(=z+7)$ unde fit $24z+4z\times\overline{z+1}$; terminum tertium $z\times\overline{z+1}$ duco in $5+\overline{z+2}$ $(=z+7,)$ unde fit $5z\times\overline{z+1}+z\times\overline{z+1}\times\overline{z+2}$. Factis itaque in unum collectis ut prius, fit $\overline{z+2}\times\overline{z+3}\times\overline{z+4}=42+30z+9z\times\overline{z+1}+z\times\overline{z+1}\times\overline{z+2}$. Et ad eundem modum procedere licet in aliis casibus.

2. Sit

2. Sit autem exemplum Propositionis in fractione $\frac{1}{z \times z+2 \times z+5}$. Restituendo factores $z+1$, $z+3$, $z+4$ in Denominatore deficientes, fractio fit $\frac{z+1 \times z+3 \times z+4}{z \times z+1 \times z+2 \times z+3 \times z+4 \times z+5}$. Revocandus itaque est Numerator $z+1 \times z+3 \times z+4$ ad formam requisitam. Itaque per methodum jam traditam fit primo $z+1 \times z+3 = 1 \times 3+z+z \times 2+z+1$ $= 3+z+2z+z \times z+1 = 3+3z+z \times z+1$. Deinde $z+1 \times z+3 \times z+4 = 3 \times 4+z+3z \times 3+z+1+z \times z+1 \times 2+z+2 = 12+3z+9z+3z \times z+1+2z \times z+1+z \times z+1 \times z+2 = 12+12z+5z \times z+1+z \times z+1 \times z+2$. Applicando hoc factum ad Denominatorem $z \times z+1 \times$ &c. $\times z+5$ fractio tandem revocatur ad hanc formam $\frac{12}{z \times z+1 \times z+2 \times z+3 \times z+4 \times z+5}$ $+\frac{12}{z+1 \times z+2 \times z+3 \times z+4 \times z+5}$ $+\frac{5}{z+2 \times z+3 \times z+4 \times z+5}+\frac{1}{z+3 \times z+4 \times z+5}$ Cujus denique Integrale est $\frac{-12}{5z \times z+1 \times z+2 \times z+3 \times z+4}$ $+\frac{-12}{4 \cdot z+1 \times z+2 \times z+3 \times z+4}+\frac{-5}{3 \cdot z+2 \times z+3 \times z+4}$ $\frac{-1}{2 \cdot z+3 \times z+4}$.

3. Quando duo tantùm sunt factores z & $z+a$, exhibebitur etiam Integrale per formulam $\frac{1}{2}-\frac{1-a}{2z \times z+1}$ $-\frac{1-a \times 2-a}{3z \times z+1 \times z+2}-\frac{1-a \times 2-a \times 3-a}{4z \times z+1 \times z+2 \times z+3}$ &c. Seriem nempe continuando donec abrumpatur per evanescentiam

neſcentiam terminorum. Si Factores duo ſint z & $z-a$ exhibebitur Integrale per formulam $\frac{-1}{z-1} - \frac{-1+a}{2\cdot z-1\cdot z-2}$ $- \frac{-1+a\times-2+a}{3\cdot z-1\cdot z-2\cdot z-3}$ — &c. Poteſt idem Integrale exprimi utroque modo, prout fractionis oblatæ factor vel minor vel major ſumatur pro z.

4. Si primus valor z ſit $a+1$, migrabit formula poſterior in hanc $\frac{-1}{a}\times\frac{1}{1}\times\frac{1}{2}\times\frac{1}{3}+$ &c. uſque $\frac{1}{a}$ incluſivè, quâ, cum ſigno contrario, exhibetur ſumma Seriei $\frac{1}{1\times1+a}+\frac{1}{2\times2+a}+\frac{1}{3\times3+a}+$ &c. in infinitum continuatæ. Sit *e. gr.* $a=1$, atque Series erit $\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+$ &c. $=\frac{1}{1}\times\frac{1}{1}=1$. Si $a=2$, erit Series $\frac{1}{1\times3}+\frac{1}{2\times4}+\frac{1}{3\times5}+$ &c. $=\frac{1}{2}\times\frac{1}{1}+\frac{1}{2}=\frac{3}{4}$; Si $a=3$, Series erit $\frac{1}{1\times4}+\frac{1}{2\times5}+\frac{1}{3\times6}+\frac{1}{4\times7}$ &c. $=\frac{1}{3}\times\frac{1}{1}+\frac{1}{2}+\frac{1}{3}=\frac{11}{18}$.

5. Ex eâdem Serie $\frac{1}{1\times1+a}+\frac{1}{2\times2+a}+\frac{1}{3\times3+a}$ $+$ &c. pro diverſo valore a oriuntur Series plures formâ ſatis elegantes, quarum nonnullas Lectori ob oculos ſiſtere, credo, ingratum non erit.

Si pro a ſumantur ſucceſſivè numeri pares, 2, 4, 6, 8, &c. Series erunt

Si $a=2$) $\frac{1}{1\times1+2}+\frac{1}{2\times2+2}+\frac{1}{3\times3+2}+\frac{1}{4\times4+2}+$ &c.

4) $\frac{1}{1\times1+4}+\frac{1}{2\times2+4}+\frac{1}{3\times3+4}+\frac{1}{4\times4+4}+$ &c.

6) $\frac{1}{1\times1+6}+\frac{1}{2\times2+6}+\frac{1}{3\times3+6}+\frac{1}{4\times4+6}+$ &c.

8) $\frac{1}{1\times1+8}+\frac{1}{2\times2+8}+\frac{1}{3\times3+8}+\frac{1}{4\times4+8}+$ &c.

Vel

Vel $\frac{1}{4-1} + \frac{1}{9-1} + \frac{1}{16-1} + \frac{1}{25-1} + \&c.$

$\frac{1}{9-4} + \frac{1}{16-1} + \frac{1}{25-4} + \frac{1}{36-4} + \&c.$

$\frac{1}{16-9} + \frac{1}{25-9} + \frac{1}{36-9} + \frac{1}{49-9} + \&c.$

$\frac{1}{25-16} + \frac{1}{36-16} + \frac{1}{49-16} + \frac{1}{64-16} + \&c.$

Vel $\frac{1}{4-1} + \frac{1}{9-1} + \frac{1}{16-1} + \frac{1}{25-1} + \&c.$

$\frac{1}{4+1} + \frac{1}{9+3} + \frac{1}{16+5} + \frac{1}{25+7} + \&c.$

$\frac{1}{4+3} + \frac{1}{9+7} + \frac{1}{16+11} + \frac{1}{25+15} + \&c.$

$\frac{1}{4+5} + \frac{1}{9+11} + \frac{1}{16+17} + \frac{1}{25+23} + \&c.$

Si pro *a* ſumantur ſucceſſive numeri impares 1, 3, 5, 7, *&c.* Series erunt

$a=1) \frac{1}{1\times1+1} + \frac{1}{2\times2+1} + \frac{1}{3\times3+1} + \frac{1}{4\times4+1} + \&c.$

$3) \frac{1}{1\times1+3} + \frac{1}{2\times2+3} + \frac{1}{3\times3+3} + \frac{1}{4\times4+3} + \&c.$

$5) \frac{1}{1\times1+5} + \frac{1}{2\times2+5} + \frac{1}{3\times3+5} + \frac{1}{4\times4+5} + \&c.$

$7) \frac{1}{1\times1+7} + \frac{1}{2\times2+7} + \frac{1}{3\times3+7} + \frac{1}{4\times4+7} + \&c.$

Vel $\frac{1}{2}\times\frac{1}{1} + \frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \&c.$

$\frac{1}{2}\times\frac{1}{3-1} + \frac{1}{6-1} + \frac{1}{10-1} + \frac{1}{15-1} + \&c.$

$\frac{1}{2}\times\frac{1}{6-3} + \frac{1}{10-3} + \frac{1}{15-3} + \frac{1}{21-3} + \&c.$

$\frac{1}{2}\times\frac{1}{10-6} + \frac{1}{15-6} + \frac{1}{21-6} + \frac{1}{28-6} + \&c.$

Vel $\frac{1}{2}\times\frac{1}{1+0} + \frac{1}{3+0} + \frac{1}{6+0} + \frac{1}{10+0} + \&c.$

$\frac{1}{2}\times\frac{1}{1+1} + \frac{1}{3+2} + \frac{1}{6+3} + \frac{1}{10+4} + \&c.$

$\frac{1}{2}\times\frac{1}{1+2} + \frac{1}{3+4} + \frac{1}{6+6} + \frac{1}{10+8} + \&c.$

$\frac{1}{2}\times\frac{1}{1+3} + \frac{1}{3+6} + \frac{1}{6+9} + \frac{1}{10+12} + \&c.$

6. Ante aliquot annos D. *Jac. Bernoulli* Geometra insignis invenit summam Seriei cujuslibet, cujus Numeratores constituunt Seriem æqualium, Denominatores verò constituunt, vel Seriem quadratorum dato aliquo quadrato Q minutorum, vel Seriem Triangulorum, dato aliquo Triangulo T minutorum. Hæc invenit ille observando quod hujusmodi Series oriantur ex ablatione Seriei Harmonicè proportionalium truncatæ ab eâdem Serie integrâ; nempe ita ut numerus terminorum deficientium in Serie truncata, sit, vel duplus lateris dati quadrati Q, vel duplus unitate auctus lateris dati Trianguli T. Idem etiam observavit frustrà quæri summam Seriei reciprocæ Quadratorum. Hoc idem etiam verum est de reciprocis Cuborum, vel aliarum quarumlibet dignitatum numerorum in progressione Arithmeticâ. Ratio est, quòd nulla intercedit differentia inter factores denominatorum, quod ad hujusmodi summationes semper requiri constat ex Methodo sumendi differentias in *Scholio Prop.* I. jam explicatâ. Nam si per formulam aliquam exhiberi posset summa quæsita, differentia istius formulæ exhiberet terminos Seriei propositæ: sed in tali differentiâ denominator semper afficitur per factores ab invicem diversos, quod quoniam in Seriebus prædictis non obtinet, summæ Serierum hujusmodi in terminis finitis haberi nequeunt. Ad eundem ferè modum, argumento petito à *Prop.* III. & IV. demonstrari potest summas Serierum exhiberi non posse in terminis numero finitis, quarum Numeratores constituunt Seriem æqualium. Denominatores vero constant ex certo numero terminorum in progressione Arithmeticâ, maximo factore cujusvis termini minore existente quàm factor minimus in termino proxime insequenti, cujusmodi est Series $\frac{1}{1.2} + \frac{1}{3.4} + \frac{1}{5.6} + \&c.$

7. Jam liceret regulas nonnullas tradere quas pro casibus quibusdam singularibus concinnavi; sed hæc nos

nos longius abducerent. Sufficiat itaque quæ generaliora sunt explicasse, & simul monuisse, ad novæ hujusce Serierum infinitarum doctrinæ provectionem nihil magis facere, quam si excogitentur formulæ generaliores summarum, ex quarum differentiis, per regulas supra traditas computatis, deinde conficiantur Canones quantitatum summabilium; ita ferè ut jam factum est in Calculo Integrali, *h. e.* in Stylo *Newtoniano*, in Methodo Fluxionum.

8. Restituendo factores in Denominatore deficientes potuisset præsens Problema revocari ad *Propositionem* II. Sed & in terminis generalioribus proponi potest, nempe pro Numeratore sumptâ quâvis Formulâ, cujus differentia aliqua datur. Sub eâ tamen conditione ut dimensiones Denominatoris ad minimum binario superent Dimensiones Numeratoris; aliàs enim summa Seriei in terminis numero finitis haberi nequit. Sit hujus rei exemplum in Serie $\frac{1}{1.3.5.7} + \frac{4}{3.4.6.8} + \frac{9}{3.5.7.9} + \frac{16}{4.6.8.10} + \&c.$ ubi Numeratores sunt numerorum naturalium quadrata. Applicando tum Numeratores tum Denominatores ad numeros naturales, Series revocatur ad formam simpliciorem $\frac{1}{3.5.7} + \frac{2}{4.6.8} + \frac{3}{5.7.9} + \frac{4}{6.8.10} + \&c.$ Per p designatis numeris naturalibus 1, 2, 3, 4, *&c.* terminus Seriei designabitur per formulam $\frac{p}{p+2 \times p+4 \times p+6}$; vel per formulam $\frac{z-2}{z \times z+2 \times z+4}$, nempe pro $p+2$ scripto z. Quoniam progrediendo de termino in terminum augetur z per unitates, restituendi sunt factores in denominatore deficientes $z+1$, $z+3$, & hoc pacto revocatur terminus Seriei ad formulam $\frac{z-2 \times z+1 \times z+3}{z \times z+1 \times z+2 \times z+3 \times z+4}$

Per methodum in hâc Propositione jam explicatam re-

 vocatur

vocatur numerator ad formam $\frac{-6-6z-z\times z+1}{+z\times z+1\times z+2}$. Unde habita ratione denominatoris Terminus revocatur ad formam: $\frac{-6}{z\times z+1\times \&c\times z+4}$ $+\frac{-6}{z+1\times z+2\times z+3\times z+4}+\frac{-1}{z+2\times z+3\times z+4}$ $+\frac{1}{z+3\times z+4}$. Adeoque sumendo Integrale fit $\frac{6}{4z\times z+1\times z+2\times z+3}+\frac{6}{3\times z+1\times z+2\times z+3}$ $+\frac{1}{2\times z+2\times z+3}+\frac{-1}{z+3}$; quo, sub signo contrario, exhibetur summa Seriei in infinitum continuatæ, incipientis à termino $\frac{z-2}{z\times z+2\times z+4}$. Summa itaque Seriei integræ incipientis à termino $\frac{1}{3\cdot 5\cdot 7}$ est $\frac{31}{240}$.

Si per *Prop.* II. procedere esset animus, ex formulâ $z-2\times z+1\times z+3$ collectis numeratoribus primis 24, 70, 144, 252, sumendo eorum differentias haberentur $46=b$, $28=c$; $6=d$, $e=0=\&c$. existente $M=24$; unde per *Lem.* 2. prodiret formula $-6-6z-z\times z+1+z\times z+1\times z+2$, quâ designatur Terminus, eadem ac supra; atque pergendo per *Prop.* II. haberetur summa.

Prop. VI. *Prob.*

Invenire summam quotlibet terminorum Seriei Fractionum, quarum Numeratores & Denominatores constituunt lineas duas quasvis transversas in Triangulo Arithmetico *Paschalii*; nempe cujus generatores sunt unitates.

Solutio. Per n designetur Ordo Seriei Numeratorum in Triangulo Arithmetico, & sit p differentia inter ordinem Numeratorum & Denominatorum, & per q designetur numerus terminorum quorum summa requiritur

quiritur. Tum ſi Denominatores ſint plurium dimenſionum quàm ſunt Numeratores, Summa exhibebitur per formulam primam ſequentem; ſi dimenſiones Numeratorum plures ſint quàm dimenſiones Denominatorum, Summa exhibebitur per formulam ſecundam.

Formula I.

$$\frac{n+p-1}{p-1} - \frac{n.n+1.n+2.\&c.n+p-1}{p-1\times n+q.n+q+1.\&c.n+q+p-2}$$

Formula II.

$$-\frac{n-p-1}{p+1} + \frac{q+n-1.q+n-2.\&c.q+n-p-1}{p+1\times n-1.n-2.\&c.n-p}$$

Ex. 1. Inveniendum ſit aggregatum ſex primorum terminorum Seriei $\frac{1}{1}+\frac{4}{7}+\frac{10}{28}+\frac{20}{84}+\frac{35}{210}+\frac{56}{462}+\&c.$ ubi Numeratores conſtituunt lineam quartam, Denominatores conſtituunt lineam ſeptimam in Triangulo Arithmetico. Sunt itaque $n=4$, $p=3$; $q=6$; & quoniam dimenſiones Denominatorum ſuperant dimenſiones Numeratorum, dabitur ſumma per Formulam primam; nempe $\frac{4+3-1}{3-1}-\frac{4.5.6}{3-1\times4+6\times4+7}$ ſive $3-\frac{6}{11}=2\frac{5}{11}$.

Ex. 2: Quæratur ſumma ſex primorum terminorum Seriei $\frac{1}{1}+\frac{7}{4}+\frac{28}{10}+\frac{84}{20}+\frac{210}{35}+\frac{462}{56}+\&c.$ cujus termini ſunt terminorum Seriei prioris reciproci. Sunt itaque $n=7$, $p=3$, $q=6$, adeoque per formulam ſecundam ſumma fit $-\frac{3}{4}+\frac{12.11.10.9}{4\times6.5.4}=24$.

Scholium 1. Formulas in hac propoſitione exhibitas ante biennium communicavi cum Viris celeberrimis *Moivreo* & *Bernoullîis*. Facile autem derivari poſſunt ex præceptis in *Prop.* I. traditis. Sit exemplum in Serie priori $\frac{1}{1}+\frac{4}{7}+\frac{10}{28}+\&c.$ Per p deſignato loco

Ter-

Termini in Serie hâc, exhibetur Terminus per formulam $\frac{4 \cdot 5 \cdot 6}{p+3 \cdot p+4 \cdot p+5}$; Unde regrediendo ad Integrale, ſumma Seriei incipientis à termino illo exhibetur per formulam $\frac{4 \cdot 5 \cdot 6}{2 \times p+3 \times p+4}$; adeoque pro p ſumpto 1, Series integra fit $\frac{4 \cdot 5 \cdot 6}{2 \cdot 4 \cdot 5} = 3$, atque ſumma primorum ſex terminorum fit $3 - \frac{4 \cdot 5 \cdot 6}{2 \cdot 10 \cdot 11}$, omninò ut per formulam jam exhibetur.

2. In formulâ primâ ſumma Seriei in infinitum continuatæ eſt $\frac{n+p-1}{p-1}$, evaneſcente jam parte alterâ formulæ. Sed in caſu formulæ ſecundæ ſumma hæc eſt infinitum quid, cujus ſpecies, reſpectu numeri infiniti q, exhibetur per formulæ partem alteram, quæ in hoc caſu fit $\frac{q^p+1}{p+1 \times n-1 \cdot n-2 \cdot \&c. \, n-p}$.

3. De hujuſmodi Seriebus in epiſtolâ datâ menſe *Maio* 1716, ſic ad me ſcripſit Vir. Ill. *D. Leibnitius*, quem magno Scientiarum damno nobis nuper ereptum lugemus. " Il me ſemble qu'autrefois j'ay auſſi ſommé " quelques Series ou ſuittes comme $\frac{1}{1} + \frac{3}{4} + \frac{3}{10} + \frac{4}{20}$ " $+ \frac{5}{35} + \frac{6}{56} + \&c.$ Le terme de cette ſuitte exprimé " Analytiquement eſt $\frac{x}{x \cdot x+1 \cdot x+2 \times \frac{1}{1} \cdot \frac{1}{2} \cdot \frac{1}{3}}$ " $= \frac{1 \cdot 2 \cdot 3}{x+1 \cdot x+2} = \frac{6}{xx+3x+2}$. On demande donc " la ſomme d'une ſuitte donneé, dont un terme ſoit " $\frac{ll}{xx+3lx+2ll}$ ou x ſignifie les nombres naturales " 1, 2, 3, 4, *&c.* & l ſignifie l'Unité, ou la difference " des x. Suppoſons que le terme de la ſuitte ſom-

" matrice

" matrice demandée ſoit $\frac{fx}{mx+nl} = \frac{\odot}{\mathfrak{D}}$. Or Diff. $\frac{\odot}{\mathfrak{D}} =$
" $-\frac{\odot}{\mathfrak{D}} + \frac{\odot + d\odot}{\mathfrak{D} + d\mathfrak{D}} = \frac{\mathfrak{D}\, d\odot - \odot\, d\mathfrak{D}}{\mathfrak{D}\,\mathfrak{D} + \mathfrak{D}\, d\mathfrak{D}}$: ſed $d\odot = f\,dx$;
" & $d\mathfrak{D} = m\,dx = m\,l$; donc la Difference de $\frac{\odot}{\mathfrak{D}}$ eſt =
" $\frac{nfll}{\begin{array}{l}mmxx + 2mnlx + nnll \\ + \ mmlx + mnll\end{array}}$. Maintenant il faut faire
" $\frac{nfll}{\begin{array}{l}mmxx + 2mnlx + nnll \\ + \ mmlx + mnll\end{array}} = \frac{mfll}{mmxx + 3mmlx + 2mmll}$
" c'eſt a dire, il faut identifier ces deux formules, ou la
" donnée eſt Multipliée per $\frac{nf}{mm}$: donc égalant les
" termes reſpectifs, puiſque les xx conviennent, on
" aura par les x, $2n + m = 3m$, c'eſt adire il y aura
" $m = n$, & par les abſolus on aura $nn + mn = 2mm$,
" ce qui donne encore $m = n$; donc l'identification
" reuſſit, & nous pouvons faire $n = m = l = 1$, &
" $f = 1$ (car f demeure arbitraire) & le terme de la
" ſuitte ſommatrice ſera $\frac{x}{x+1}$, car diff. $\frac{x}{x+1}$ donne
" $-\frac{x}{x+1} + \frac{x+1}{x+2} = \frac{1}{xx+3x+2}$, & par conſequente
" $\frac{6x}{x+1}$ donne la ſomme des $\frac{x}{x.x+1.x+2 \times \frac{1}{1}.\frac{1}{2}.\frac{1}{3}}$
" 3, 4, $\frac{9}{2}$, $\frac{24}{5}$, 5, $\frac{36}{7}$, *&c. Series ſummatrix, cujus ter-*
" *minus* $\frac{6x}{x+1}$.
" $\frac{1}{1} + \frac{2}{4} + \frac{3}{10} + \frac{4}{20} + \frac{5}{35} +$ *&c. Series ſummanda, cu-*
" *jus terminus* $\frac{x}{x.x+1.x+2 \times \frac{1}{1} \times \frac{1}{2}.\frac{1}{3}}$ Et pour
" ſ'en ſervir aux ſommations, les 5 termes, *par Ex.* de

" la ſuitte donnée ſeront $\frac{36}{7} - 3 = \frac{15}{7}$. Et generalle-
" ment la ſomme des termes juſqu'a quelque terme
" $\frac{x}{x \cdot x+1 \cdot x+2 \times \frac{1}{1} \cdot \frac{1}{2} \cdot \frac{1}{3}}$ excluſivement, ſera $\frac{6x}{x+1}$
" -3: Et pour la ſomme de la ſuitte entiere a l'infi-
" nie, x devient infini, & $\frac{6x}{x+1} = 6$: donc la ſomme
" de toute la ſuitte eſt $6 - 3 = 3$, comme vous
" l'avez trouvé.

" Cette methode eſt le calcul des differences ap-
" pliqué aux Nombres; & il faut vous avouer qu'a-
" vant que de l'appliquer aux Figures, & même avant
" que d'avoir été Geometre, Je le prattiquai en quel-
" que façon dans les nombres; ayant trouvé encore
" jeune garçon que les ſuittes dont les Numerateurs
" fuſſent des Unites, & dont les Denominateurs fuſſent
" les Nombres figurés, comme Triangulaires Pyrami-
" daux *&c.* etoient les differences 1^{eres}, 2^{es}, 3^{emes}, *&c.*
" multipliées par les conſtantes de la ſuitte $\frac{1}{1} + \frac{1}{2} + \frac{1}{3}$
" $+ \frac{1}{4} +$ *&c.* & par conſequent ſommables. Mais
" quand je devins un peu Geometre & Analyſte, Je
" vis qu'il y avoit moyen de venir a bout de telles
" ſommations par une Methode generalle, autant qu'il
" etoit poſſible; & que le calcul des differences eſtoit
" encore plus commode dans la Geometrie que dans
" les Nombres, puis qu'il y a plus d'evanouiſſements,
" & que les differences repondent aux Tangentes, les
" ſommes aux Quadratures. Cette methode generalle
" de chercher la ſuitte ſommatrice de la ſuitte donnée,
" quand elle eſt poſſible, reuſſit toujours, quand le terme
" de la ſuitte donnée exprimé Analytiquement n'a
" point la quantité variable enveloppé dans une racine,
" ny entrant dans l'expoſant; & alors, on peut tou-
" jours

" jours determiner la ſuitte ſommatrice, ou prouver " qu'il eſt impoſſible d'en trouver. Et la choſe reuſſit " méme bien ſouvent, lors méme que la variable en- " tre dans l'Expoſant. Mais comme il y a quelque- " fois des Quadratures particulieres de quelques por- " tions d'une Figure, dont ou ne ſçauroit donner la " Quadrature generalle ou la Figure quadratrice ; de " méme on peut trouver quelquefois la ſomme de " toute la ſuitte, ou d'un certaine partie, quoy qu'on " ne puiſſe pas trouver la ſomme de chaque partie ; & " alors il faut avoir recours a des Methodes particulieres, " dont on n'eſt pas toujours le maiſtre, noſtre Analyſe " n'eſtant pas encore portée a ſa perfection.

Prop. VII. *Prob.*

Invenire ſummam Seriei cujus Numeratores conſtituunt lineam quamlibet erectam in Triangulo Arithmetico *Paſchalii*, Denominatores vero conſtituunt lineam quamlibet tranſverſam.

Solutio. Deſignetur ordo lineæ erectæ per p, ordo lineæ tranſverſæ per q, & ſit m aggregatum tot terminorum primorum in lineâ erectâ ordinis $p+q-1$ quot ſunt unitates in $q-1$, atque ſumma quæſita erit $\overline{2^{p+q-2}} - m \times \frac{1.2.3.\&c.\overline{q-1}}{\overline{p.p+1.\&c.p+q-2}}$.

Ex. 1. Proponatur Series $\frac{1}{1} + \frac{5}{4} + \frac{10}{10} + \frac{10}{20} + \frac{5}{35} + \frac{1}{56}$ Ubi Numeratores conſtituunt lineam ſextam erectam, Denominatores occupant lineam quartam tranſverſam. In hoc itaque caſu ſunt $p = 6$, $q = 4$, $p+q-1 = 9$, $q-1 = 3$, adeoque $m = 1+8+28 = 37$ *i. e.* tribus terminis primis lineæ nonæ erectæ. Unde fit ſumma quæſita $\overline{2^8} - 37 \times \frac{1.2.3}{6.7.8} = \frac{219}{56}$.

Ex. 2. Conſtituant Numeratores lineam centeſimam erectam, & ſint Denominatores Numeri Trigonales, qui occupant lineam tertiam tranſverſam. Tum erunt

$p = 100$, $q = 3$, $m = 102$ atque adeo ſumma quæſita fit $\overline{2^{101} - 102} \times \frac{1 \cdot 2}{100 \cdot 101}$.

Cor. Si $q = 2$, formula fit $\frac{2^p - 1}{p}$, quâ exhibetur aggregatum primi termini, unâ cum ſemiſſe ſecundi, triente tertii, quadrante quarti, & ſic porrò, lineæ cujuſvis erectæ ordinis p Trianguli Arithmetici *Paſchalii*. Sic *v. gr.* eſt $\frac{1}{1} + \frac{5}{2} + \frac{10}{3} + \frac{10}{4} + \frac{5}{5} + \frac{1}{6} = \frac{2^6 - 1}{6} = 10\frac{1}{2}$.

Prop. VIII. *Prob.*

Invenire ſummam ejuſdem Seriei, quando terminorum ſigna ſunt alternatim + & —.

Solutio. Summa quæſita exhibetur per formulam ſimpliciſſimam $\frac{q-1}{p+q-2}$.

Ex. Invenienda ſit ſumma Seriei $\frac{1}{1} - \frac{6}{9} + \frac{15}{45} - \frac{20}{165} + \frac{15}{495} - \frac{6}{1287} + \frac{1}{3003}$, ubi Numeratores conſtituunt lineam ſeptimam erectam, Denominatores conſtituunt nonam tranſverſam. In formulâ itaque pro p & q ſcriptis 7 & 9, fit ſumma $\frac{8}{14}$.

Manente eâdem Serie Numeratorum (nempe lineâ ſeptimâ erectâ), ſi pro Serie Denominatorum ſumantur ſucceſſivè lineæ tranſverſæ 2^da, 3^tia, 4^ta, *&c.* Summæ erunt $\frac{1}{7}$, $\frac{2}{8}$, $\frac{3}{9}$, $\frac{4}{10}$, $\frac{5}{11}$, *&c.* quæ ſic poſſunt ſcribi, $\frac{1}{7}$, $\frac{7}{28}$, $\frac{28}{84}$, $\frac{84}{210}$, $\frac{210}{462}$, *&c.* ubi tam Numeratores, quàm Denominatores excerpuntur ex lineâ tranſverſâ ordinis ſeptimi. Idem eveniret ſi loco ſeptimæ, Numeratores conſtituiſſent aliam quamlibet lineam erectam ordinis p; Summæ quippe orirentur ex applicatione terminorum

lineæ

lineæ tranſverſæ ejuſdem ordinis p ad terminos proximè ſequentes in eadem lineâ.

Propoſitiones hæ duæ noviſſimæ potius elegantes ſunt quàm utiles; quare Formularum noſtrarum demonſtrationem Lectoris ſolertia inveſtigandam relinquimus, ad Propoſitionem ultimam jam properantes, quæ tertiam continet Serierum ſpeciem, ob uſum multiplicem ſatis inſignem.

Lemma 5.

Sit Series quævis $\frac{M}{h}, \frac{N}{h^2}, \frac{O}{h^3}, \frac{P}{h^4}$, *&c.* cujus terminorum Denominatores conſtituunt progreſſionem quamlibet Geometricam h, h^2, h^3, h^4, *&c.* Sint etiam Numeratorum primus $A\ (=M)$, prima differentiarum primarum B, prima ſecundarum C, prima tertiarum D, quartarum E, & ſic porrò; & ſint $\frac{\alpha}{h}, \frac{\beta}{h^2}, \frac{\gamma}{h^3}, \frac{\delta}{h^4}$, *&c.* reſpectivè, aggregata, Unius, Duorum, Trium, Quatuor, vel plurium terminorum Seriei $\frac{M}{h}, \frac{N}{h^2}, \frac{O}{h^3}$, *&c.* atque ſint Numeratorum primus $a\ (=\alpha)$ prima differentiarnm primarum b, prima ſecundarum c, prima tertiarum d, & ſic porrò: & ſit $h - 1 = q$. Tum ipſorum a, b, c, d, *&c* valores erunt.

$$a = A = \alpha = M$$
$$b = hA + B$$
$$c = qhA + hB + C$$
$$d = q^2hA + qhB + hC + D$$

& ſic porrò.

Demonſtratio.

Satis conſtat eſſe $a = \alpha = A = M$.

Termini $\frac{M}{h}, \frac{N}{h^2}, \frac{O}{h^3}, \frac{P}{h^4}$ *&c.* Numeratoribus M, N, O, P, *&c.*

&c. expreſſis per *A*, *B*, *C*, *D*, *&c.* transformantur in terminos $\frac{A}{h}$, $\frac{A+B}{h^2}$, $\frac{A+2B+C}{h^3}$, $\frac{A+3B+3C+D}{h^4}$ *&c.* Unde colligendo ſummas terminorum, inveniuntur Numeratores α, β, γ, δ, *&c.* nempe

$$\alpha = A$$
$$\beta = \overline{h+1}A + B$$
$$\gamma = \overline{h^2+h+1}A + \overline{h+2}B + C$$
$$\delta = h^3+h^2+h+1\,A + h^2+2h+3\,B + h+3\,C + D$$

&c.

Unde ſumendo differentias fiunt

$$b = hA + B$$
$$c = qhA + hB + C$$
$$d = qqhA + qhB + hC + D$$

& ſic porrò, ut in Propoſitione exhibentur.

Cor. 1. Si Numeratorum *M*, *N*, *O*, *P*, *&c.* differentia vel prima, vel ſecunda, vel alia quædam detur, terminis omnibus poſt primos aliquot in Serie *A*, *B*, *C*, *D*, *&c.* evaneſcentibus, Differentiæ *b*, *c*, *d*, *&c.* tandem incurrent in Progreſſionem Geometricam in ratione 1 ad *q*. Exempli gratiâ ſi detur Numeratorum *M*, *N*, *O*, *P* *&c.* differentia prima *B*, erunt *c*, *d*, *&c.* in ratione continuâ Geometricâ 1 ad *q*; ut conſtat per ipſorum valores $qhA + hB$, $qqhA + qhB$, *&c.* exiſtentibus $C = 0 = D = \&c.$

Cor. 2. Ordo autem primæ differentiarum *B*, *C*, *D*, *&c.* quæ hoc modo evaneſcunt, idem eſt ac ordo differentiæ vel *b*, vel *c*, *&c.* unde incipit Progreſſio illa Geometrica. Sic ſi $B = 0 = C = \&c.$ erunt *b*, *c*, *d*, *&c.* in Progreſſione Geometricâ; ſi $C = 0 = D = \&c.$ erunt *c*, *d*, *&c.* in Progreſſione Geometricâ. Et ſic porrò.

Lemma 6.

Iiſdem poſitis ſit *r* terminus unde incipit Progreſſio Geometrica in Serie differentiarum *b*, *c*, *d*, *&c.* & per

$p+1$ designetur ordo Termini in Serie $\frac{\alpha}{h}, \frac{\beta}{h^2}, \frac{\gamma}{h^3}, \frac{\delta}{h^4}$, &c. Tum Terminus ille designabitur per fractionem cujus Denominatore existente h^{p+1} Numerator est

$$\frac{a+bp+cp\times\frac{p-1}{2}+dp\times\frac{p-1}{2}\times\frac{p-2}{3}+\&c.+\frac{r}{q^n}}{\times h^p-1-qp-q^2p\times\frac{p-1}{2}-q^3p\times\frac{p-1}{2}\times\frac{p-2}{3}-\&c.}$$

nempe per n designato ordine differentiæ evanescentis in Serie B, C, D, &c. ut & Numero terminorum $a+bp$, &c. item terminorum $-1-qp$, &c.

Demonstratio. Per *Lemma* 1. Termini istius Numerator exhibetur per formulam

$$a+bp+cp.\frac{p-1}{2}+dp\times\frac{p-1}{2}.\frac{p-2}{3}+\&c.(p+1$$

subeunte vices x in Lemmate isto)

Ergò si sit, *ex. gr.* $n=2$, per *Lemm.* 5. *Cor.* 2. erunt c, d, &c. in ratione continuâ 1 ad q. Numerator itaque in hoc casu est

$$a+bp+cp\times\frac{p-1}{2}+cqp\times\frac{p-1}{2}\times\frac{p-2}{3}+cq^2p$$

$$\times\frac{p-1}{2}\times\frac{p-2}{3}\times\frac{p-3}{4}+\&c.$$ Sed si termini $cp\times\frac{p-1}{2}$

$+cqp\times\frac{p-1}{2}\times\frac{p-2}{3}+\&c.$ ducantur in $\frac{q^2}{c}$, & productui addantur termini $1+qp$, prodibit Series quâ exprimitur binomii $1+q$ dignitas $\overline{1+q}^p=h^p$. Ergo productum illud æquale est h^p-1-qp; adeoque termini $cp\times\frac{p-1}{2}+cqp\times\frac{p-1}{2}\times\frac{p-2}{3}+\&c.=\frac{c}{q^2}\times\overline{h^p-1-qp}$. Quo pacto Numerator fit $a+bp+\frac{c}{q^2}\times\overline{h^p-1-qp}$, existentibus duobus terminis $a+bp$, ut & duobus $-1-qp$, juxta sensum Propositionis, quoniam $n=2$. Atque eadem est demonstratio in aliis casibus. De Denominatore verò per se satis constat.

Prop.

Prop. IX. *Prob.*

Invenire ſummam quotlibet terminorum Seriei cujuſvis $\frac{M}{h}, \frac{N}{h^2}, \frac{O}{h^3}, \frac{P}{h^4}$, **&c. cujus terminorum Denominatores conſtituunt progreſſionem quamlibet Geometricam** h, h^2, h^3, h^4, *&c.* **Numeratores autem ſunt quantitates differentiâ aliquâ conſtanti gaudentes.**

Solutio. Sunto Numeratorum M, N, O, P, *&c.* primus A, prima differentiarum primarum B, prima ſecundarum C, prima tertiarum D, & ſic porrò; & ſit ipſorum A, B, C, D, *&c.* numerus n, atque $h - 1 = q$, Tum fiat $a = A (= M)$. $b = hA + B$, $c = qhA + hB + C$, $d = q^2 hA + qhB + hC + D$, *&c.* ut ſint tot termini a, b, c, d, *&c*, quot ſunt unitates in $n + 1$. Terminorum iſtorum ultimus dicatur r, atque per $p + 1$ deſignetur numerus terminorum $\frac{M}{h}, \frac{N}{h^2}, \frac{O}{h^3}, \frac{P}{h^4}$, *&c.* quorum ſumma requiritur; Dico ſummam illam exhiberi per fractionem, cujus Denominatore exiſtente h^{p+1}, Numerator eſt

$$\frac{a + bp + cp \times \frac{p-1}{2} + dp \times \frac{p-1}{2} \times \frac{p-2}{3} + \&c. + \frac{r}{q^n}}{\times h^p - 1 - qp - q^2 p \times \frac{p-1}{2} - q^3 p \times \frac{p-1}{2} \times \frac{p-2}{3} - \&c.}$$

$$\overline{- q^{n-1} p \times \frac{p-1}{2} \times \&c.}$$

Demonſtratio. Nam (per *Lem.* 6.) per hanc formulam repræſentatur terminus ordine $p + 1$ Seriei $\frac{\alpha}{h}, \frac{\beta}{h^2}, \frac{\gamma}{h^3}, \frac{\delta}{h^4}$, *&c.* qui terminus (per conſtructionem *Lemmatis* 5.) æqualis eſt aggregato terminorum numero $p + 1$ Seriei propoſitæ $\frac{M}{h}, \frac{N}{h^2}, \frac{O}{h^3}, \frac{P}{h^4}$. Q. E. D.

Ex. I.

Ex. 1. Invenienda ſit ſumma novem terminorum Seriei $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{8}$, $\frac{4}{16}$, *&c.* Sunt in hoc caſu $h=2$, $q(=h-1)=1$, $p+1=9$, $p=8$, $A=1$, $B=1$, $C=0$, $=D=$ *&c.* adeoque $n=2$, (quoniam ſunt duo A,B,) Hinc fit $a(=A)=1$, $b(=hA+B=2\times1+1)=3$, $c(=qhA+hB+C=2\times1+2\times1+0)=4=r$, Adeoque per formulam fit ſumma quæſita $\frac{1+3\times8+\frac{4}{1^2}\times\overline{3^8-1-1\times8}}{2^9}=\frac{1013}{512}$.

Ex. 2. Quæratur ſumma ſex terminorum Seriei $1\times3+3\times3^2+6\times3^3+10\times3^4+15\times3^5+21\times3^6+$ *&c.* In hoc caſu ſunt $h=\frac{1}{3}$, $q=\frac{-2}{3}$, $p+1=6$, $p=5$, $A=1$, $B=2$, $C=1$, $D=0=E=$ *&c.* adeoque $n=3$, atque $a=1$, $b=\frac{1}{3}+2=\frac{7}{3}$, $c=\frac{-2}{9}+\frac{2}{3}+1=\frac{13}{9}$, $d=\frac{4}{27}-\frac{4}{9}+\frac{1}{3}=\frac{1}{27}=r$. Unde ſumma quæſita fit $=1995\frac{6}{\frac{1}{3}}$. ſive $\frac{1+\frac{7}{3}\times5+\frac{13}{9}\times5\times\frac{4}{2}+\frac{-1}{8}\times\overline{\frac{1}{3^5}-1+\frac{2}{3}\times5-\frac{4}{9}\times5\times\frac{4}{2}}}{\frac{1}{3}\Big|^6}$

Cor. 1. Ejuſdem Seriei, à termino primo $\frac{M}{h}$ in infinitum continuatæ, ſumma exhibetur per formulam ſimpliciſſimam $\frac{A}{\overline{b-1}}+\frac{B}{\overline{b-1}|^2}+\frac{C}{\overline{b-1}|^3}+\frac{D}{\overline{b-1}|^4}$ *&c.*

Cor. 2. Si $h=2$, Seriei totius in infinitum continuatæ ſumma habetur ſolâ additione terminorum A, B, C, D, *&c.* Et hæc ſumma eadem eſt ac ſumma lineæ erectæ reſpondentis termino primo A, in Triangulo Arithmetico, cujus lineam tranſverſam occupant Numeratores

ratores *M*, *N*, *O*, *P*, *&c.* Quod facile conſtat ex contemplatione Trianguli. Si itaque fuerint *M*, *N*, *O*, *&c.* Numeri figurati cujuſvis ordinis *n*, ſumma Seriei $\frac{M}{2} + \frac{N}{4} + \frac{O}{8} + \frac{P}{16} + \&c.$ æqualis erit Numeri binari dignitati $\overline{2|}^{n-1}$. Sic Series $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \&c. = 2^{1-1} = 1$, ut vulgò notum; Series $\frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \&c. = 2^{2-1} = 2$; Series $\frac{1}{2} + \frac{3}{4} + \frac{6}{8} + \frac{10}{16} + \&c. = 2^{3-1} = 2^2 = 4$, & ſic porrò.

Scholium. Celeb. D. *Jac. Bernoulli*, in Tractatu ſuo de Seriebus infinitis, ſolvit illud Problema. " Invenire " ſummam Seriei infinitæ Fractionum quarum Denomi- " natores creſcunt in Progreſſione quacunque Geome- " tricâ, Numeratores verò progrediuntur vel juxta Nu- " meros naturales, 1, 2, 3, 4, *&c.* vel Trigonales 1, " 3, 6, 10, *&c.* vel Pyramidales 1, 4, 10, 20, *&c.* " aut juxta Quadratos 1, 4, 9, 16, *&c.* aut Cubos 1, " 8, 27, 64, *&c.* eorumve multiplices." Ipſius ſolutionem conſulat Lector. Aliam verò, & quidem multo generaliorem invenit D. *Nic. Bernoulli* illius Nepos, eamque (poſtquam ei hæc miſeram, ſed ſine demonſtratione) mecum communicare dignatus eſt, in epiſtolâ datâ 18° *Septembris* 1715, miris quidem inventis refertiſſimâ, qualibus me crebro dignatur vir Doctiſſimus. De hoc vero Problemate ſic ſcribit. " Pour la ſomme " d'un nombre determiné *n* de termes de la ſuitte de " voſtre Theoreme 7. [*Corollarium* primum eſt hujus Propoſitionis] j'ay trouvé cette formule $\frac{1}{m^n} \times$

$$\times \frac{m-1}{m-1}a + \frac{A-n}{m-1}b + \frac{B-n.\frac{n-1}{2}}{m-1}c + \frac{C-n.\frac{n-1}{2}.\frac{n-2}{3}}{m-1}d$$

" + *&c.* ou les Lettres *A*, *B*, *C*, *&c.* marquent " les

" les Coefficients des termes immediatement prece-
" dents. Et en mettant dans cette formule $p + 1$
" pour n, h^m pour m, & en multipliant tout encore
" par e^{m-1}, on a la ſolution de voſtre *Prob.*
" IX^{me}". Et me monuit Vir peritiſſimus hanc ſuam formulam generalem in noſtram particularem (*Cor.* 1. hujus propoſitionis) migrare quando $n = \infty$; quippe tum evaneſcunt 1, n, $n \cdot \frac{n-1}{2}$, $n \cdot \frac{n-1}{2} \cdot \frac{n-2}{3}$, *&c.* reſpectu ipſorum m^n, A, B, C, *&c.* adeo ut Series in eo caſu ſit $\frac{1}{m-1} a + \frac{A}{m-1} b + \frac{B}{m-1} c +$ *&c.* quæ omnino coincidit cum noſtrâ $\frac{a}{m-1} + \frac{b}{\overline{m-1}|^2} + \frac{c}{\overline{m-1}|^3} +$ *&c.*

Adhuc aliam hujus Problematis ſolutionem, & quidem ab hiſce admodum diverſam, invenit D. *Taylor* ope Methodi ſuæ Incrementorum. Viri doctiſſimi rogatu, ad eum miſeram formulam meam ſecundam pro ſolutione Problematis II^{di}, item formulas alias ſpectantes ad Propoſitiones tertiam, quartam & quintam, ſed ſine demonſtrationibus: quippe non dubitabam quin Vir acutiſſimus, atque ipſe Methodi iſtius Incrementorum Inventor, hiſce, vel ſaltem paribus inveniendis par eſſet. Reſcripſit ſe harum ſolutiones inveniſſe, & ſimul alia quædam communicavit ad hujus methodi profectum multum facientia, quæ jam noſtro hortatu inductus hiſce ſubjungere dignatur.

Appendix

APPENDIX

Quâ methodo diversâ eadem materia tractatur: *Auctore* Brook Taylor, *LL. D. R. S. Secr.*

HOrtatu Viri Clariſſ. cui nos innumeris officiis devinctiſſimos eſſe libenter fatemur, ſequentes jam Propoſitiones exhibemus, quas quidem in aliam occaſionem reſervandas eſſe decreviſſemus, ni æquum viſum fuiſſet parendum eſſe imperio amici qui, dum Propoſitiones quaſdam præcedentes ſuas olim nobis inveſtigandas propoſuit, earum inveniendarum occaſionem dedit.

Definitiones.

1. Quantitatis cujuſvis variabilis valorem præſentem deſigno literâ ſimpliciter ſcriptâ, ut x; valores præcedentes diſtinguo lineolis eidem literæ ex parte ſuperiori poſitis, ſequentes lineolis ex parte inferiori ſcriptis. Ut vi hujus Definitionis ſint x'', x', x, $x_{,}$, $x_{,,}$, ejuſdem variabilis valores quinque continui, exiſtente x valore præſenti, x' proximè præterito, x'' ſecundò præterito; $x_{,}$ proximè, atque $x_{,,}$ ſecundò futuro. Et ſic de aliis.

Ad eundem modum ſunt interpretandæ lineolæ quæ incrementis apponuntur. Sic ſunt $\ddot{x}$, $\dot{x}$, $x_{..}$, $x_{,..}$, $x_{,,..}$, ipſius $\dot{x}$ valores quinque continui; ut ſit $\ddot{x}$ incrementum ſecun-

ſecundum ipſius x, ſit $\dot{x}$ incrementum ſecundum ipſius $\dot{x}$. Et ſic de aliis.

Cor. Vi hujus Definitionis, $x + \dot{x} = x$, $\dot{x} + \dot{x} = \dot{x}$, $x + \dot{x} = x$. Et ſic de aliis hujuſmodi.

Quando uſu venit ut variabilis quantitas, puta x. ſpectanda ſit tanquam Incrementum, ejus Integrale deſigno literâ inter uncos [ ] inclusâ. Iſtius etiam Integralis $[x]$ Integrale (vel ipſius x Integrale ſecundum,) deſigno numero binario uncorum priori ſuperimpoſito, ut $\overset{2}{[}x]$. Iſtius etiam Integralis Integrale (vel ipſius x Integrale tertium,) ad eundem modum deſigno numero ternario, ut $\overset{3}{[}x]$. Et ſic deinceps. Unde vi hujus Definitionis conſtituunt $\overset{3}{[}x]$, $\overset{2}{[}x]$, $[x]$, x Seriem terminorum, quorum quilibet eſt ipſum immediatè præcedentis incrementum primum, ut ſit $\overset{2}{[}x] = \overset{3}{[}\dot{x}]$, $[x] = \overset{2}{[}\dot{x}]$, $x = [\dot{x}]$.

Lemma.

Facti xv ex Multiplicatione duorum variabilium x & v, incrementum eſt $\dot{x}v + x\dot{v}$.

Nam auctis variabilibus per propria incrementa, fit novum productum $\overline{x + \dot{x}} \times \overline{v + \dot{v}}$, ſive $xv + \dot{x}v + x\dot{v} + \dot{x}\dot{v}$, hoc eſt $xv + \dot{x}v + x\dot{v} + \dot{x}\dot{v}$ (pro $x + \dot{x}$ ſcripto $\dot{x}$ *per Def.* 1.)

Unde dempto priori producto xv, reſtat Incrementum $\dot{x}v + x\dot{v}$.

Prop. I. *Theor.*

Ejuſdem Facti $x\,v$ Incrementum, vel primum, vel ſecundum, vel tertium, vel aliud quodvis, cujus ordo deſignatur per ſymbolum n, exhibetur per formulam hanc generalem

$$x\,v + n\,x\,v + n \times \frac{n-1}{2}\,x\,v + n \times \frac{n-1}{2} \times \frac{n-2}{3}\,x\,v + \&c.$$

In hâc formulâ hæc ſunt obſervanda, 1^mò Terminorum numeri coefficientes 1, n, $n \times \frac{n-1}{2}$, $n \times \frac{n-1}{2} \times \frac{n-2}{3}$ &c. iidem ſunt ac in binomii dignitate n, 2^do Numeri n, $n-1$, $n-2$, $n-3$, &c. ipſis x infraſcripti deſignant numeros punctorum quibus definiuntur Incrementa. 3^tio Lineolæ $,$ $,,$ $,,,$ &c. ipſis x infraſcriptæ, interpretandæ ſunt per Def. 1. 4^to. In quovis Termino numerus punctorum ipſis x & v ſimul infraſcriptorum, eſt n. Sit *v. g.* $n = 4$: tum per formulam, ipſius $x\,v$ incrementum quartum prodit $x\,v + 4\,x\,v + 6\,x\,v + 4\,x\,v + x\,v$.

Theorema hoc generale demonſtrari poteſt per Inductionem, incrementis continuò ſumptis juxta formam in Lemmate præcedenti traditam. Sed & collectâ formâ Seriei ex hujuſmodi calculo, Theorema etiam demonſtrari poteſt per Methodum Incrementorum, ad eum modum cujus ſpecimen mox dabimus in demonſtratione Propoſitionis tertiæ.

Prop. II. *Theor.*

Ipſius $x\,v$ Integrale primum $[x\,v]$ exhibetur per Seriem $[x]\,v - \overset{2}{[x]}\,v + \overset{3}{[x]}\,v - \overset{4}{[x]}\,v$ &c.

Series autem ita terminatur, ut ſit $[x\,v] = [x]\,v$

$$- \left[\left[x \right]_{,} v \right]_{\cdot} = [x]\, v - \overset{2}{[}x]_{,} v_{\cdot} + \left[\overset{2}{[}x]_{,,} v_{\cdot\cdot} \right] = \&c.$$

Nam ſumendo incrementa reſtituitur propoſitum xv.

Cor. 1. Datis duobus ex iſtis $[x]$, $[xv]$, $\left[\left[x \right]_{,} v \right]_{\cdot}$, datur tertium. Item datis tribus ex iſtis $[x]$, $[x]_{,}$, $\overset{2}{[}x]$, $[xv]$, $\left[\overset{2}{[}x]_{,} v \right]_{\cdot}$, datur quartum, Et ſic porrò.

Cor. 2. Si $v_{\cdot} = 0$, datur $[xv]$ ex dato $[x]$. Si $v_{\cdot\cdot} = 0$ datur $[xv]$ ex datis duobus $[x]$, & $\overset{2}{[}x]$, Si $v_{\cdot\cdot\cdot} = 0$, datur $[xv]$, ex datis tribus $[x]$, $\overset{2}{[}x]$, $\overset{3}{[}x]$. Et ſic porrò.

Ex. 1. Sit exemplum hujus formulæ in inventione Integralis ipſius $\frac{v}{z z_{,} z_{,,} z_{,,,}}$, dato nempe $z_{\cdot}$, atque exiſtente $v_{\cdot\cdot} = 0$, qui caſus eſt ſpecialis Propoſitionis ſecundæ Tractatus præcedentis D^ni *Monmort*. Facto itaque $x = \frac{1}{z z_{,} z_{,,} z_{,,,}}$, ſunt $[x] = \frac{-1}{3 z_{\cdot} z_{,} z_{,,}}$, $\left[x \right]_{,} = \frac{1}{2 z_{\cdot} \times 3 z_{\cdot} z_{,} z_{,,}}$, atque $\left[x \right]_{,,} = \frac{-1}{1 z_{\cdot} \times 2 z_{\cdot} \times 3 z_{\cdot} z_{,,}}$. Unde per formulam fit $[xv]$, hoc eſt $\left[\frac{v}{z z_{,} z_{,,} z_{,,,}} \right] = - \frac{v}{3 z_{\cdot} z_{,} z_{,,}} =$

$$- \frac{v_{\cdot}}{2 z_{\cdot} \times 3 z_{\cdot} z_{,} z_{,,}} - \frac{v_{\cdot\cdot}}{1 z_{\cdot} \times 2 z_{\cdot} \times 3 z_{\cdot} z_{,,}}.$$

Ex.

Ex. 2. Sit aliud exemplum in inventione Integralis ipſius na^z, ubi eſt $z = 1$, atque datur a, Tum pro x ſumpto a^z, & pro v ſumpto n, fit $\underline{x = a^z}$ hoc eſt $\dot{x} = ax$, ſeu $x + \dot{x} = ax$, adeoque $\dot{x} = a - 1\,x$, atque $x = \frac{\dot{x}}{a-1}$. Regrediendo itaque ad Integralia fit $[x] = \frac{x}{a-1}$; item $[\overset{2}{x}] = \frac{[x]}{a-1} = \frac{x}{\overline{a-1}|^2}$, item $[\overset{3}{x}] = \frac{x}{\overline{a-1}|^3}$; & ſic porrò. Adeoque (quoniam $\dot{x} = ax$,) ſunt $[x] = \frac{x}{a-1}$, $[\dot{x}] = \frac{ax}{\overline{a-1}|^2}$, $[\ddot{x}] = \frac{a^2x}{\overline{a-1}|^3}$, *&c.* Unde per formulam prodit $[na^z] = \frac{a^z n}{a-1} - \frac{a^{z+1} n}{\overline{a-1}|^2} + \frac{a^{z+2} n}{\overline{a-1}|^3}$ *&c.*

In hoc exemplo continetur Solutio Problematis, de quo agit D^nus^ de *Monmort* in Propoſitione nona. Coincidit autem formula cum ea quam exhibet ille in Corollario primo ejuſdem Propoſitionis.

Scholium. Poſſunt etiam ex hâc formulâ alii derivari valores Integralis quæſiti, pro vario modo quo interpretantur Incrementi propoſiti factores. Sic in exemplo ſecundo integrale ipſius na^z exhiberi poteſt per formulam $a^z[n] - \overline{a-1}\,a^z[\dot{n}] + \overline{a-1}|^2\,a^z[\ddot{n}] - \&c.$ pro x nempe ſumpto n, & pro v ſumpto a^z. Sed de his fortaſſe aliâ occaſione fuſius dicemus.

Prop. III. *Theor.*

Ejuſdem xv Integrale, vel primum, vel ſecundum, vel tertium, vel aliud quodvis cujus ordo deſignatur ſymbolo n, exhibetur per Seriem in hâc formâ generali prodeuntem $[\overset{n}{xv}] = [\overset{n}{x}]\,v - n\,[\overset{n+1}{x}]\,\dot{v} +$

$+ n\,x$

$$+ n \times \frac{n+1}{2} \overset{n+2}{[} \overset{}{x} \overset{}{]} \overset{v}{..} - n \times \frac{n+1}{2} \times \frac{n+2}{3} \overset{n+3}{[} \overset{}{x} \overset{}{]} \overset{v}{...} + \&c.$$

Collectâ formâ Seriei ex Propoſitione præcedenti, Coefficientes $1, -n, n \times \frac{n+1}{2}, -n \times \frac{n+1}{2} \times \frac{n+2}{3}$, &c. ſic inveniuntur per Methodum Incrementorum. Pone

$$\overset{n}{[}x\,v] = A\overset{n}{[}x]v + B\overset{n+1}{[}x]\overset{v}{.} + C\overset{n+2}{[}x]\overset{v}{..} + D\overset{n+3}{[}x]\overset{v}{...} + \&c.$$

Tum aucto n incremento ſuo $\dot{n} = 1$, atque ipſis A, B, C, D, &c. incrementis ſuis contemporaneis $\dot{A}, \dot{B}, \dot{C}, \dot{D}$, &c. ut jam evadant n, A, B, C, D, &c. fiet novum

Integrale (quod Integrale eſt ipſius $\overset{n}{[}x\,v]$,) $\overset{n+1}{[}x\,v] = A\overset{n+1}{[}x]v + B\overset{n+2}{[}x]\overset{v}{.} + C\overset{n+3}{[}x]\overset{v}{..} + D\overset{n+4}{[}x]\overset{v}{...} + \&c.$ Hujus itaque Incrementum primum coincidere debet cum Integrali prius poſito. Sumptis ergo incrementis, fit

$$\overset{n}{[}x\,v] = A\overset{n}{[}x]v + A\overset{n+1}{[}x]\overset{v}{.} + B\overset{n+2}{[}x]\overset{v}{..} + C\overset{n+3}{[}x]\overset{v}{...} + \&c.$$
$$+ B \quad + C \quad + D$$

idem ac Integrale prius poſitum. Itaque terminos homologos inter ſe comparando fit 1^{mo} $A = A$. Unde eſt A datum quid. Sed ubi $n = 0$, eſt $A = 1$, ergo $A = 1$. 2^{do}. $B = B + A$, hoc eſt $B = B + \dot{B} + 1$, ſeu $\dot{B} = -1 = -n$. Ergo regrediendo ad Integralia, fit $B = -n + a$. Sed ubi $n = 0$, eſt $B = 0$. Ergo $a = 0$, atque $B = -n$. 3^{tio}. $C = C + B$, hoc eſt $\dot{C} = n$. Regre-

diendo itaque ad Integralia fit $C = \frac{n\,n}{2} + b$. Sed ubi $n = 0$, eſt $C = 0$. Ergo $b = 0$, atque $C = \frac{n\,n}{2}$, hoc eſt, $n \times \frac{n+1}{2}$. 4^to. Ad eundem modum invenitur $D = -n \times \frac{n+1}{2} \times \frac{n+2}{3}$. Et ſic pergendo inveniuntur cæteri Coefficientes.

Scholium. 1. In hâc Propoſitione comparatâ cum Propoſitione primâ, cernitur ſingularis quædam relatio Incrementa inter & Integralia. Ut enim in Arithmeticâ vulgari, Multiplicatio & Diviſio ſunt invicem ita contrariæ. ut ſi Multiplicatio deſignetur per Indicem affirmativum, Diviſio deſignabitur per Indicem cum ſigno negativo; ſic etiam in Methodo Incrementorum, ſi Incrementum deſignetur per Indicem affirmativum, Index negativus Integrale ſiſtet. Sic in Propoſitione primâ, ſi pro n ſumatur Numerus binarius 2, per formulam exhibebitur ipſius $x\,v$ incrementum ſecundum, nempe $\ddot{x}\,v + 2\dot{x}\,\dot{v} + x\,\ddot{v}$; Sed ſi pro n ſumatur numerus negativus -2, ut jam quæratur ipſius $x\,v$ incrementum (ita loqui liceat) negativè ſecundum, (quod idem eſt ac Integrale ſecundum) prodeunt coefficientes iidem ac ſi ſumatur n affirmativè in Propoſitione præſenti: atque interpretatis inſuper ipſis x, $\dot{x}$, $\ddot{x}$, &c. per $[x]^{-2}$, $[\dot{x}]^{-3}$, $[\ddot{x}]^{-4}$, &c. Series fit omnino eadem ac per Propoſitionem præſentem prodit, ubi quæritur Integrale ſecundum.

2. Ex his autem formulis quaſi ſuâ ſponte procedunt formulæ Propoſitionum undecimæ atque duodecimæ Libri de Methodo Incrementorum. Nam pro incre-

incrementis ſcribe Fluxiones, atque evaneſcentibus incrementis fiant jam omnes x, $\dot{x}$, $\ddot{x}$, $\dddot{x}$, &c. inter ſe æquales, atque migrabit ſtatim hæc Propoſitio ſecunda in illam undecimam, atque præſens tertia in illam duodecimam. Quod quidem exemplum ſatis inſigne eſt Methodi *Newtonianæ*, quâ colligit ille rationes Fluxionum ex rationibus ultimis Incrementorum evaneſcentium, vel ex primis naſcentium.

Additamentum.

PRæcedentium impreſſioni intentus dum Typothetarum erroribus corrigendis do operam, atque eâ occaſione in animo illa ſœpius revolvo, ſubiit Artificium illud quo jam olim uſus eſt D. *Jac Bernoulli* in inventione quarundam Serierum, ope Progreſſionis Harmonicæ cujus meminit D. de *Monmort* in *Scholio* 6. *Prop.* V. præcedente commodè etiam applicari poſſe ad inventionem ipſius *Monmortii* Propoſitionum $2^{dæ}$, $3^{tiæ}$, $4^{tæ}$, $5^{tæ}$, atque id genus aliarum aliquanto fortaſſe generaliorum. Hoc in ſequentibus paucis oſtendiſſe, credebam Lectori non fore ingratum.

Theorema.

Sit Progreſſio Arithmetica p, $p + n$, $p + 2n$, &c. cujus termini ſinguli ſucceſſivè deſignentur per x, & ſunto b, c, d, &c. quivis multiplices differentiæ datæ n terminorum Progreſſionis iſtius Arithmeticæ. Sint A, B, C, D, &c. Numeri quilibet dati, & conſtituantur fractiones quotvis $\frac{A}{x}$, $\frac{B}{x+b}$, $\frac{C}{x+c}$, $\frac{D}{x+d}$, &c. Pro x ſucceſſive ſcriptis valoribus ſuis p, $p + n$, $p + 2n$, &c. ex

ex harum fractionum quâlibet, oritur Series Harmonicè proportionalium Sic *v g.* ex fractione primâ $\frac{A}{p}$, oritur Series $\frac{A}{p}$, $\frac{A}{p+n}$, $\frac{A}{p+2n}$, *&c.* Dico quod aggregatum quotlibet hujuſmodi Serierum in infinitum continuatarum in terminis numero finitis exhiberi poteſt, ſi modo fuerit numeratorum *A, B, C, D, &c.* aggregatum æquale nihilo. Duobus exemplis hoc fiet manifeſtum.

Ex. Sint duæ tantùm fractiones $\frac{A}{x}$, atque $\frac{-A}{x+3n}$, exiſtente $b = 3n$. Scribantur Series harmonicæ ex his formulis ortæ, eo ordine, ut termini, in quibus ſunt denominatores æquales, ſibi invicem reſpondeant, & collectis ſummis terminorum homologorum, prodibit aggregatum Serierum in terminis numero finitis, ut in calculo appoſito videre eſt.

$$\frac{A}{p}+\frac{A}{p+n}+\frac{A}{p+2n}+\frac{A}{p+3n}+\frac{A}{p+4n}+\&c. = \text{Seriei ortæ ex } \frac{A}{x}$$

$$+\frac{-A}{p+3n}+\frac{-A}{p+4n}+\&c. = \text{Seriei ex } \frac{-A}{x+3n}$$

$$\frac{A}{p}+\frac{A}{p+n}+\frac{A}{p+2n}+\ 0\ +\ 0\ +\&c. = \text{Aggreg. Serierū.}$$

Ex. 2. Sint tres fractiones $\frac{A}{x}$, $\frac{B}{x+2n}$, $\frac{C}{x+3a}$, exiſtentibus $b = 2n$, $c = 3n$, atque $A+B+C = 0$. In hoc caſu Calculus ſic ſe habet.

$$\frac{A}{p}+\frac{A}{p+n}+\frac{A}{p+2n}+\frac{A}{p+3n}+\ldots+\&c. = \text{Seriei ortæ ex } \frac{A}{x}$$

$$+\frac{B}{p+2n}+\frac{B}{p+3n}+\ldots.+\&c. = \text{Seriei ex } \frac{B}{x+2n}$$

$$+\frac{C}{p+3n}+\ldots..+\&c. = \text{Seriei ex } \frac{C}{x+3n}$$

$$\frac{A}{p}+\frac{A}{p+n}+\frac{A+B}{p+2n}+\frac{A+B+C=0}{p+3n}+\&c. = \text{Aggregato Serierum.}$$

Ubi

Ubi etiam prodit aggregatum Serierum in terminis numero finitis, nempe $\frac{A}{p} + \frac{A}{p+n} + \frac{A+B}{p+2n}$, ob Numeratorum A, B, C, aggregatum æquale nihilo. Et ad eundem modum demonſtratur Theorema in aliis caſibus quibuſvis.

Cor. 1. Ex his principiis derivari poſſunt innumeræ Series in infinitum continuatæ, in terminis tamen numero finitis ſummabiles.

Caſ. 1. Sint $\frac{A}{x}$ & $\frac{-A}{x+b}$ formulæ duarum Serierum harmonicarum quarum aggregatum prodit in terminis numero finitis per ſuperius demonſtrata, Tum, formulis iſtis in unam ſummam collectis, fit $\frac{Ab}{x \times x+b}$ formula Seriei ſummabilis. Sint *v. gr.* $A = \frac{1}{6}$, $p = 1$, $n = 2$, atque $b = 3n = 6$. Tum formulæ Serierum harmonicarum erunt $\frac{1}{6x}$, & $\frac{-1}{6 \times x+6}$, formula Seriei compoſitæ ſummabilis erit $\frac{1}{x \times x+6}$, Serie illa exiſtente $\frac{1}{1 \times 7} + \frac{1}{3 \times 9} + \frac{1}{5 \times 11} + \frac{1}{7 \times 13} + \&c.$ atque ſumma Seriei, per calculum in præmiſſis demonſtratum, erit $\frac{1}{6 \times 1} + \frac{1}{6 \times 3} + \frac{1}{6 \times 5}$. Sint tres formulæ Serierum harmonicarum $\frac{A}{x}$, $\frac{B}{x+b}$, $\frac{C}{x+c}$, (exiſtente $A+B+C=0$, ut ſit Serierum aggregatum finitum per præmiſſa.) Tum formulis in unam ſummam collectis fit $\frac{A \times x+b \times x+c + B \times x \times x+c + C \times x \times x+b}{x \times x+b \times x+c}$, ſeu (terminis revocatis ad formam factorum x, $x \times x+b$, $x \times x+b \times x+c$,) $\frac{Acb + Ac + c-b\,B \times x + A+B+C \times x \times x+b}{x \times x+b \times x+c}$, hoc eſt

(ob $A+B+C=0$) $\frac{Acb+\overline{Ac+B\times c-b}\times x}{x\times x+\overline{b\times x+c}}$, formula Seriei ſummabilis. Si quatuor ſint Fractiones $\frac{A}{x}$, $\frac{B}{x+b}$, $\frac{C}{x+c}$, $\frac{D}{x+d}$, (exiſtente $A+B+C+D=0$) ad eundem modum invenietur formula Seriei ſummabilis $\frac{\overline{Abcd+Acd+B\times c-b}\times d-b\times x+\overline{Ad+B\times d-b+C\times d-c}\times x\times x+b}{x\times x+\overline{b\times x+c}\times x+d}$

Et ſic pergere licet ad formulas adhuc magis compoſitas.

Caſ. 2. Et ſi plures ſint formulæ Serierum hujuſmodi ſummabilium, quarum denominatorum factores excerpantur ex diverſis progreſſionibus Arithmeticis, ex iſtarum formularum quotvis in unam ſummam additione, conficietur formula nova Seriei ſummabilis. Sint *e. gr.* formulæ duæ Serierum ſummabilium $\frac{1}{x\times x+3}$ & $\frac{1}{z\times z+2}$, excerptis x ex Progreſſione Arithmeticâ 1, 2, 3, 4, *&c.* z ex Progreſſione Arithmeticâ 1, 3, 5, *&c.* Tum ex his formulis in unam ſummam collectis fiet formula nova $\frac{z\times\overline{z+2}+x\times\overline{x+3}}{\overline{x\times x+3}\times z\times\overline{z+2}}$, vel, (expoſito z per x & numeros datos) $\frac{\overline{2x-1\times 2x+1}+x\times\overline{x+3}}{\overline{x\times x+3}\times\overline{2x-1\times 2x+1}}$

Cor. 2. Hinc omnis Series in infinitum continuata ſummabilis eſt, cujus termini deſignantur per Fractionem, cujus denominatoris factores excerpuntur ex datâ quâlibet Progreſſione Arithmeticâ, numerator autem eſt multinomium, cujus dimenſiones ſunt ad minimum binario pauciores, quam ſunt dimenſiones Denominatoris. Nam omnis hujuſmodi fractio reſolvi poteſt in tot fractiones ſimplices, quot ſunt dimenſiones (hoc eſt, quot ſunt factores) Denominatoris, quarum numeratorum aggregatum eſt nihil. Sit exempli gratiâ,

formula

formula oblata $\frac{\alpha+\beta x+\gamma x\times\overline{x+b}}{x\times\overline{x+b}\times\overline{x+c}\times\overline{x+d}}$. Pone hanc formulam æquari aggregato fractionum $\frac{A}{x}+\frac{B}{x+b}+\frac{C}{x+c}+\frac{D}{x+d}$. Tum fractionibus istis in unam summam collectis fiet $\frac{Abcd+\overline{Acd+Bc-b\times d-b}\times x}{+\overline{Ad+B\times d-b}+C\times\overline{d-c}\times x\times\overline{x+b}}$ $+\overline{A+B+C+D\times x\times x+b}\times\overline{x+c}$ applicatum ad $x\times\overline{x+b}\times\overline{x+c}\times\overline{x+d}=\frac{\alpha+\beta x+\gamma x\times\overline{x+b}}{x\times\overline{x+b}\times\overline{x+c}\times\overline{x+d}}$.

Unde per comparationem terminorum homologorum fit $Abcd=\alpha$, $Acd+B\times c-b\times d-b=\beta$, $Ad+B\times d-b+C\times d-c=\gamma$, $A+B+C+D=0$. adeoque $A=\frac{\alpha}{bcd}$, $B=\frac{\beta-Acd}{c-b\times d-b}$, $C=\frac{\gamma-Ad-B\times d-b}{d-c}$, $D=-A-B-C$, Quo pacto formula oblata resolvitur in fractiones simplices $\frac{\alpha}{bcdx}$ $+\frac{\beta-Acd}{c-b\times d-b\times x+b}+\frac{\gamma-Ad-B\times d-b}{d-c\times x+c}$ $+\frac{-A-B-C}{x+d}$, ex quibus ortarum Serierum aggregatum, hoc est, summa Seriei ortæ ex formulâ oblatâ $\frac{\alpha+\beta x+\gamma x\times\overline{x+b}}{x\times\overline{x+b}\times\overline{x+c}\times\overline{x+d}}$, per jam dicta prodit in terminis numero finitis. Quod verò dimensiones numeratoris in formulâ oblatâ, debeant esse binario ad minimum pauciores, quam sunt dimensiones Denominatoris, hinc constat, quòd in reductione fractionum $\frac{A}{x}$, $\frac{B}{x+b}$, $\frac{C}{x+c}$, $\frac{D}{x+d}$, quilibet numerator $A, B, C, D,$

ducitur

ducitur in omnes denominatores excepto uno, nempe ſuo; unde prodeunt Numeratoris Dimenſiones unitate pauciores quam ſunt dimenſiones Denominatoris. Sed per æquationem $A+B+C+D=0$ perit altiſſima dimenſio in numeratore; Unde ſuperſunt Numeratoris Dimenſiones ad minimum binario pauciores quam ſunt dimenſiones Denominatoris. Ad hoc verò Corollarium revocari poſſunt D. de *Monmort* Propoſitiones 2^da & 5^ta.

Cor. 3. Item oblatâ formulâ juxta *Caſ.* 2. *Cor.* 1. adhuc magis compoſitâ, ex iidem principiis perſpici poteſt an ſit Series ſummabilis. Sint progreſſiones duæ Arithmeticæ 1, 3, 5, *&c.* 2, 4, 6, *&c.* quarum termini homologi deſignentur per x & z, & ſit formula Seriei oblata $\frac{\alpha+\beta x+\gamma x^2}{x\times x+2\times z\times z+2}$, vel (pro z ſcripto $x+1$, & factoribus Denominatoris in ordinem coactis) $\frac{\alpha+\beta x+\gamma x^2}{x\times x+1\times x+2\times x+3}$. Pone formulam hanc æquari aggregato formularum $\frac{P}{x\times x+2}$, $\frac{Q}{x+1\times x+3}$, Serierum per ſuperius dicta ſummabilium, ut (formulis his noviſſimis in unam ſummam collectis) ſit $\frac{P\times x+1\times x+3+Q\times x\times x+2}{x\times x+1\times x+2\times x+3}$ ſeu $\frac{3P+4P+2Q x+P+Q x^2}{x\times x+1\times x+2\times x+3}=\frac{\alpha+\beta x+\gamma x^2}{x\times x+1\times x+2\times x+3}$. Hinc comparando terminos homologos oriuntur æquationes $3P=\alpha$, $4P+2Q=\beta$, $P+Q=\gamma$. Unde eliminatis P & Q per debitas operationes Analyticas, prodit æquatio $2\alpha-3\beta+\gamma=0$, qua definitur relatio quæ inter coefficientes α, β, γ intercedere debet, ut Series orta ex formulâ oblatâ $\frac{\alpha+\beta x+\gamma x^2}{x\times x+1\times x+2\times x+3}$

ſit

ſit ſummabilis. Ad eundem modum ſi formulæ oblatæ Denominatoris factores excerpantur ex tribus Progreſſionibus Arithmeticis, invenientur duæ æquationes quibus definiantur relationes coefficientium Numeratoris, ut ſit Series ſummabilis. Si quatuor ſint Progreſſiones Arithmeticæ, Coefficientium relatio definietur per tres æquationes. Et ſic porrò. Et in hujuſmodi formulis ut ſint Series ſummabiles, hæc inſuper obſervanda ſunt, Primò ut Numeratorum dimenſiones ſint ad minimum binario pauciores quam ſunt dimenſiones Denominatorum, Deinde ut ex ſingulis Progreſſionibus Arithmeticis excerpantur ad minimum duo factores Denominatoris. Denique, quod ſi ſint duo vel plures factores Denominatoris inter ſe æquales, ponendum ſit tot etiam Progreſſiones Arithmeticas, ex quibus excerpuntur, eſſe inter ſe æquales. Præmiſſis attentius perpenſis, hæc obvia erunt. Ad hoc vero Corollarium facile revocantur D. de *Monmort* Propoſitiones 3^{tia} & 4ta.

F I N I S.

ERRATUM in N°. 352.

PAge 586, after the end of line 15, add *black Cloud, from behind which there iſſued a.*

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Jos: Banks

PHILOSOPHICAL TRANSACTIONS.

For the Months of *Octob. Nov.* and *Decemb.* 1717.

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I. *An Advertiſement to* Aſtronomers, *of the Advantages that may accrue from the Obſervation of the* Moon's *frequent Appulſes to the* Hyades, *during the Three next enſuing Years.*

OF all the Methods hitherto propoſed for finding the *Longitudes* of Places for Geographical Uſes, none ſeems more adapted to the purpoſe, than that by the *Occultations* of the fixt Stars by the *Moon* obſerved in diſtant Parts: For thoſe Immerſions of the Stars which happen on the dark Semicircle of the Moon, and their Emerſions from the ſame, are perfectly momentaneous, without that Ambiguity, to which the Obſervations of the Eclipſes of the *Moon* and thoſe of *Jupiter*'s *Satellites* are ſubject. Beſides, whilſt the Moon is horned, and her weaker Light leſs dazling, an ordinary ſhort Teleſcope, ſuch as by Experience is found to be manageable on Ship-board, ſuffices to obſerve thoſe Moments, even in the Occultations of very minute Stars: On which account, this way ſeems to bid faireſt for the deſired Solution of the grand Problem of finding the Longitude at ſea. But ſince it would be needleſs to enquire exactly what Longitude a Ship is in, when that of the Port to which ſhe is bound is ſtill unknown it were to be wiſht that the Princes of the Earth would cauſe ſuch Obſervations to be made, in the Ports and on the principal Head-Lands of their Dominions, each for his own, as might

might once for all ſettle truly the Limits of the Land and Sea. This Work however being likely to be left to the Care and Curioſity of private Perſons, it may not be amiſs hereby to give notice of the preſent Opportunity of performing it, in this our Northern Hemiſphere, by help of the frequent Appulſes of the Moon to the more Southerly of the *Hyades*, many of which ſhe eclipſes in each monthly Revolution, and will continue ſo to do, during the Years 1718, 1719, and 1720.

Theſe Stars are but Three or Four in all former Catalogues, but the *Britiſh* of Mr *Flamſteed* encreaſes them to Sixteen; to them we have added Three others ſomewhat ſmaller, *viz.* *c*, *i*, and *n* in the Figure of the *Hyades* hereto annext. In it the principal Stars are markt with *Bayer*'s Marks, and the reſt with the Letters of the *Italick* Alphabet; their Longitudes are fitted to the beginning of the Year 1718, and being truly laid down, may ſerve to inſtruct the curious Obſerver, when and where to look for them, when the Moon is among them.

It appears by this Scheme that the Diſtance between *a* and *α* or *Palilicium*, is about Nine Hours Motion of the Moon, in which time ſuppoſing her to paſs from one to the other, ſhe muſt eclipſe *γ* and *e*, and Four or Five of thoſe about *θ*, and muſt apply very cloſe, with her Southern Limb, to all thoſe which have about Six Degrees South-Latitude; which would be a very entertaining Sight for the Lovers of theſe Arts. But if the Times of the Occultations of any One of theſe Stars, or even of any Two of them in the ſame Night, be accurately obſerved under diſtant Meridians, the difference of thoſe Meridians may be truly obtain'd thereby; eſpecially ſince the *Moon*'s *Parallax*, and all other parts of her Theory thereto required, are at preſent ſufficiently ſtated and known.

For the ſake of ſuch as are willing to make uſe of this Method, we have added the Places of all the *Hyades* fitted to the preſent Time, and chiefly taken from the *Britiſh* Catalogue, which being faulty in the Stars we call *k* and *l*, we have here rectified them.

Catalogus

Catalogus Hyadum, *ineunte Anno* 1718.

Stellarum NOMINA.		Long. ♊			Lat. Auſt.			Ma.
Quæ præcedit γ Tauri ——	a	0	51	3	5	50	14	7
In naribus Tauri, Bayero	γ	1	50	54	5	46	22	3
Quæ ſub γ —— ——	b	1	56	31	6	19	57	7
In Origine Naſi Tauri ——	c	2	54	25	4	47	5	7
Inter nares & oculum Tauri Boreum —— ——	δ	2	54	47	4	0	34	3 7
Huic contigua ad Auſtrum.···	d	3	10	33	4	9	4	6
Præcedentium θ Borealis ——	e	3	17	21	5	41	50	8
Earum Auſtralis clarior. ——	f	3	25	32	6	2	44	6
Quæ ſequitur δ —— ——	g	3	35	2	3	43	27	5
Contiguarum inter nares & Palilicium Borea ——	θ	3	59	45	5	47	16	4
Earundem Auſtralior ——	θ	4	0	11	5	52	55	4
Duarum ſupra θ Borea ——	h	4	2	32	5	23	43	7
Earundem Auſtralior ——	i	4	7	44	5	36	40	8
Sub θ trium in recta præcedens	k	4	19	27	6	9	45	4 7
Earum media —— ——	l	4	26	55	6	7	35	5 7
2^rum^ *θ ſequentium Borea* ——	m	4	30	26	5	37	49	7
Oculus Boreus Tauri ——	ε	4	30	31	2	35	58	3
Sequentium θ Auſtralis ——	n	4	32	35	5	41	00	8
Trium ſub θ ſequens ——	o	4	45	55	6	0	35	7
Palilicium Bayero	α	5	50	20	5	29	50	1
Quæ hanc ſequitur proxime —	p	6	17	35	6	3	20	7
Contig. ſequentium Auſtralis —	q	6	30	34	6	19	19	6
Borea & clarior —— ——	σ	6	33	12	6	12	35	7

II. *Solutio Problematis à Dom^no G. G. Leibnitio, Geometris Anglis nuper propositi. Per Brook Taylor, LL. D. & R. S. Secr.*

CUM Dom. *G. G. Leibnitius* nuper defunctus, in controversiâ jampridem ortâ circa Inventionem Methodi Fluxionum, (quam is Differentialem vocare maluit, sibique pertinaciter appropriari nisus est,) nihil omnino responsi dederit argumentis, quibus inclyti istius Inventi gloria Dom^no *Neutono* vendicatur ; en tandem, hortante Dom^no *Joh. Bernoulli*, Problema Geometris *Anglis* solvendum proposuit ; quo scilicet vires eorum in Methodo istâ experiretur ; quasi Problematis istius Solutioni si cæteri istius Nationis deprehendantur impares, rectè concludatur, nec ipsum *Neutonum*, qui, fatente etiam *Leibnitio*, ab hujusmodi contemplationibus jam jure immunis esse debet, olim fuisse parem inventioni istius Methodi. Sive Problema solvatur, sive insolutum maneat, nihil exinde consequetur quod *Neutonum* afficiat ; nec istis certè *Leibnitii* Fautoribus, qui Problematis solutionem etiamnum continenter efflagitant, jus ullum est nos ad certamen ingeniorum tantâ cum licentiâ provocandi ; adeoque Problema eorum jure merito negligi posset. Verùm ne aliquando exinde occasionem triumphandi arripiant, si hoc Problema maneat ab *Anglis* omninò intactum, ipse. Geometra longè non summi inter nostrates subsellii, inducor, ut solutionem edam qualem qualem Problematis, nec usu, nec difficultate adeò insignis.

Problema à *Leibnitio* primò propositum, ita fuit intellectum quasi nihil aliud requisitum fuisset, quam ut secarentur ad angulos rectos Hyperbolæ Conicæ iisdem Centro & Verticibus descriptæ. Verùm cum illi nuncia-

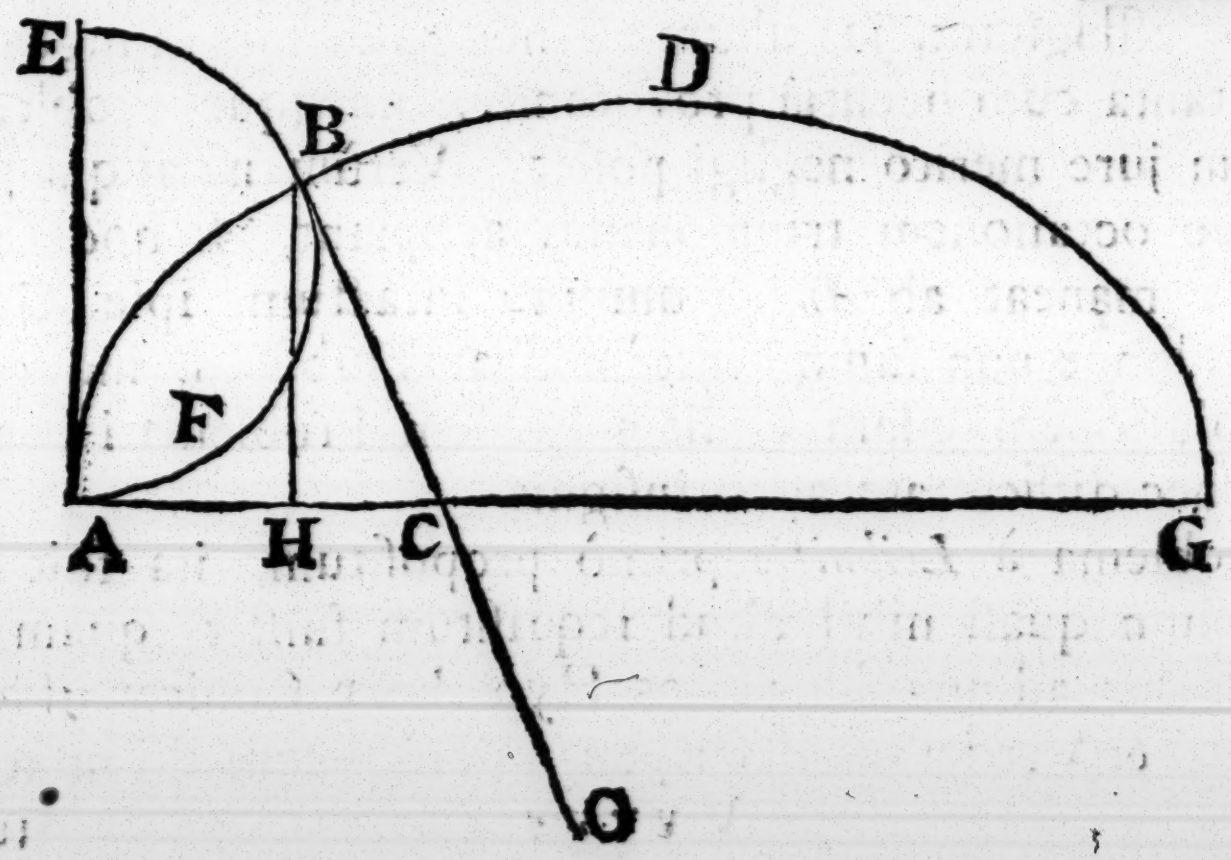
tum fuerat hunc casum à quibusdam *Anglis* fuisse illicò solutum, rescripsit, non solutionem casûs particularis, sed generalem requiri. Quo factum est ut solutiones istæ particulares non editæ fuerint; verùm in Transactione Philosophicâ N° 347. subinde prodiit Solutio maximè generalis. Sed nec illâ contenti fuerunt *Leibnitius* & Fautores ejus, quin illam derisui habuêre, quasi qui illam excogitaverat non potuisset eam ad casum specialem applicare. Si nondum viderint quomodo ex illâ æquationes sint deducendæ, id profectò illorum imperitiæ tribuendum erit. Paulò ante *Leibnitii* obitum prodiit tandem Problema sequens; quod quidem diversimodè solvi potest, premendo vestigia Solutionis generalis modò citatæ, sed quod in præsentia solvimus ut sequitur.

Problema.

Super rectâ A G *tanquam axe, ex puncto* A *educere infinitas Curvas, qualis est* A B D, *ejus naturæ, ut radii Osculi, in singulis punctis* B *& ubique ducti,* B O *secentur ab axe* A G *in* C, *in datâ ratione, ut nempe sit* B O *ad* B C *ut* 1 *ad* n.

Deinde construendæ sunt Trajectoriæ E B F *primas Curvas* A B D *normaliter secantes.*

Solutionis



Solutionis Pars prima;

Nempe Inventio Curvarum ſecandarum A B D.

1. DUctâ ordinatâ BH ad axem AG normali, ſint, Abſciſſa $AH=z$, Ordinata $HB=x$, Curva $AB=v$. Tum per Methodum Fluxionum directam erit $BC=\frac{\dot{v}}{\dot{z}}\dot{x}$, & fluente uniformiter v, $BO=\frac{\dot{v}\dot{x}}{\dot{z}}$. Unde per conditionem Problematis fit $BO\left(\frac{\dot{v}\dot{x}}{\dot{z}}\right) : BC\left(\frac{\dot{v}}{\dot{z}}\dot{x}\right) :: 1 : n$; adeoque $\ddot{z}x-n\dot{z}\dot{x}=0$.

2. Collatâ hâc æquatione cum formulâ Fluxionum ſecundâ, in calce *Prop.* 6. Methodi Incrementorum, invenitur $\dot{z}x^{-n}=\dot{v}\alpha^{-n}$; exiſtente α linea data, per cujus valorem poteſt Curva ABD accommodari conditioni alicui Problemati annexæ.

3. Pro v ſcripto ipſius valore $\sqrt{x^2+z^2}$, migrat æquatio $\dot{z}x^{-n}=\dot{v}\alpha^{-n}$ in hanc $z=\frac{\dot{x}x^n}{\sqrt{\alpha^{2n}-x^{2n}}}$. Unde datur z ex datâ x, per quadraturam Curvæ cujus abſciſſâ exiſtente x eſt ordinata $\frac{x^n}{\sqrt{\alpha^{2n}-x^{2n}}}$.

4. Sint σ & τ numeri integri, vel affirmativi vel negativi, tales ut ſit Curvarum iſto modo provenientium ſimpliciſſima, ea cujus eſt Abſciſſa y, & Ordinata $y^{\frac{1-n+2\sigma n}{2n}}\overline{|x^{2n}-y^{2n}|}^{\tau-\frac{1}{2}}$; tum erit ea omnium Curvarum ſimpliciſſima, per quarum Quadraturam datur Abſciſſa z ex datâ Ordinatâ x.

5. Eſt Curva ABD Geometrica, quoties pro n ſumitur reciprocum numeri cujuſvis imparis.

6. In prædictis Curvam *A B D* conſideravimus ut verſus axem *A G* concavam, quo in caſu maxima ordinata *x* æqualis eſt lineæ datæ *a*, quam Parametrum Curvæ commodè vocare licet. Et in hoc caſu Curva actu occurret Axi. Unde fluente ipſius $\frac{\dot{x}\,x^n}{\sqrt{a^{2n}-x^{2n}}}$ debitè ſumptâ, hoc eſt, ita ut ſimul evaneſcant *z* & *x*, tranſibit Curva per punctum datum *A*, ſicut poſtulat Problema.

7. Sed ſi quæratur Curva *A B D*, quæ ſit versùs axem convexa, ad eundem modum pervenietur ad æquationem $\dot{z} = \frac{a^n \dot{x}}{\sqrt{x^{2n}-a^{2n}}}$; quæ etiam ex æquatione priori derivari poteſt mutando ſignum ipſius *n*. Et in hoc caſu eſt curva *A B D* Geometrica, quoties pro *n* ſumitur reciprocum cujuſvis numeri paris. In hoc verò caſu Ordinata omnium minima *x* æqualis eſt Parametro *a*; adeoque Curva nuſquam occurrit Axi. Quare limitatur Problema ad caſum priorem.

8. Ex præmiſſis facilè colligitur Curvas omnes *A B D* eſſe inter ſe ſimiles, & circa punctum datum *A* ſimiliter poſitas, lateribus earum homologis exiſtentibus proportionalibus Parametris *a*.

Solutionis Pars altera;

Nempe Inventio Curvæ ſecantis.

9. Ex § 2. fit $\dot{v} : \dot{z} :: a^n : x^n$. Sed eſt $BC : BH :: \dot{v} : \dot{z}$. Unde fit $BC : BH :: a^n : x^n$. Ex conditione verò Problematis eſt *B C* tangens Curvæ quæſitæ *E B F*. Quare ſi jam ſumantur *A H* (*z*) & *B H* (*x*) pro coordinatis Curvæ *E B F*, Curvâ ipsâ *E B* exiſtente *r*, erit, per Meth. Flux. direct. $r : -\dot{x} :: (BC : BH ::)\ a^n : x^n$. Unde fit $\frac{x^n}{a^n} = \frac{-\dot{x}}{\dot{r}}$.

10. In

10. In Curva ABD finge æquationem $\dot{z} = \frac{\dot{x}\,x^n}{\sqrt{a^{2n} - x^{2n}}}$ transformari in æquationem signis radicalibus non affectam $\dot{z} = A\dot{x}\frac{x^n}{a^n} + B\dot{x}\frac{x^{3n}}{a^{3n}} +$, &c. Tum regrediendo ad Fluentes fiet $z = \frac{1}{n+1}A\frac{x^{n+1}}{a^n} + \frac{1}{3n+1}B\frac{x^{3n+1}}{a^{3n}} +$, &c. coefficiente novâ introductâ nullâ, quoniam per conditionem Problematis debent simul nasci z & x. Hinc vice $\frac{x^n}{a^n}$ substituto ipsius valore $\frac{-\dot{x}}{\dot{r}}$ in § 9 invento, fit $z = \frac{1}{n+1}Ax\frac{-\dot{x}}{\dot{r}} + \frac{1}{3n+1}Bx\frac{-\dot{x}^3}{\dot{r}^3} +$, &c. quæ æquatio fluxionalis est primi gradûs ad Curvam quæsitam EBF. Revocatur autem ad formulam simpliciorem in terminis numero finitis, modo sequenti.

11. Fluat uniformiter r, & existente a quantitate non fluente, sit $\frac{-\dot{x}}{\dot{r}} = \frac{s^n}{a^n}$. Substituto hoc valore ipsius $\frac{-\dot{x}}{\dot{r}}$ in æquatione novissimè inventâ, atque ductâ æquatione in $\frac{s}{x}$, transformatur ea in hanc $\frac{z\,s}{x} = \frac{1}{n+1}A\frac{s^{n+1}}{a^n} + \frac{1}{3n+1} \times B\frac{s^{3n+1}}{a^{3n}} +$ &c. Unde capiendo Fluxiones fit $\frac{\dot{s}z\dot{x} + s\dot{z}\dot{x} - sz\ddot{x}}{\dot{x}^2} = A\dot{s}\frac{s^n}{a^n} + B\dot{s}\frac{s^{3n}}{a^{3n}} = \frac{\dot{s}\,s^n}{\sqrt{a^{2n} - s^{2n}}}$. Quod ultimum constat ex Analogia Serierum $A\dot{x}\frac{x^n}{a^n} +$ &c. & $A\dot{s}\frac{s^n}{a^n} +$ &c. Hinc pro s & $\dot{s}$ substitutis eorum valoribus ex æquatione $\frac{-\dot{x}}{\dot{r}} = \frac{s^n}{a^n}$ collectis, elicitur æquatio $n\dot{x}^2\dot{z}z - \ddot{x}\dot{x}z\dot{z} - n\dot{x}\dot{x}\dot{z}^2 - \ddot{x}\dot{x}x^2 = 0$. Quæ ad Fluxiones primas revocatur modo sequenti.

fit

12. In termino ultimo $-\ddot{x}\dot{x}x^2$ vice $\ddot{x}\dot{x}$ ſcripto ipſius valore $-\ddot{z}z$, & æquatione deinde applicata ad z, fit $n\dot{x}^2z-\ddot{x}xz-n\dot{x}\dot{x}z+\ddot{x}xz=0$. Quæ æquatio in x^{-n-1} ducta eſt Fluxio æquationis $-\dot{x}x^{-n}z+x^{1-n}\dot{z}=a^{1-n}\dot{r}$; exiſtentibus a & r non fluentibus. Eſt ergo $-\dot{x}x^{-n}z+x^{1-n}\dot{z}=a^{1-n}\dot{r}$, ſeu $\dot{z}x-z\dot{x}\times a^{n-1}=\dot{x}x^n$, æquatio fluxionalis primi gradûs ad Curvam quæſitam EBF.

13. In iſtâ autem æquatione eſt a valor Ordinatæ BH, quando incidit punctum H in punctum A.

14. Haud proclive eſt æquationem $\dot{z}x-z\dot{x}\times a^{n-1}=\dot{r}x^n$, manente n in terminis generalibus, revocare ad æquationem Fluentes tantùm involventem, vel ad quadraturam Curvarum. Sed puncta curvæ EBF poſſunt commodè inveniri per deſcriptionem Curvæ ABD, & Curvæ cujuſdam Geometricæ. Per Geometricam hic intelligo Curvam, cujus æquationem non ingrediuntur Fluxiones, nec fluentes in Indicibus dignitatum. Secetur enim Curva ABD, cujus Parameter ſit α, in B, à Curvâ geometricâ cujus æquatio eſt $a\alpha^{n-1}x^n-za^{n-1}x^n=xa^{n-1}\sqrt{a^{2n}-x^{2n}}$; atque erit punctum illud interſectionis B ad unam ex Trajectoriis quæſitis, nempe quæ tranſit per punctum E, exiſtente $AE=a$ & normali ipſi AG.

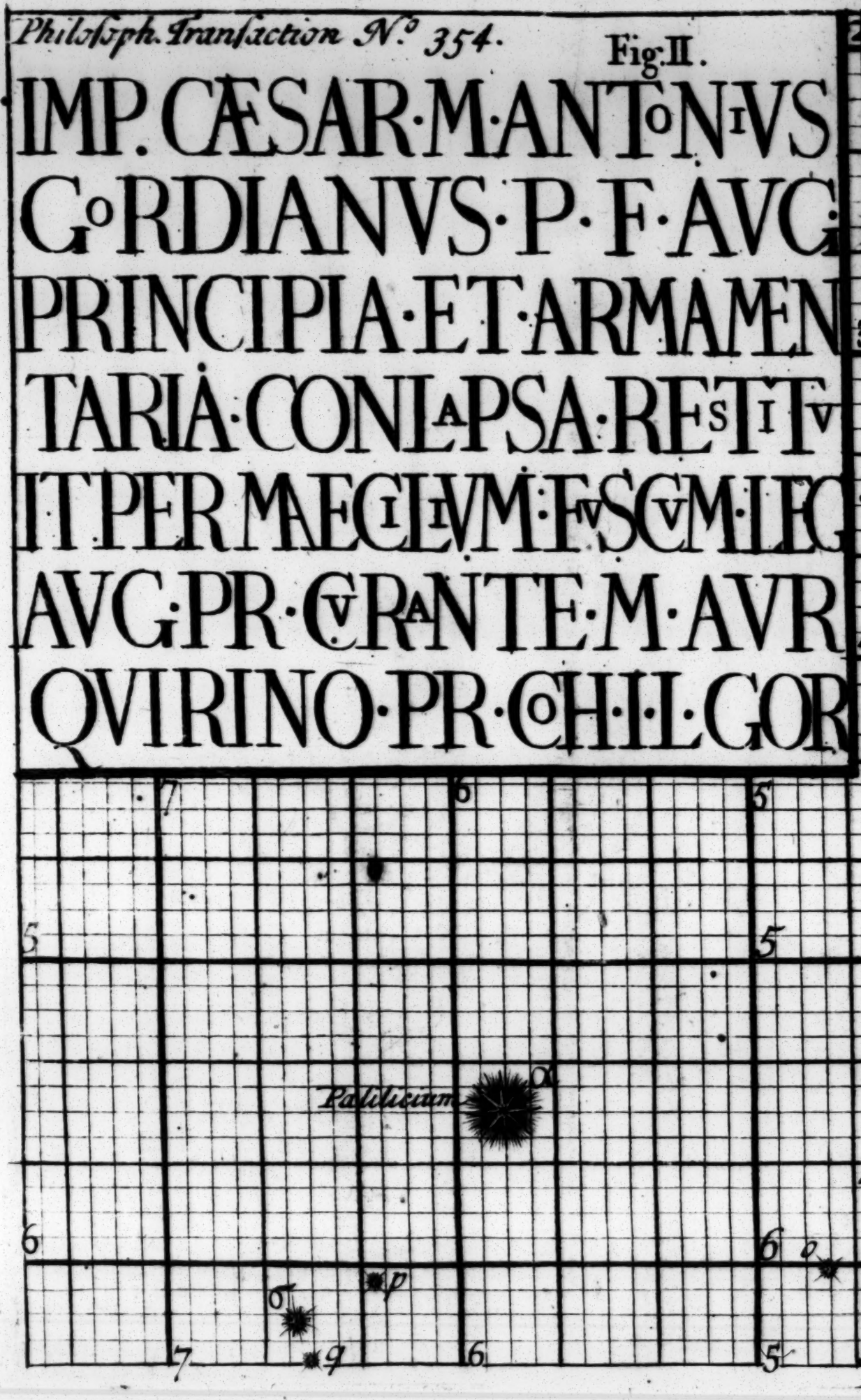
15. Hinc ſi ABD ſit Curva Geometrica, erit etiam EBF geometrica.

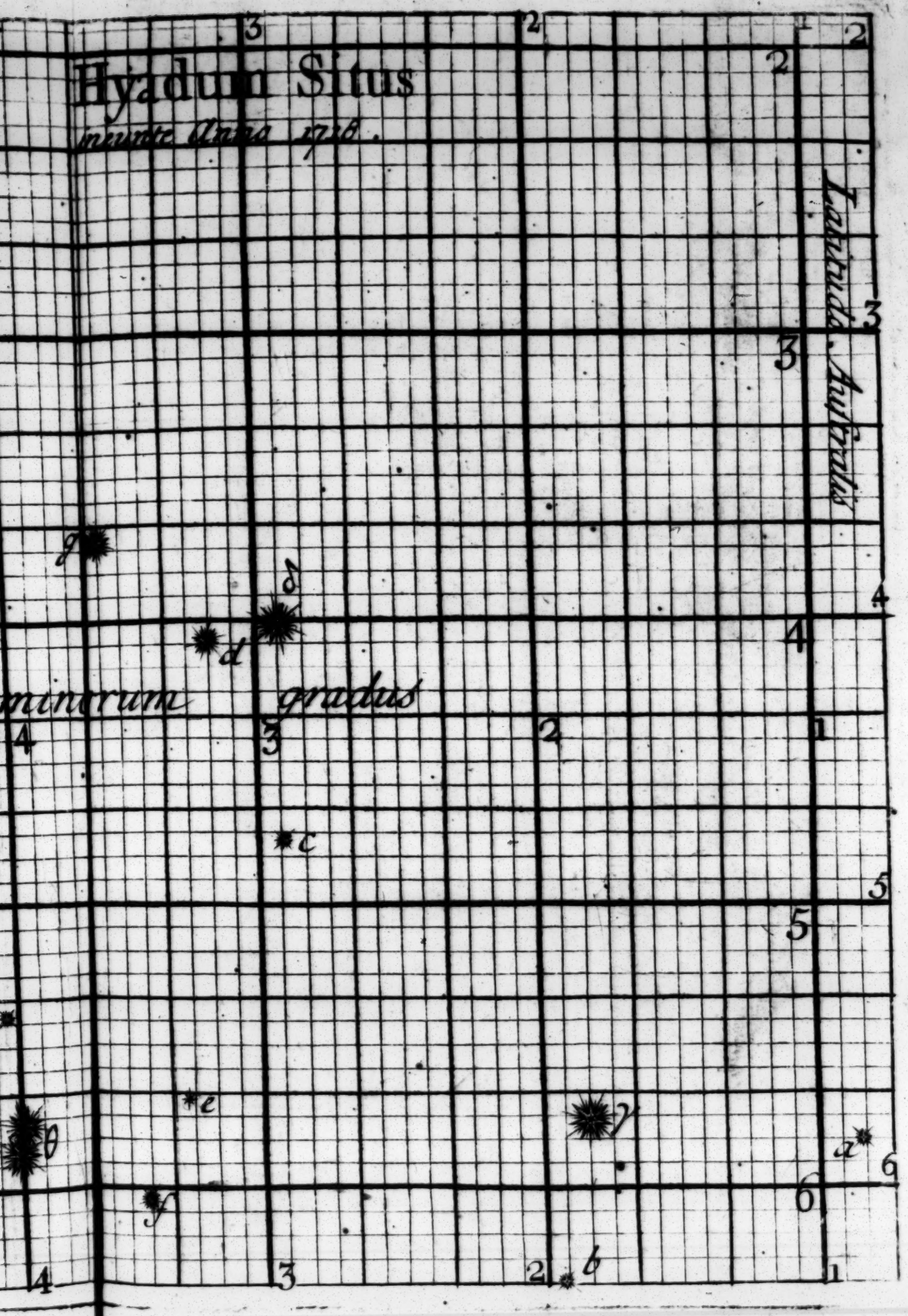
Scholium. Poteſt & alio modo inveniri æquatio $\dot{z}x-z\dot{x}\times a^{n-1}=\dot{r}x^n$. Nam certâ quâdam Analyſi quam nunc celare ſtatuo, inveni æquationem $\frac{\dot{a}}{a}=\frac{\dot{r}r}{zz+xx}$. Quâ comparatâ cum æquatione $\frac{x^n}{a^n}=\frac{-x}{r}$ (§ 9) eliminando $\dot{a}$ & a, tandem pervenitur ad prædictam æquationem $\dot{z}x-z\dot{x}\times a^{n-1}=\dot{r}x^n$.

* *Exemplum*

Philosoph. Transaction N.o 354. Fig. II.

IMP. CÆSAR·M·ANTONIVS
GORDIANVS·P·F·AVG·
PRINCIPIA·ET·ARMAMEN
TARIA·CONLAPSA·RESTITV
IT PER MAECLIVM·FVSCVM·LEG
AVG·PR·CVRANTE·M·AVR
QVIRINO·PR·CHIL·GOR




Hyadum Situs
minute Anno 1716.
Latitud. Australis
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g
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Philosoph

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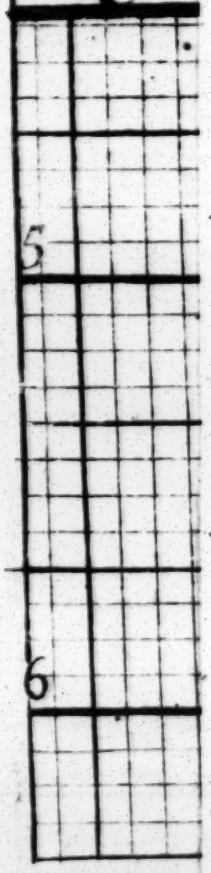

Exemplum. Ad demonſtrationem Solutionis noſtræ ſufficerit exemplum ſimpliciſſimum. Sit itaque $n=1$; quo in Caſu eſt ABD ſemicirculus diametro AG deſcriptus, atque eſt EBF item ſemicirculus deſcriptus diametro AE. Eſt autem in hoc Caſu $\frac{\dot{x}\dot{x}^n}{\sqrt{a^{2n}-x^{2n}}}=\frac{\dot{x}\dot{x}}{\sqrt{a^2-x^2}}$. Unde in § 3. fit $\dot{z}=\frac{\dot{x}\dot{x}}{\sqrt{a^2-x^2}}$; adeoque $z=a-\sqrt{a^2-x^2}$, quæ æquatio eſt ad Circulum diametro $AG=a$ deſcriptum, ut fieri debuit. Item pro n ſcripto 1, æquatio $\dot{z}x-z\dot{x}\times a^{n-1}=r\dot{x}^n$ (§ 12.) migrat in hanc $\dot{z}x-z\dot{x}=r\dot{x}$. Unde exterminando r ope æquationis $rr=xx+zz$, fit $\frac{2\dot{z}zx-\dot{x}z^2}{x^2}=-\dot{x}$; adeoque regrediendo ad Fluentes $\frac{zz}{x}=-x+a$, quæ æquatio eſt ad Circulum diametro $AE=a$ deſcriptum, ut etiam fieri debuit.

III. *Extract of a Letter of Dr.* Chr. Hunter, *M.D. to Dr.* J. Woodward, *R. S. S. from* Durham, *giving an Account of a* Roman *Inſcription, lately dug up in the North of* England; *with ſome Hiſtorical and Chronological Remarks thereon.*

THE Inſcription which comes herewith, (*Fig.* II.) was dug up, two Years ago, in the *Roman CASTRUM*, near *Lanchester:* The Inſcription is very legible, and gives me reaſon to hope, a Search after the firſt Fortifying this Place will not be unneceſſary; eſpecially, being able to fix the Time of *Gordian*'s Repair-

Repairing this Fortreſs, to the 243d Year of Chriſt. We may reaſonably aſcribe the Foundation to the prudent Adminiſtration of *Julius Agricola*, in the Reign of *Fl. Veſpaſian*, about 169 Years before. In Confirmation of this, I find the following Particulars very material, and think it not unbecoming to begin my Enquiry with *Veſpaſian*'s firſt Appearance upon the Theatre of Fame in *Britain*.

In the Second Year of the Emperor *CLAUDIUS*, *Ann. Dom.* 44. the *Romans* invaded *Britain*, under the Command of *Aulus Plautius*, in which Expedition *Veſpaſian* *, then Legate of the Second Legion, made a glorious Figure; having been engaged in no leſs than thirty Battels, and reduced two powerful Provinces, above twenty Towns, and the Iſle of *Wight*. All theſe Succeſſes, tho' continued with good Improvements in ſome of the following Years and Governments, could not frighten the Natives into an entire Submiſſion; eſpecially, no Advance being made into the Country of the *Brigantes*, till the Advancement of *Veſpaſian* to the Imperial Throne, about 26 Years after, *Ann. Dom.* 70. Then the whole Empire was deliver'd from the Miſeries of *Nero*'s, and the ſhort but lamentable Devaſtations of the three ſucceeding Reigns: *Veſpaſian* then reſolv'd to puſh on his begun Conqueſts in *Britain*; choice Armies, commanded by experienced Generals, are ſent over; and the XX^th^ Legion, having in the preceding Troubles acted ſeditiouſly, (not without Difficulty) was reduced to ſubmit to *Veſpaſian* (moſt of the Officers as well as Soldiers having been advanced by *Vitellius*). *Julius Agricola* is conſtituted Legate, who under the Governour *Petilius Cerealis*, bore a conſiderable Share

* *Suetonius*, *Veſpaſian*, Cap. 4.

in

in the Succeſſes againſt the *Brigantes*; * " Sed primò
" *Cerealis* modò labores & diſcrimina, mox & gloriam
" communicabat: Sæpe parti Exercitus in experimen-
" tum, aliquando majoribus copiis ex eventu præfecit.
Tacitus afterwards in a few Words ſums up the Whole
of *Cerealis* his Acquiſitions, † " Terrorem ſtatim intu-
" lit *Petilius Cerealis*, *Brigantum* Civitatem, quæ nu-
" meroſiſſima Provinciæ totius perhibetur, aggreſſus;
" multa prælia, & aliquando non incruenta; magnam-
" que *Brigantum* partem aut victoria amplexus, aut
" bello. Notwithſtanding theſe Advantages, I dare not
ſuppoſe the *Romans* to have then penetrated ſo far into
this Province as our *Longovicum*, which is ſituate ſo
near the Northern Bounds of the *Brigantes*, that at pre-
ſent it's not diſtant above twelve Miles from *Carbridge*,
the Roman *Curia*, the chief Town of the adjoining Peo-
ple the *Otadini*. I now advance to my principal Mo-
tive, (I hope its Length may deſerve Pardon, being un-
der no Obligation to account for the Government of
Jul. Frontinus Succeſſor to *Cerealis*) to fix upon the ſe-
cond Year of *Julius Agricola*'s Government for this Work,
which *Tacitus* thus deſcribes, || " Sed ubi Æſtas adve-
" nit contracto Exercitu — loca Caſtris ipſe capere,
" æſtuaria ac ſylvas ipſe prætentare: & nihil interim
" apud Hoſtes quietum pati, quo minus ſubitis Excur-
" ſibus popularetur; atque ubi ſatis terruerat, parcen-
" do rurſus irritamenta Pacis oſtentare. Quibus rebus
" multæ Civitates quæ in illum diem ex æquo egerant,
" datis Obſidibus iram poſuere, & Præſidiis Caſtelliſ-
" que circumdatæ, tantâ ratione curâque, ut nulla an-
" tè Britanniæ nova pars illaceſſita tranſierit. This ex-
cellent Conduct *Tacitus* further confirms from the Ob-

* *Tacit. Vit. Agric.* 8. † Cap. 17. || Cap. 20.

 ſervation

ſervation of Others. "Adnotabant periti, non alium "Ducem Opportunitates locorum ſapientius legiſſe, "nullum ab *Agricola* poſitum Caſtellum aut vi Ho- "ſtium expugnatum, aut pactione aut fugâ deſertum.

Agricola, this Summer, having quieted ſo large a Tract, and finiſhed ſo many Fortreſſes, it cannot be expected all ſhould be built with the moſt exquiſite Art, ſufficient to perpetuate them. I proceed to *Gordian*'s Repairs; whoſe Hiſtorian *Julius Capitolinus* having never once named *Britain*, yet giving ſo many Hints of the excellent Oeconomy of his Government, under the prudent Adminiſtration of his Father in-Law *Miſitheus*, I dare not fix this Work till the Third Year of his Reign, He having before been under the Direction of the Eunuchs and Officers of the Court, whom *Capitolinus* repreſents, in *Miſitheus* his Letter to *Gordian*, to have proſtituted all Employments to their own Covetouſneſs and mercenary Creatures.

Durham, July 5. 1717.

IV. *A New*

IV. *A new* Genus *of* PLANTS, *call'd* Araliaſtrum, *of which the famous* Nin-zin *or* Ginſeng *of the* Chineſes, *is a* Species. *Communicated by Mr.* Vaillant *Prædemonſtrator at the Royal Garden at* Paris, *to the Learned Dr.* Will. Sherrard, *LL. D. late Conſul at* Smyrna, *and by him to the* Royal Society.

Araliaſtrum is a *Genus* of Plants, whoſe *Flower* A* is complete †, regular, polypetalous, and hermaphrodite, ſtanding on the *Ovary* B. The *Ovary*, which is crown'd by a *Calyx* cut into ſeveral Parts, becomes a Berry D, in which are, for the moſt part, two flat Seeds, like a Semicircle, which both together repreſent a ſort of a Heart. Add to this the Stalk, which is ſingle, ending in an Umbel of which each Ray bears but one Flower. Above the Middle of the Stalk come out ſeveral *Pedicles*, (as on that of the *Anemone*) on the Extremities of which grow ſeveral Leaves like Rays, or like an open Hand.

The Species *of this* Genus *are*,

1. *Araliaſtrum Quinquefolii folio, majus, Nin-zin vocatum D.* Sarrazin. *Gin-ſeng.* Des lettres edifiantes & curieuſes, *Tom.* X. *pag:* 172.
2. *Araliaſtrum Quinquefolii folio, minus.* D. Sarrazin. *Plantula Marilandica, foliis in ſummo caule ternis, quorum unumquodque quinquefariam dividitur, circa margines ſerratis.* N° 36. Raii Hiſt. III. 658.

* *vid.* ARALIA *Inſt. rei herb.* Tab. 154.
† Complete, *that is to ſay, that has a* Calyx.

3. *Araliaſtrum Fragrariæ folio, minus.* D. Vaillant. *Naſturtium Marianum Anemones ſylvaticæ foliis, enneaphyllon, floribus exiguis.* Pluk. Mantiſſ. 135. Tab. 435. Fig. 7.

To ſhew wherein *Araliaſtrum* differs from *Aralia*, (from whence it takes its Name) 'tis convenient to give alſo the Character of this laſt *Genus*, ſuch as Mr. *Vaillant* eſtabliſh'd it, in his Demonſtrations of the Year 1717.

Aralia * is altogether like the *Araliaſtrum*, as to the Structure and Situation of its Flower; but its Berry conſiſts of Five Seeds plac'd round an *Axis*. Moreover, its Leaves are branched, almoſt like thoſe of *Angelica*; and its Stalks (which in ſome *Species* are naked, and in others have Leaves ſet alternately) bear each ſeveral Umbels at their top, in the Form of a Bunch of Grapes.

The Species *of* Aralia, *are*

1. *Aralia caule aphyllo, radice repente.* D. Sarrazin. *Chriſtophoriana Virginiana Zarzæ radicibus ſurculoſis & fungoſis, Sarſaparilla noſtratibus dicta.* Pluk. Almag. 98. Tab. 238. Fig. 5. *Zarſaparilla Virginienſibus noſtratibus dicta, lobatis umbelliferæ foliis, Americana.* Ejuſd. Almag. 396.
2. *Aralia caule folioſo lævi,* D. Sarazzin. *Aralia Canadenſis.* Inſt. rei Herb. 300.
3. *Aralia caule folioſo & hiſpido* D. Sarazzin.
4. *Aralia arboreſcens ſpinoſa,* D. Vaillant. *Angelica arboreſcens, ſpinoſa, ſeu Arbor Indica, Fraxini folio, cortice ſpinoſo* Raii Hiſt. II. 1798. *Chriſtophoriana arbor aculeata Virginienſis* Pluk. Almag. 98. Tab 20.

* vid. *Inſt. rei Herb.* 300. Tab. 154.

All

All the *Species* of theſe two *Genera*, except the laſt of each of them, are common in *Canada*, whence Mr. *Sarrazin*, Counſellor in the upper Council, Phyſician to his Majeſty, and Correſpondent of the Royal Academy of Sciences, ſent them to the Royal Garden firſt in 1700.

The Inhabitants of that Colony, and thoſe of *Virginia*, call the firſt *Species* of *Aralia* by the Name of *Sarſaparilla*, becauſe its Roots have almoſt the ſame Figure and Vertues.

Mr. *Sarrazin* writes, that he had a Patient who had been cured of an *Anaſarca*, about two Years before, by the uſe of a Drink made of theſe Roots. That able Phyſician aſſures us alſo, that the Roots of the ſecond *Species*, well boyl'd and apply'd by way of *Cataplaſme*, are very excellent for the curing of old Ulcers; as alſo the Decoction of them, with which they bath and ſyringe the Wounds. He does not at all doubt, but the Virtues of the third *Species* (which I ſhall briefly deſcribe) are the ſame with thoſe of the ſecond.

Its Roots creep, and ſend forth Stalks, which riſe commonly to the Height of a Foot and half, and ſometimes to two Foot; the bottom part of them is rough, with reddiſh, ſtiff and prickling Hairs. Theſe Stalks are ſet from the Bottom to almoſt the Top (which are divided ſucceſſively into ſeveral naked Branches charg'd with Umbels) with branch'd alternate Leaves, almoſt like thoſe of *Podagraria hirſuta Angelicæ folio & odore* D. Vaillant; which Plant is grav'd in the ſecond Tome of *Boccone's Muſæum*, by the Name of *Cerefolium rugoſo Angelicæ folio, Aromaticum.* Tab. 19. and in *Rivini* by that of *Myrrhis folio Podagrariæ*

See the Account of the Chineſe *Gin-ſeng*, in Phil. Tranſact. *Anni* 1713. p. 237.

V. *Extract*

V. *Extract of a Letter of Mr.* Edw Berkeley *from* Naples, *giving ſeveral curious Obſervations and Remarks on the Eruptions of Fire and Smoak from Mount* VESUVIO. *Communicated by Dr.* John Arbuthnot, *M. D. and* R. S. S.

APril 17. 1717. with much Difficulty I reach'd the top of Mount *Veſuvius*, in which I ſaw a vaſt Aperture full of Smoak, which hinder'd the ſeeing its Depth and Figure. I heard within that horrid Gulf certain odd Sounds, which ſeem'd to proceed from the Belly of the Mountain; a ſort of Murmuring, Sighing, Throbbing, Churning, Daſhing (as it were) of Waves, and between whiles a Noiſe like that of Thunder or Cannon, which was conſtantly attended with a Clattering, like that of Tiles falling from the Tops of Houſes on the Streets. Sometimes, as the Wind changed, the Smoak grew thinner, diſcovering a very ruddy Flame, and the Jaws of the Pan or *Crater*, ſtreak'd with Red, and ſeveral Shades of Yellow. After an Hour's ſtay, the Smoak, being moved by the Wind, gave us ſhort and partial Proſpects of the great Hollow, in the flat Bottom of which I could diſcern two Furnaces almoſt contiguous; that on the Left, ſeeming about three Yards in Diameter, glow'd with red Flame, and threw up red-hot Stones with a hideous Noiſe, which, as they fell back, caus'd the fore mentioned Clattering. *May* 8. in the Morning, I aſcended to the Top of *Veſuvius* a ſecond time, and found a different face of things. The Smoak aſcending upright,

gave

gave a full Proſpect of the *Crater*, which, as I could judge, is about a Mile in Circumference, and an Hundred Yards deep. A conical Mount had been formed ſince my laſt Viſit, in the middle of the Bottom. This Mount I could ſee was made of the Stones thrown up and fallen back again into the *Crater*. In this new Hill remained the two Mouths or Furnaces already mention'd; that on our Left Hand was in the *Vertex* of the Hill which it had formed round it, and raged more violently than before, throwing up every three or four Minutes, with a dreadful Bellowing, a vaſt Number of red-hot Stones, ſometimes in appearance above a Thouſand, and at leaſt 300 Foot higher than my Head as I ſtood upon the Brink. But there being little or no Wind, they fell back perpendicularly into the *Crater*, increaſing the conical Hill. The other Mouth to the Right was lower in the ſide of the ſame new formed Hill. I cou'd diſcern it to be fill'd with red hot liquid Matter, like that in the Furnace of a Glaſs-Houſe, which raged and wrought as the Waves of the Sea, cauſing a ſhort abrupt Noiſe like what may be imagin'd to proceed from a Sea of Quickſilver daſhing among uneven Rocks This Stuff wou'd ſometimes ſpew over and run down the convex ſide of the conical Hill, and appearing at firſt red-hot, it changed Colour, and harden'd as it cool'd, ſhewing the firſt Rudiments of an Eruption, or if I may ſo ſay, an Eruption in Miniature. Had the Wind driven in our Faces, we had been in no ſmall Danger of ſtifling by the ſulphurous Smoak or being knock'd on the Head by Lumps of molten Minerals, which we ſaw had ſometimes fallen on the Brink of the *Crater*, upon thoſe Shots from the Gulf at Bottom. But as the Wind was favourable, I had an opportunity to ſurvey this odd Scene for above an Hour and a half together;

together; during which it was very obſervable, that all the Volleys of Smoak, Flame, and burning Stones, came only out of the Hole to our Left, while the liquid Stuff in the other Mouth wrought and overflow'd as hath been already deſcribed. *June* 5. After a horrid Noiſe, the Mountain was ſeen at *Naples* to ſpew a little out of the *Crater*: The ſame continu'd the 6th. The 7th, nothing was obſerv'd till within two Hours of Night, when it began a hideous bellowing, which continued all that Night and the next Day till Noon, cauſing the Windows, and as ſome affirm, the very Houſes in *Naples* to ſhake From that time it ſpew'd vaſt Quantities of molten Stuff to the South, which ſtream'd down the ſide of the Mountain, like a great Pot boyling over. This Evening I return'd from a Voyage thro' *Apulia*, and was ſurpriſed, paſſing by the North ſide of the Mountain, to ſee a great Quantity of ruddy Smoak lie along a huge Tract of Sky over the River of molten Stuff, which was it ſelf out of ſight. The 9th, *Veſuvius* raged leſs violently; that Night we ſaw from *Naples* a Column of Fire ſhoot between whiles out of its Summit. The 10th, when we thought all wou'd have been over, the Mountain grew very outragious again, roaring and groaning moſt dreadfully. You cannot form a juſter idea of this Noiſe in the moſt violent Fits of it, than by imagining a mix'd Sound made up of the raging of a Tempeſt, the Murmur of a troubled Sea, and the Roaring of Thunder and Artillery, confuſed all together. It was very terrible as we heard it in the further End of *Naples*, at the Diſtance of above twelve Miles. This moved my Curioſity to approach the Mountain. Three or four of us got into a Boat, and were ſet aſhoar at *Torre del Greco*, a Town ſituate at the Foot of *Veſuvius* to the South-

South-West, whence we rode four or five Miles before we came to the burning River, which was about Mid-Night. The Roaring of the *Volcano* grew exceeding loud and horrible as we approach'd. I obſerved a Mixture of Colours in the Cloud over the *Crater*, green, yellow, red and blue; there was likewiſe a ruddy diſmal Light in the Air over that Tract of Land where the burning River flowed; Aſhes continually ſhower'd on us all the way from the Sea-Coaſt. All which Circumſtances, ſet off and augmented by the Horror and Silence of the Night, made a Scene the moſt uncommon and aſtoniſhing I ever ſaw; which grew ſtill more extraordinary as we came nearer the Stream. Imagine a vaſt Torrent of liquid Fire rolling from the Top down the Side of the Mountain, and with irreſiſtible Fury bearing down and conſuming Vines, Olives, Fig-trees, Houſes, in a word, every thing that ſtood in its way. This mighty Flood divided into different Channels, according to the Inequalities of the Mountain. The largeſt Stream ſeem'd half a Mile broad at leaſt, and five Miles long. The Nature and Conſiſtence of theſe burning Torrents hath been deſcribed, with ſo much Exactneſs and Truth, by *Borellus*, in his Latin Treatiſe of *Mount Ætna*, that I need ſay nothing of it. I walked ſo far before my Companions, up the Mountain along the ſide of the River of Fire, that I was oblig'd to retire in great haſte, the ſulphureous Steam having ſurpriz'd me, and almoſt taken away my Breath. During our Return, which was about Three-a-Clock in the Morning, we conſtantly heard the Murmur and Groaning of the Mountain, which between whiles would burſt out into louder Peals, throwing up huge Spouts of Fire and burning Stones, which falling down again reſembled the Stars in our Rockets. Sometimes I ob-

ſerv'd two, at others three diſtinct Columns of Flame, and ſometimes one vaſt one that ſeem'd to fill the whole *Crater*. Theſe burning Columns, and the fiery Stones ſeem'd to be ſhot a 1000 Foot perpendicular above the Summit of the *Volcano*. The 11th at Night, I obſerv'd it, from a Terraſs in *Naples*, to throw up inceſſantly a vaſt Body of Fire and great Stones to a ſurpriſing Height. The 12th in the Morning, it darken'd the Sun with Aſhes and Smoak, cauſing a ſort of Eclipſe. Horrid Bellowings this and the foregoing Day were heard at *Naples*, whither part of the Aſhes alſo reached. At Night I obſerved it throw up Flame, as on the 11th. On the 13th, the Wind changing, we ſaw a Pillar of black Smoak ſhot upright to a prodigious Height. At Night I obſerved the Mount caſt up Fire as before, tho' not ſo diſtinctly becauſe of the Smoak. The 14th, A thick black Cloud hid the Mountain from *Naples*. The 15th, in the Morning, the Court and Walls of our Houſe in *Naples* were cover'd with Aſhes. In the Evening, Flame appear'd on the Mountain thro' the Cloud The 16th, the Smoak was driven by a Weſterly Wind from the Town to the oppoſite ſide of the Mountain. The 17th, the Smoak appear'd much diminiſh'd, fat and greaſy. The 18th, the whole Appearance ended, the Mountain remaining perfectly quiet without any viſible Smoak or Flame. A Gentleman of my Acquaintance, whoſe Window look'd toward *Veſuvius*, aſſur'd me, that he obſerv'd this Night ſeveral Flaſhes, as it were of Lightening, iſſue out of the Mouth of the *Volcano*. It is not worth while to trouble you with the Conjectures I have formed concerning the Cauſe of theſe Phænomena, from what I obſerved in the *Lacus Amſancti*, the *Solfatara*, &c. as well as in Mount *Veſuvius*. One thing I may venture to ſay, that I ſaw the fluid Matter riſe out of the Centre

of

of the Bottom of the *Crater*, out of the very middle of the Mountain, contrary to what *Borellus* imagines, whoſe Method of explaining the Eruption of a *Volcano* by an inflexed Syphon, and the Rules of *Hydroſtaticks*, is likewiſe inconſiſtent with the Torrent's flowing down from the very *Vertex* of the Mountain. I have not ſeen the *Crater* ſince the Eruption, but deſign to viſit it again before I leave *Naples*. I doubt there is nothing in this worth ſhewing the Society; as to that, you will uſe your Diſcretion.

E. Berkeley.

VI. *An Account of an extraordinary* TUMOUR *or* WEN *lately cut off the Cheek of a Perſon in* Scotland. *Communicated to the Royal Society by Dr.* Thomas Bower, *M. D.* and *F. R. S.*

IT is generally acknowledg'd, that the exact Obſervation of internal Diſeaſes, and the faithful Accounts of external Tumours, and extraordinary Caſes in Chirurgery, have contributed very much to the Advancement of Medicine. *Hippocrates* and *Galen*, and other ancient Fathers of Medicine, have ſet us fair Copies of this; and the Moderns, happily following their Footſteps, have illuſtrated this Matter by many curious Obſervations and Reflections. The Royal Societies and Colleges of *Virtuoſi*, that are now over all *Europe*, have taken much pains in this Affair, and have given us many Inſtances and Examples of Extraordinary Caſes in Medicine, which are of great uſe to all the Practiſers of Phyſick and Chirurgery. According to theſe laudable Examples I ſhall, for the Satisfaction

of the Curious and Ingenious, give a true and faithful Account of an extraordinary Excreſcence cut off from the Cheek of a Man, which weighed Nineteen Pounds, and the Patient entirely recover'd in few Weeks time. I never before ſaw the like, nor never read of it in any Author, tho' I have conſulted many on the Head. This Excreſcence is preſerved among the Rarities of the College of Phyſicians at *Edinburgh*. The Phyſician concern'd in this Affair, was Dr. *Alexander Ruſſel* of *Elgine*, a learned and ingenious Man, and the Operator was the ingenious Mr. *George Gordon* of *Keith*, from whom I had the following Information.

Alexander Palmer, of the Pariſh of *Keith*, in the County of *Bamff*, in the North of *Scotland*, now about Fifty Four Years of Age, obſerved, when about Twenty Seven, a little hard Swelling in the Muſcle of the lower Jaw on the Left Side, without any Hurt or manifeſt Occaſion, which at firſt went on ſlowly, but afterwards it proceeded more quickly, and the older it grew, it ſtill came on the faſter; until it increaſed to a prodigious Bulk and Weight: From the firſt Appearance of this Tumour to the total Exciſion of it, there were about Twenty Seven Years. He had exceſſive Pains and Uneaſineſs in it, and at laſt it mightily extenuated and emaciated him, who was otherwiſe a ſtrong and robuſt Man.

This Excreſcence was of the natural Colour of the Skin, and ſeem'd to be an *Atheroma*, being a glandulous Subſtance with ſeveral big Blood-Veſſels in it, and had Hair growing on it, as on the other Parts of the Body, as may yet be ſeen. It was almoſt round and very hard, and was as ſenſible as any other Part of the Body; for, when the poor Man was working in the Fields,

Fields, ſome ſix or ſeven Years ago, he accidentally made a great Gaſh or Wound in it with a ſharp Iron, which was very painful, but was cured by a Surgeon, after the manner of an ordinary Wound; the Cicatrice is ſtill to be ſeen in it.

This Excreſcence having grown ſo big, was attach'd to the Muſcle under the Left Eye, call'd *Obliquus minor* or *inferior*, to the Ear and its Muſcles, and to the Muſcle of the lower Jaw, named *Deprimens*. By reaſon of its great Bulk and Weight, it could not hang down freely without ſome Support, therefore it reſted on the top of the Shoulder, which made a conſiderable Dimple in it, that is yet very obſervable; beſides, it was holden up by the Man's Hand in the Day-time, and laid on a Pillow in the Night-ſeaſon.

Some three or four Days before the total Exciſion was made, the Patient obſerved this Tumour begin to mortify at the lower end, which made him ſo uneaſy, that he took a Knife and cut off a good part of it. This occaſion'd a great Hæmorrhage; ſo that he reckon'd there was loſt a *Scots* Pint or four Pounds of Blood, before it could be ſtopt. The Patient, after ſo great Trouble and Pain, at laſt applied himſelf to Mr *Gordon*, Surgeon of the Place, who made a total Extirpation of it, on the 19*th* of *January*, 1717.

He made a cloſe Ligature, taking in the Baſis of the Excreſcence, and all the looſe Skin, and contracting it as much as poſſible, he cut it entirely off with a ſharp Raſour. There guſh'd out of the Excreſcence, after it was cut off, and was lying on the Ground, as near as could be gueſs'd, two Pounds of Blood; for it was nouriſh'd by ſeveral large Blood-Veſſels, perhaps by ſome Branches of the Carotide Artery much inlarged, and other Blood-Veſſels coming from

from the Ear, and the Muſcles of the Eye and lower Jaw abovementioned. When Mr. *Gordon* brought it to us, which was full three Months after it was cut off, we cut off with a Knife about a quarter of an Inch broad of the Baſis of it; and in that ſmall Space we obſerved four big Blood-Veſſels. The Baſis, as it now appears, is five Inches Diameter, which ſeems too large for the whole ſide of the Face: So that after the Exſection, I think the looſe Skin has turn'd backwards, which may make the Baſis now appear ſo big.

After all this Blood was loſt, the Excreſcence was weighed, and was full Nineteen Pound Weight; ſo that before his own Inciſion and this Operation, it behoved to be ſeveral Pounds heavier, which is a moſt prodigious Weight to be depending on ſuch a place. This Tumour was of a Spheroidical Figure, and when meaſured, was Thirty four Inches about by the longeſt way, and Twenty eight by the broadeſt.

The Hæmorrhage, which was great, was ſtopped by the Vitriolic Powders and other Aſtringents, and the ordinary Dreſſing was uſed. So this great Cure was completed in ſix Weeks time, and the Patient entirely recover'd, and goes about his Buſineſs, to the great Admiration and Aſtoniſhment of every body. The Lid of his Left Eye continues ſtill downwards a little, as does that ſame ſide of the Mouth, which was occaſion'd by the great Weight depending on that ſide of the Face; but it may be expected they may come again to their right Poſture; for the Head, at firſt after cutting, enclined much to the Right ſide, by reaſon of the great Weight on the Left Cheek having been removed; but it now begins to ſtand upright, ſince he is perfectly recovered. Tho' the Skin, and even a deal of the muſculous part of the Cheek and

lower

lower Jaw was cut away, yet, according to the Information I have from Mr. *Gordon* the Operator, it is grown up again, and is of the ordinary Colour of the Skin, and like the other ſide of the Face; ſo that there grows Hair on that ſide of the Face as well as on the other, which he ordinarily ſhaves; and this is as ſurprizing as any thing in the whole Affair.

I have given a true and plain Account of this extraordinary Caſe from certain Information; I have contented my ſelf to relate only Matters of Fact, without making any Obſervations or Reflections on it; for I leave it to the Philoſophers and *Virtuoſi* to make their own Reaſonings and Refinements as ſeems beſt to themſelves.

VII. *An Account of an Experiment to prove an* interſpers'd Vacuum; *or to ſhew that all Places are not equaly full.*

THIS Experiment was *made before the* KING, and HER Royal Highneſs the Princeſs of *Wales*, at *Hampton-Court*, *in the Month of* September 1717. *afterwards before the* ROYAL-SOCIETY, *on* Thurſday, December 5. 1717. *and ſince that, in* Channel-Rowe, Weſtminſter, *before ſome Members of the* Royal-Society, *by* I. T. Deſaguliers, *M. A. F. R. S.* as follows:

Having had the Honour to make ſome Experiments laſt Year before his Majeſty and their *Royal Highneſſes* the *Prince and Princeſs of* Wales; among others, I ſhew'd that of a Guinea and a Piece of fine Paper; then of a Guinea and a Feather dropt together from the top of an exhauſted Glaſs Receiver about 20 Inches high; both

which

which fell to the Bottom at the ſame Inſtant of Time: Now ſince the chief *Reſiſtance* of a *Medium* (and indeed almoſt all of it) depends upon the † Quantity of its Matter; therefore this Diminution of Reſiſtance, whereby the Feather fell as ſoon as the Guinea, ſhew'd a Diminution of the Quantity of Matter, and conſequently prov'd an *interſpers'd Vacuum*. Some time after this, I was inform'd that ſome *Pleniſts* here in *England* objected againſt the Shortneſs of the Glaſs-Receiver; as if the Difference of Time in the Fall of the two Bodies, which they affirm'd to be real, could not be perceiv'd in ſuch a Glaſs; and that ſome Philoſophers from abroad affirm'd that in a Glaſs Receiver 7 or 8 Foot long, there would be ſuch a manifeſt Difference in the Time of the Fall of the ſaid Bodies, as to ſhew this Experiment no Proof of a *Vacuum*; though at the ſame time, ſome of the Objectors well knew that there could be no Receivers of half that Length made at the Glaſs Houſe, and therefore thought the Experiment impracticable. To obviate this, I contriv'd a Machine for the purpoſe, which conſiſted of a ſtrong wooden Frame 15 Foot high, that held the Air-Pump and four Cylindric Glaſs-Receivers of about two Foot long each, and ſix Inches Diameter: Of theſe, having ſet the firſt upon the Air-Pump Plate, I laid on the Top of it a Braſs-Plate of ſeven Inches Diameter, that had an oil'd Leather fix'd to it above and below, with an Hole through the middle, of between four and five Inches Diameter; then on that Plate I ſet the next Receiver, with a like Plate at top; and after the ſame manner fix'd the other two with Plates between them: The upper Receiver being a little narrower at the Neck,

† See Sir *Iſ. Newton's Principia*, Book II. Prop. 40.

went

went into the Hole of a Board, whereby it was ſcrew'd down pretty hard on the other Glaſſes, and fix'd to the whole Machine. On the top of this upper Receiver I laid the Braſs Plate, wet Leather, and Braſs Springs which contain'd the Bodies to be dropt

Having acquainted His Majeſty with what I had prepar'd, he order'd me to ſhew him the Experiment with this long compounded Receiver, at *Hampton-Court*; and when I made it before him and her Royal Highneſs, he was pleas'd (by pulling down a String fix'd to a Leaver at the top of the Machine) to let looſe the Bodies himſelf, to ſee that the Experiment was fair.

When the Receiver was full of common Air before Pumping, the Guinea came to the Bottom, juſt as the Paper was about the Middle of the ſecond Glaſs; but when the Receiver was exhauſted, the Guinea and Paper came to the Bottom preciſely in the ſame Inſtant of Time.

Upon my giving an account of the Succeſs of this Experiment to the *Royal Society*, they order'd me to repeat it before them on the 5th Day of *December* 1717, being the *Thurſday* next after the Yearly Meeting for chooſing Officers on St. *Andrew*'s Day; on which Day an annual Experiment is appointed to be made, in Conformity to the Will of their late worthy Member and Benefactor Sir *Godfrey Copley*.

I made the Experiment firſt with two of the Receivers; then with all the four; dropping a Guinea and a ſmall Piece of Paper together; and the Succeſs anſwer'd Expectation: But not being willing to try with a Down-Feather, becauſe I fear'd the Air might Inſinuate between ſome of the Glaſſes, by reaſon the Number of Perſons preſent ſhak'd the Room, the Society order'd me to make the Experiment at home before one or more of their Members.

Martin Foulkes, Esq; a very ingenious Member of the Society, did me the favour to be present when I made the Experiment at my House; where we made four Tryals in the following manner.

The whole Machine being fix'd, as above mention'd, we first let fall a Guinea and two Papers; the one placed over, and the other under it, (before any Air was pump'd out) and the Guinea came to the Bottom when the Papers were only in the Middle of the second Glass from the Top. Then having laid a Feather on the Brass-Springs close by the Guinea, we let them loose both together; and the Feather was fallen only down to the 4th part of the Length of the first Glass or $\frac{1}{16}$ of the whole Distance, when the Guinea was got down to the Bottom of the Receiver. We then laid two Papers and two Feathers, one of each under, and the other over the Guinea between the Springs; and having drawn out so much of the Air as to bring up the Mercury in the Gage-Tube within a quarter of an Inch of the greatest Height to which it could be then rais'd by the Pressure of the external Air, we caus'd the Bodies to fall all at once: And tho' the Papers came down to the Bottom at the same time as the Guinea, yet the Feathers, being much lighter, wanted about three Inches. But at last, having laid the Papers, Feathers, and Guinea, as before, we pump'd out all the Air, and then the Feathers, as well as the Papers, came to the Bottom of the Receiver at the same instant of time as the Guinea.

VIII. An

VIII. *An Account of a ſmall* Teleſcopical Comet *ſeen at* London *on the* 10th *of* June 1717. *by* Edm. Halley, *LL. D.* R. Soc. Secr.

THAT the Number of Comets traverſing our Solar Syſtem is much greater than ſome, on account of the late rareneſs of their Appearance, have ſuppoſed it, may be collected from ſeveral ſmall ones which have within few Years been deſcribed in the Memoirs of the *French* Royal Academy of Sciences; thoſe diligent Obſervers aſſuring us that they diſcover'd one in *Sept.* 1698. another in *Febr.* 1699. a third in *April* 1702: and again a fourth in *Novemb.* 1707. none of which, as far as I can learn, were ever ſeen in *England*; all of them having been very obſcure and without Tails, by means whereof Comets uſually firſt ſhew themſelves. And beſides theſe, two other Comets with remarkably long Tails, the one in *Novemb.* 1689. the other in *Febr.* 1702. paſt by unobſervable in theſe our Northern Climats, they having great South Latitude, and their Motions directed toward that Pole. Hence we may juſtly conclude that the Returns of Comets are much more frequent than is vulgarly reckoned, and that it is only contingent that for theſe 35 Years no one of them has been ſeen and obſerved by our Aſtronomers.

But there may be ſtill a much greater Number of theſe Bodies, which by reaſon of their Smallneſs and Diſtance are wholly inviſible to the naked Eye; ſo that unleſs Chance do direct the Teleſcope of a proper Obſerver, almoſt to the very Points where they are (againſt which there are immenſe Odds) it will not be poſſible

for them to be discovered: And that this is not barely a Conjecture, take the following Instance.

On *Monday*, *June* 10 in the Evening, the Sky being very serene and calm, I was desirous to take a View of the Disk of *Mars* (then very near the *Earth*, and appearing very glorious) to see if I could distinguish, in my 24 Foot Telescope, the Spots said to be seen on him. Directing my Tube for that purpose, I accidentally fell upon a small whitish Appearance near the Planet, resembling in all respects such a *Nebula* as I lately described in *Philos. Transact.* N° 347. but smaller. It seemed to emit from its upper part a very short kind of Radiation directed towards the East, but Northerly withal; which, considering its Situation, was nearly towards the Point opposite to the Sun. The great Light of the Moon, then very near it and also near the Full, hinder'd this *Phænomenon* from being more distinctly seen; but its Place in the Heavens was sufficiently ascertained from the Neighbourhood of *Mars*, from whom it was but about half a Degree distant towards the Southwest, the difference of Latitude being somewhat more than that of Longitude; and *Mars* being at that time in ♐ 17°. 30′ with 3°. 48′ South Latitude, I concluded the place thereof in ♐ 17°. 12′ with 4°. 12′. Lat. South, or thereabouts; the which may yet be more securely determined by help of two small fixt Stars I found near it, the more northerly of which I judged to have the same Latitude with it, and to follow it at about the Distance of six Minutes; the other Star was about four Minutes more southerly than the former, and about one Minute in consequence thereof; the Angle at the Northern Star was a little obtuse, as of about 100 Degrees, and the Distance of our *Nebula* from it *sesquialter* to the Distance of the two Stars, or rather a little more. The Reverend Mr. *Moses Williams*, Mr. *Alban Thomas*, and my self, contem-

contemplated this Appearance for above an Hour, *viz.* from $10\frac{1}{2}^{h}$ to near twelve, and we could not be deceiv'd as to its Reality; but the Slowneſs of its Motion made us at that time conclude that it had none, and that it was rather a *Nebula* than a *Comet*.

However, ſuſpecting that it might have ſome Motion, I attended the next Night, *June* 11th, at the ſame Hours and in the ſame Company, when with ſome Difficulty by reaſon of the Thickneſs of the Air, we found the two little Stars, but the *Nebula* could not at that time be ſeen, which we then imputed to the want of a clearer Sky. But on *Saturday*, *June* 15. the Moon being abſent, and the Air perfectly clear, we had again a diſtinct View of the two Stars, with an entire Evidence that there remained no Footſtep or Sign of it, in the place where we had firſt ſeen this *Phænomenon*, which we therefore now found to be a Comet, and that being far without the Orb of the Earth, and in it ſelf a very ſmall Body, it appeared only like a little Speck of a Cloud, ſuch as would ſcarce have been diſcerned in an ordinary Teleſcope, much leſs by the naked Eye.

IX. *An Account of Books*, I. Joannis Poleni in Gymnaſio Patavino Phil. Ord. Prof. & Scient. Societatum Regalium, quæ Londini & Berolini ſunt, Sodalis, *De Motu Aquæ mixto*, Libri duo, *&c.* 4^to^. Patavii 1717.

THE Subject here treated of not having hicherto fallen under the Conſideration of Mathematical Writers, the Learned Author is obliged to make uſe of ſeveral Terms, which are either wholly new, or at leaſt

leaſt are apply'd in a ſenſe ſomewhat different from their common Acceptation; for which reaſon he begins his Work with a Sett of Definitions.

Aqua mortua, or a dead Water, is that, whoſe Surface being every where equally diſtant from the *Centrum gravium*, no part of it can deſcend any lower, without forcing ſome other upward, and conſequently the Whole is without Motion.

Aqua viva, or a running Water, is that which is put into motion by the Preſſure of the Incumbent Water, and whoſe Motion is oppoſed by no other Water lying in its way.

The motion of a running Water is call'd *Motus ſimplex*, or the ſimple Motion.

If a running Water moving over the Surface of a dead Water, do, by its Preſſure, communicate part of its Motion to the dead Water; the compound Motion with which the whole Body of the Water flows, is called *Motus mixtus*, or the mixt Motion.

If a Water at different Depths from the Surface run with different Velocities, the *mean Velocity* is that, which being the ſame at all Depths, will diſcharge the ſame Quantity of Water.

Next follows a ſhort Hiſtory of the Original, and Progreſs of the Doctrine of running Waters, the Invention of which our Author juſtly aſſerts to the Learned *Caſtellus*, and defends him againſt *Fabretti*, who has maintain'd that *Caſtellus*'s fundamental Propoſition of the Quantity diſcharged being *cæteris paribus* in proportion to the Velocity, was known, and publickly taken notice of before him by *Frontinus*.

The Author allows *Caſtellus* to have been miſtaken in determining the Velocity of Water running out at the bottom of a Veſſel, he having aſſerted that Velocity to be as the Depth of the Water.

Three

Three Years after *Caſtellus*'s Book came out, this Miſtake was corrected by the famous *Torricellius*, who was the firſt that maintain'd, that the Velocity of the Water running out was in a ſubduplicate *Ratio* of the Depth, but gave no Demonſtration of it.

This Propoſition, ſays our Author, was confirm'd by the Experiments of *Maggiotti*, *Mariotte*, and *Guglielmini*, and has ſince been demonſtrated by Mr. *Varignon*, by *Herman* in his *Phoronomia*, and *John Bernoulli*, as reported by *Herman* in the *Acta Lipſienſia*.

Here it may not be improper to take notice, that the Demonſtrations of thoſe three Learned Perſons are all grounded upon this Suppoſition, that the Water running out from the Hole is preſt upon by the Column of Water incumbent upon it, which may eaſily be demonſtrated to be a Miſtake. Likewiſe, if their Demonſtrations be juſt, it will follow, that the firſt Drops of Water, which iſſue out from the Hole, muſt run with the ſame Velocity, as after the Water has been running ſome time; the Contrary of which appears to be true in Fact by the Experiments of the famous Mr. *Mariotte*.

The Author might have found a juſter Account of this matter in the Writings of a Great Man, whom he cites on another Occaſion; the ſecond Edition of whoſe Book was come out ſome time before *Herman* publiſh'd either of thoſe Demonſtrations, and had been ſeen by him, as appears by his quoting it frequently, and mentioning the Difference in this very Particular between the firſt and ſecond Edition.

Our Author goes on to conſider the ſimple Motion of Water running out by a Section perpendicular to the Horizon, in the ſide of a Receptacle, which is always entertain'd at the ſame Height. He ſhews, that the Velocities, with which the Water iſſues out at different

Depths,

Depths, being as the Roots of thoſe reſpective Depths, may be repreſented by the Ordinates of a *Parabola*, whoſe Axis repreſents the entire Depth of the Water. Conſequently, ſince the Quantities of Water, running out at different Depths, are as thoſe Velocities, they likewiſe may be repreſented by the ſame Ordinates, and the Quantity of Water diſcharg'd from the whole Section, will be repreſented by the Parabolick Space; and the mean Velocity by that ſame Space divided by the Abſciſſe.

The Times, being as the Quantities of Water diſcharged, may be repreſented in the ſame manner as thoſe Quantities.

Hence he derives his general Theorem, That the Quantities of Water diſcharged, are in a *ratio* compounded of the ſeſquiplicate *ratio* of the Depths of the Water, the *ratio* of the Breadths of the Section, and of that of the times of the Efflux.

The Author proceeds now to the mixt Motion of Water; in order to diſcover the Nature of which he has made ſome curious Experiments, after the following manner:

A large cylindrical Veſſel, with a perpendicular Section through the ſide of it, was placed upright in a dead Water; ſo that the bottom of the Veſſel was a conſiderable Depth below the Surface of the Water; and the Veſſel was kept immovable in this ſituation.

Above this was fixt another Veſſel, full of Water, whoſe Bottom was pierced with 16 Holes, exactly round, and of the ſame Bore, and ſo order'd, as to be open'd, or ſtopt at pleaſure. The Water in this Veſſel was always kept at the ſame Height, by means of a third Veſſel, which ſupply'd the Water, as faſt as it ran out at the round Holes in the Bottom; and a large Aperture, in the ſide of the ſecond Veſſel near the Top,

prevented

prevented the Water in it from exceeding the due Height. To break the Force of the Water running into the two lowermoſt Veſſels, they were each of them divided by a Board, placed perpendicular, but not reaching the Bottom, which ſeparated the Part where the Water came in, from that where it went out.

The *Apparatus* being thus fixt, three of the round Holes in the Bottom of the ſecond Veſſel were unſtopt, to let the Water run into the lower Veſſel. Where not running out at the Section in the ſide, ſo faſt as it came in from above, it roſe to a conſiderable Height above the Surface of the dead Water; after which, the Efflux of the Water becoming equal to the Influx, it roſe no higher.

In other Tryals the Water being ſuffer'd to run from 6, from 9, 12, and 15 of the round Holes, the Water roſe ſucceſſively to greater Heights, before the Section diſcharged it as faſt, as it came in.

The Experiment being repeated with opening other Numbers of the round Holes, with Sections of different Breadths, and at different Depths of the dead Water, the ſeveral Heights, to which the Water roſe in the Veſſel, were carefully obſerv'd and ſet down.

Other Experiments were made by placing the lower Veſſel on dry Ground, and the ſeveral Heights to which the Water roſe in the Veſſel, according as different Quantities were ſuffer'd to run in, were likewiſe obſerv'd, and found agreeable to the Heights deduced by Calculation from the general Theorem above-mention'd, concerning the ſimple Motion of Water.

The Learned Author comes now to apply theſe Experiments, in order to diſcover the Theory of mixt motion, to which end he lays down theſe two Hypotheſes.

First, he ſuppoſes, that the Velocity of the running Water is every where in a ſubduplicate *ratio* of the Depth, and conſequently the Quantities diſcharged may be repreſented by the Parabolick Spaces, juſt as in the caſe of the ſimple Motion of Water.

Secondly, that the Velocity of the dead Water, is the ſame at all Depths, and equal to the greateſt Velocity of the running Water. Wherefore the Quantity of dead Water diſcharged may be repreſented by a Rectangle, whoſe Height repreſents the Depth of the ſtagnant Water, and whoſe Baſe is the greateſt Ordinate of the Parabolick Space abovementioned.

Having thus contrived a way of repreſenting the Quantities of Water diſcharged by the mixt Motion, as had been done before for the ſimple Motion of Water, our Author obſerves that the Velocities of the Water iſſuing out at different Depths, and conſequently the Parabolick Spaces repreſenting the Quantities of Water expended, muſt be leſs in the mixt, than in the ſimple Motion.

In order therefore to find a general Rule for determining the Proportion between the Parabolick Spaces, which repreſent the Quantities diſcharged by the mixt and ſimple Motion, or between the Parameters of thoſe *Parabolas*, he draws ſome Obſervations from the foregoing Experiments, by the help of which he hopes ſuch a Rule may be found out;

Firſt, he obſerves, that, if the Depth of the running Water continue unchanged, a greater Depth of dead Water requires a leſs Parameter.

Secondly, That this Parameter does not decreaſe in ſo great a Proportion, as the Depth of the Water increaſes.

Thirdly, That, if the Depth of the dead Water decreaſe, or the Depth of the running Water increaſe in ſuch

ſuch manner, that the latter becomes infinitely great in proportion to the former, then the Parameter of the mixt Motion muſt become equal to that of the ſimple Motion.

Fourthly, That, if the Depth of the dead Water become infinitely great in compariſon of the Depth of the running Water, the Parameter of the mixt Motion vaniſhes, or becomes equal to nothing.

The Rule, therefore, which is to be found, ought to agree with all theſe Obſervations, and beſides muſt produce the ſame Quantities of Water by Calculation, as were found by Experiment to anſwer to the ſeveral Depths of running and dead Water, in the above mention'd Tryals.

Upon this Foundation the Learned Author proceeds, in a tentative Method, to find his Rule, and having diſcover'd it, he ſhews by Calculation, that it anſwers all the Conditions before requir'd.

This Rule is expreſt in a pretty high Equation, which, beſides other Operations, requires the extracting the Root of the ſixth Power.

From this Equation is derived another, ſerving to find either the Quantity of Water diſcharg'd, the Depth of the running, or that of the dead Water, the other two of them being given; as likewiſe a third Equation, to find the mean Velocity.

Our Author goes on to ſhew the Uſefulneſs and Neceſſity of conſidering the Doctrine of mixt Motion, in all Queſtions relating to the Courſe of Rivers, the Quantities of Water which they diſcharge, the enlarging or narrowing their Outlets, the ſcouring and deepening their Channels, and the Motion of the Tides in Harbours. Theſe he illuſtrates by ſeveral Deductions from the Equations above mention'd; to render which of greater Evidence, it were to be wiſht, that thoſe Equations

were built upon a more ſolid Foundation than a tentative *Calculus*; and that Allowance had been made for the Velocity impreſt upon the preceding Water in Rivers, by the *impetus* of that which follows, which is omitted by the Author in his Theory, both of mixt and ſimple Motion.

In the Second Book, this Learned Writer propoſes the State of the *Laguna* of *Venice*, as a proper Example, to demonſtrate the Uſefulneſs of his new Theory. He conſiders very minutely the ſeveral Cauſes of choaking up the *Laguna*, examins the Methods propoſed by various Authors for ſcouring and keeping it clear, ſome of which he rejects as impracticable on account of the Expence, others as uſeleſs, or prejudicial; and laſtly delivers his own Opinion.

The principal Cauſes, which he aſſigns, of filling up the *Laguna*, are the Rivers running into it, and the Sea.

The Rivers, which enter it, ariſing out of the *Alps*, and running down with great Rapidity, carry with them, eſpecially after Rains, great Quantities of Soil, which is eaſily ſuſpended in the Water, ſo long as that Swiftneſs of Motion continues. But when they come into the *Laguna*, the Water, upon extending it ſelf over that vaſt Surface, looſes almoſt all its Velocity, and conſequently the Soil and Filth, which before it carry'd with it, ſubſides in great Quantities to the Bottom.

The Remedy our Author propoſes for this Inconvenience, is either wholly to divert the Courſe of the Rivers and carry them, by another Way, directly into the Sea; or at leaſt, to ſecure their Outlets with Sluices, ſo as to ſuffer them to paſs into the *Laguna*, when their Waters are clear; but after great Rains, when they run foul and turbid, to ſtop their Paſſage that way, and let them out by the other Channel into the Sea.

The

The ſecond principal Cauſe of choaking up the *Laguna*, is the Sea. Concerning which our Author obſerves, that the Tide of Flood ſets into the *Laguna* from along the Coaſt of *Iſtria* and *Friuli*, where it is perpetually waſhing away the Land in great Quantities, with which, and the Sand which it raiſes upon high Winds in the Shallows near the Shore, it enters the *Laguna* excedingly turbid and foul; eſpecially, when the Wind blows hard at South-Eaſt, at which times the Tide of Flood is ſeveral Hours longer than the Ebb. This occaſions very high Tides in the *Laguna*, and a great part of the Water, which enters by the Flood, not being carry'd out by the ſubſequent Ebb, has the more time to diſcharge its Soil and Sand in the *Laguna*.

This is an Enemy very hard to deal with, however our Author propoſes ſome Works of ſtrong Piles, and large Stones thrown in between them, to be carried directly forward into the Sea, in order to break the Violence of the Waves, and prevent their waſhing and carrying away the Land.

He ſeems likewiſe to favour a Propoſal made by the late famous *Guglielmini*, and ſome others, to let the Tide enter the *Laguna* by more Paſſages than it is to go out at, in order to make it run out with a greater Velocity, and thereby ſcour and deepen the Channels. But he thinks this Contrivance will ſcarcely perform all that is expected from it; beſides that, it will be attended with great Difficulties in making Works, and Floodgates of a ſufficient Strength, to reſiſt the Violence of the Waters.

He occaſionally combats the Opinion of *Guglielmini*, and moſt other Mathematicians who have thought upon the Subject, that in order to give a greater Velocity to the Water of a River, thereby to ſcour and cleanſe the Channel, it is proper to make the Outlet narrower.

narrower. This our Author maintains to be oftner falſe, than true, and endeavours to ſhew from his Theorem above-mention'd, that making the Outlet narrower, will frequently cauſe the *mean Velocity* of the Waters to become leſs than it was before. But whether a Propoſition of ſuch Conſequence, and ſeemingly ſo well ſupported by Reaſon and Experience, ought to be condemn'd upon the Authority of a Theorem founded only upon a tentative Calculation, muſt be left to the Judgment of the Learned.

II. Apollonii Pergæi *Conicorum Libri Octo, & Sereni Antiſſenſis de Sectione Cylindri & Coni Libri duo. Fol. Reg. E Theatro* Oxon. 1710.

THE worthy Curators of the *Oxford* Preſs having obliged the Publick with a very elegant Edition of the Works of *Euclid*, *Græco-Latinè*, were pleas'd further to proceed in the laudable Intention of giving the reſt of the ancient *Greek Mathematicians* in the ſame beautiful Form: In this Deſign they were chiefly animated by the late learned and beneficent *Dean* of *Chriſt-Church*, Dr. *Henry Aldridge*, who pitching upon *Apollonius*, as moſt proper to ſucceed *Euclid*, engaged the two *Savilian* Profeſſors to take upon them the Care and Pains of the Edition: Dr. *David Gregory* promiſing his Aſſiſtance as to the firſt Four Books, which are ſtill extant in *Greek*; and Dr. *Edm. Halley* undertaking to tranſlate the Fifth, Sixth, and Seventh Books out of *Arabick* (in which Language they were only to be found) and to endeavour to reſtore the Eighth, long ſince wholly loſt. But Dr. *Gregory* ſoon after dying, the Care of the Whole devolved on Dr. *Halley*, who hath ſpared no Pains to render the Work complete.

He in his Preface tells us what Helps he had to perfect the Text, That he had the uſe of two *Greek MSS.* of the firſt Four Books, one of which was Sir *Henry Savil*'s, and is in the *Savilian Study* at *Oxford*, the other is now in the Royal

Royal Society's *Muſeum*, having been lately preſented them by that skilful Mathematician Mr. *William Jones*, F. R. S. That he had only one Manuſcript of *Eutocius*'s Commentary, out of the *Bodley* Library; and two *Greek* Copies, from the *Savilian Study*, of *Pappus*'s Collections, out of whoſe 7th Book he took the *Lemmata*, which ſerve as a Comment on the more difficult Places of his Author; and that he was forced to reviſe and correct the Miſtakes and Improprieties of the Latin Tranſlation of *Commandine*.

As to the latter Books, which were only in *Arabick*, he informs us, that he made uſe of the *Bodley* Tranſcript of a *Manuſcript* that is at *Leyden*, which it ſelf is a late Copy of that ancient *Arabick* Book of the *Conicks*, heretofore *Golius*'s, but ſince purchaſed by that great Patron of Univerſal Learning, *Nareiſſus* late Primate of *Ireland*, who was pleaſed to favour him ſo far as to ſend over into *England* this Original Book, whereby he not only amended ſeveral Faults committed by the Copyiſts in a double Tranſcription, but was alſo aſſured that this *Arabick* Book was a verbal Tranſlation from the Greek; the ſame Schemes markt with the ſame Letters, and the whole Context being the ſame in the firſt four Books of it, as in the Greek *Apollonius*. This valuable Manuſcript, with about 800 others, *Oriental* and *Greek*, has ſince, by the Donation of that moſt venerable Prelate, made a noble Acceſſion to the *Bodley* Library, wherein it is now depoſited. It appears by an *Epigraphe* at the end, to have been written in the Year of Chriſt 1303. and to have been a Copy of a Tranſlation of the Conicks, made ſome Ages before by *Thebit Ben Corah*, but then newly reviſed by that famous *Perſian* Mathematician *Naſir-eddin*, who flouriſh'd about the middle of our thirteenth Century.

Beſides this, the Editor tells us, that on occaſion he conſulted another *Arabick* Manuſcript (heretofore *Ravius*'s) of great Antiquity, being an *Epitome* of the ſame Books by *Abdolmelec* of *Schiraz*, every where agreeing in the Order and Argument with the former, but abridg'd. So that having had theſe Helps, he is in hopes that he has ſo far retrieved thoſe Three Books of *Apollonius*, that the Loſs of the *Greek* Text may henceforth be leſs lamented.

The

The Eighth Book of these Conicks, was wanting in the *Greek* Copies even before the Traduction of them into *Arabick* by *Thebit*. But it having been observ'd that there was a very near relation between the Arguments of the VIIth and VIIIth Books, for that the same *Lemmata* of *Pappus* were common to them both, which are different to all the rest, it seemed that the *Theorems Diorismica* of the VIIth Book were designed to determine the Limits of the *Problemata Diorismena* of the VIIIth; and therefore supposing what those Problems might have been, and their Order from that of the said Theorems, Dr. *Halley* has in XXXIII Propositions given the *Analyses* and *Synthesis* of them, after the Method of the Ancients, every where following the Steps of *Apollonius* to be found in his VIIth Book. This he calls *Conicorum Liber Octavus restitutus*, and may serve the turn, till such time as the Original Eighth Book come to light; if that be not now to be despair'd of.

Because of the Affinity of the Subject, he hath subjoin'd the two Books of *Serenus Antissensis*, the Greek Text of which was never before in print. This was procured by the aboveſaid Reverend *Dean* of *Christchurch* Dr. *Aldrich*, in a collated Copy of three Manuscripts extant in the King's Library at *Paris*, and by him, according to his wonted Goodness and Generosity, freely communicated for the use of the Publick. To this also is added the Latin Translation of *Commandine*, which in many Cases needed Castigation.

As to the Authors themselves little needs be said, they having stood the Test of so many Ages, and been highly valued by the Learned in all Times, especially the Conicks, justly esteemed a Masterpiece in the Geometry of the Ancients: So that it may seem strange, that a Book so excelling in its kind, should not till now have been printed in its native *Greek*, a Tongue so peculiarly adapted to Mathematical Purposes. But this present Edition may make ample Amends, the Paper and the Elegance and Correctness of the Print being remarkable. The Book is now to be had of Mr. *Christopher Bateman* in *Pater-noster Row*, *London*.

Printed for *W.* and *J. Innys*, at the *Prince*'s *Arms* in St. *Paul*'s Church-yard. 1718.

Jos: Banks

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PHILOSOPHICAL TRANSACTIONS.

For the Months of *Jan. Feb. March* and *Apr.* 1718.

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I. *Considerations on the Change of the Latitudes of some of the principal fixt Stars.* By Edmund Halley, R. S. Sec.

HAving of late had occasion to examine the quantity of the Precession of the Equinoctial Points, I took the pains to compare the Declinations of the fixt Stars delivered by *Ptolomy*, in the 3*d* Chapter of the 7*th* Book of his *Almag.* as observed by *Timocharis* and *Aristyllus* near 300 Years before *Christ*, and by *Hipparchus* about 170 Years after them, that is about 130 Years before *Christ*, with what we now find: and by the result of very many Calculations, I concluded that the fixt Stars in 1800 Years were advanced somewhat more than 25 degrees in Longitude, or that the Precession is somewhat more than 50″ *per ann.* But that with so much uncertainty, by reason of the imperfect Observations of the Ancients, that I have chosen in my Tables to adhere to the even proportion of five Minutes in six Years, which from other Principles we are assured is very near the Truth. But while I was upon this Enquiry, I was surprized to find the Latitudes of three of the principal Stars in Heaven directly to contradict the supposed greater *Obliquity* of the *Ecliptick*, which seems confirmed by the Latitudes of most of the rest; they being set down in the old Catalogue, as if the Plain of the Earths Orb had changed its Situation, among the fixt Stars, about 20′ since the time of *Hipparchus*. Particularly all the Stars in *Gemini* are put down, those to the *Northward* of the Ecliptick, with so much less Latitude than we find, and those to the *Southward* with so much more *Southerly* Lati-

Latitude. Yet the three Stars *Palilicium* or the *Bulls* Eye, *Sirius* and *Arcturus* do contradict this Rule directly: for by it, *Palilicium* being in the days of *Hipparchus* in about 10 *gr.* of *Taurus* ought to be about 15 Min. more *Southerly* than at present, and *Sirius* being then in about 15 of *Gemini* ought to be 20 Min. more *Southerly* than now; yet *è contra Ptolomy* places the first 20 Min. and the other 22 more *Northerly* in Latitude than we now find them. Nor are these errors of Transcription, but are proved to be right by the declinations of them set down by *Ptolomy*, as observed by *Timocharis*, *Hipparchus* and himsel, which shew that those Latitudes are the same as those Authors intended. As to *Arcturus*, he is too near the Equinoctial Colure, to argue from him concerning the change of the Obliquity of the Ecliptick, but *Ptolomy* gives him 33′ more *North* Latitude than he now has; and that greater Latitude is likewise confirmed by the Declinations delivered by the aboveſaid Obſervers. So then all theſe three Stars are found to be above half a degree more *Southerly* at this time than the Antients reckoned them. When on the contrary at the ſame time the bright Shoulder of *Orion* has in *Ptolomy* almoſt a degree more *Southerly* Latitude than at preſent. What ſhall we ſay then? It is ſcarce credible that the Antients could be deceived in ſo plain a matter, three Obſervers confirming each other. Again theſe Stars being the moſt conſpicuous in Heaven, are in all probability the neareſt to the Earth, and if they have any particular Motion of their own, it is moſt likely to be perceived in them, which in ſo long a time as 1800 Years may ſhew it ſelf by the alteration of their places, though it be utterly imperceptible in the ſpace of a ſingle Century of Years. Yet as to *Sirius* it may be obſerved that *Tycho Brahe* makes him 2 Min. more *Northerly* than we now find him, whereas he ought to be above as much

much more *Southerly* from his Ecliptick, (whose Obliquity he makes 2 ½ greater than we esteem it at present) differing in the whole 4 ½ Min. One half of this difference may perhaps be excused, if refraction were not allowed in this Case by *Tycho*; yet two Minutes, in such a Star as *Sirius*, is somewhat too much for him to be mistaken

But a further and more evident proof of this change is drawn from the Observation of the application of the Moon to *Palilicium Anno Christi* 509 *Mart.* 11°. when in the beginning of the Night the Moon was seen to follow that Star very near, and seemed to have Eclipsed it, ἐπέβαλε γὰρ ὁ ἀστὴρ τῷ φωτὶ τῆς διχοτομίας μέρει τῆς κύρτης περιφερείας τὸ πεφωτισμένου μέρους. *i. e Stella apposita erat parti per quam bisecabatur limbus Lunæ illuminatus*, as *Bullialdus*, to whom we are beholden for this Antient Observation, has translated it. Now from the undoubted principles of Astronomy, it was impossible for this to be true at *Athens*, or near it, unless the Latitude of *Palilicium* were much less than we at this time find it. *Vide Bullialdi Astr. Philolaica, pag.* 172.

This Argument seems not unworthy of the *Royal Society's* Consideration, to whom I humbly offer the plain Fact as I find it, and would be glad to have their Opinion.

But whether it were really true, that the Obliquity of the Ecliptick was, in the time of *Hipparchus* and *Ptolemy*, really 22 Min. greater than now, may well be questioned; since *Pappus Alexandrinus*, who lived but about 200 Years after *Ptolemy*, makes it the very same that we do. *Vide Pappi Collect. Lib.* VI. *Prop.* 35.

II. *An*

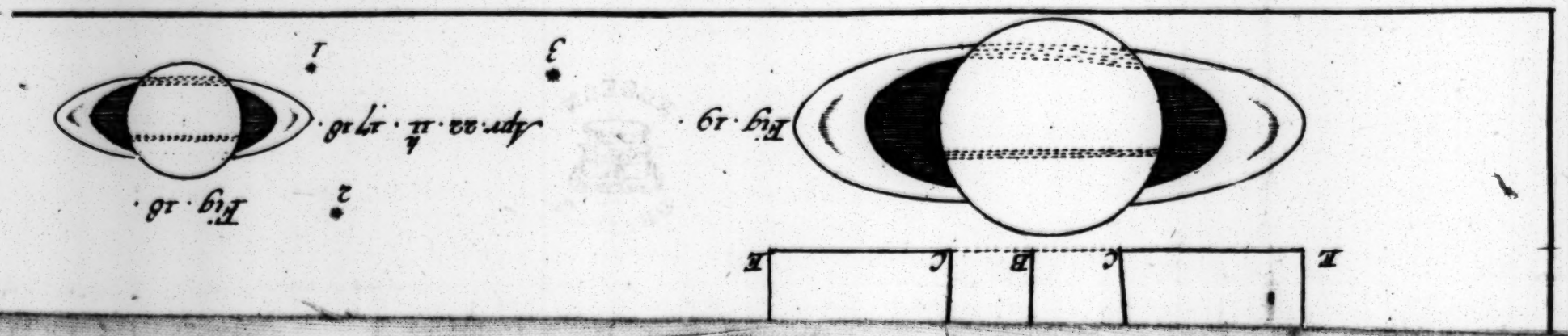
A
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Fig. 19.
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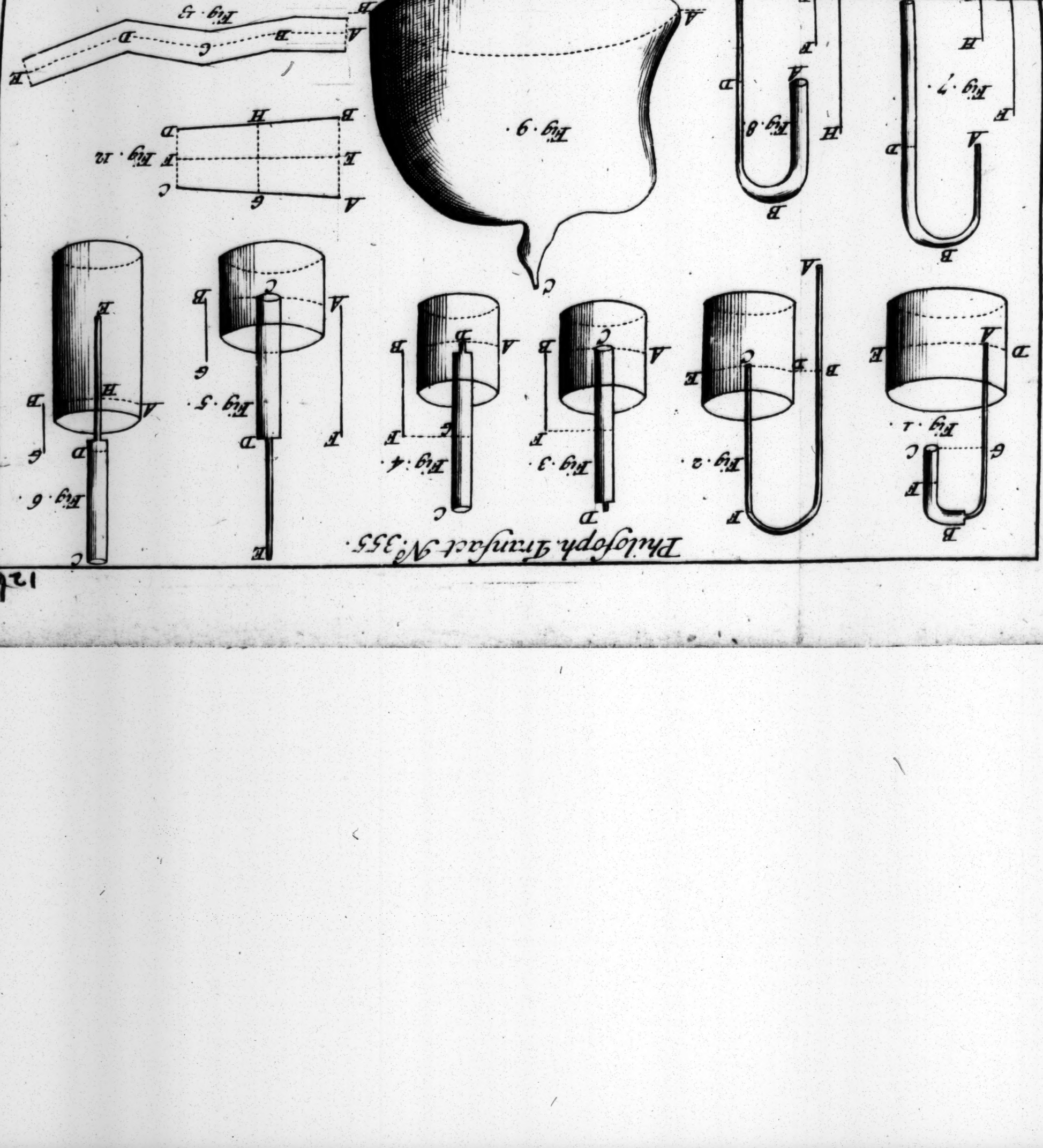

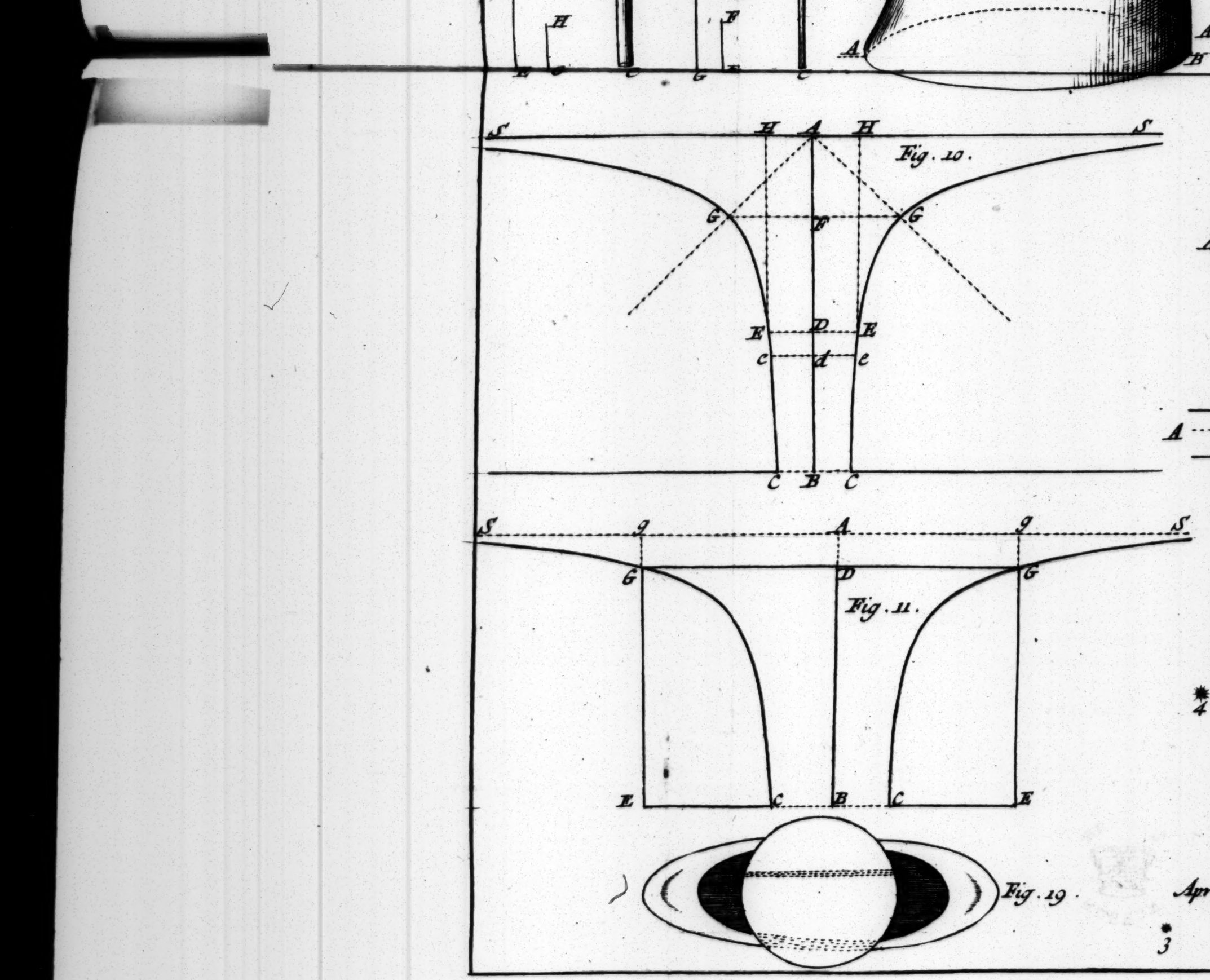

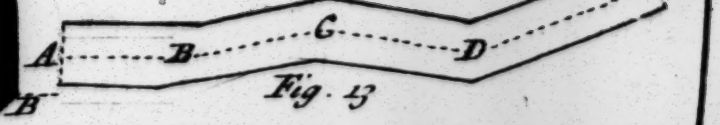

Fig. 13

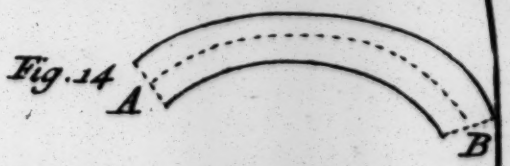

Fig. 14

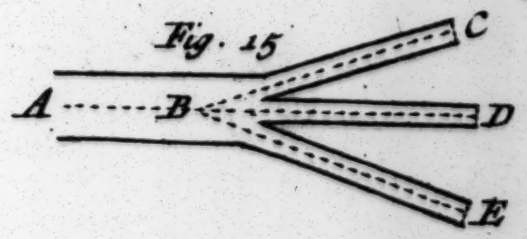

Fig. 15

Fig. 10.

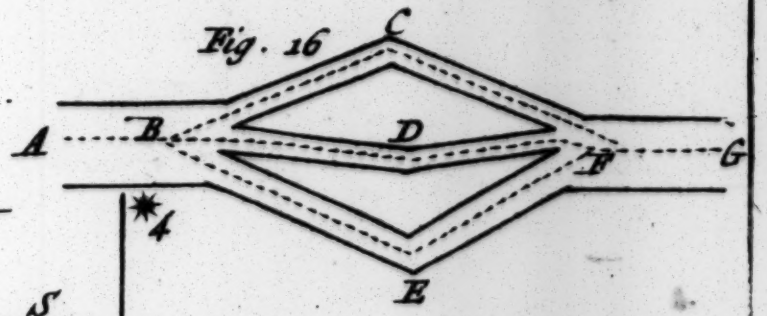

Fig. 16

Fig. 11.

Fig. 19.

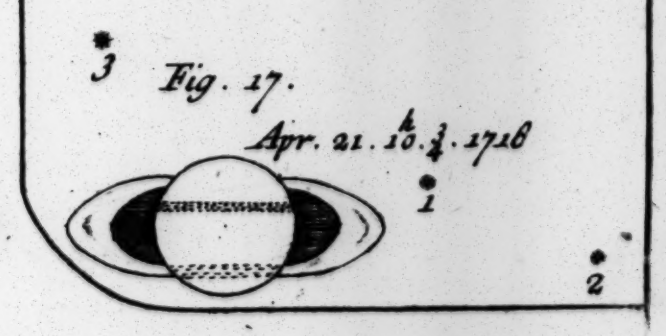

Fig. 17.

Apr. 21. 10. ¾. 1716

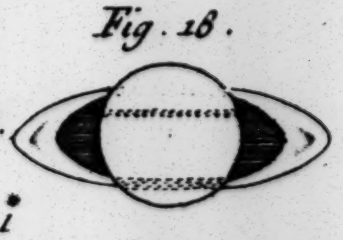

Fig. 18.

Apr. 22. 11. 1716.

II. *An Account of ſome Experiments ſhown before the* Royal Society; *with an enquiry into the cauſe of the Aſcent and Suſpenſion of Water in Capillary Tubes.* *By* James Jurin, *M. D. and* R. Soc. S.

SOme Days ago a Method was propoſed to me by an ingenious Friend, for making a perpetual Motion, which ſeem'd ſo plauſible, and indeed ſo eaſily demonſtrable from an Obſervation of the late Mr. *Hawksbee*, ſaid to be grounded upon Experiment, that, tho' I am far from having any Opinion of attempts of this Nature, yet, I confeſs, I could not ſee why it ſhould not ſucceed. Upon tryal indeed I found my ſelf diſappointed. But as ſearches after things impoſſible in themſelves are frequently obſerv'd to produce other diſcoveries, unexpected by the Inventer; ſo this Propoſal has given occaſion not only to rectify ſome miſtakes into which we had been led, by that ingenious and uſeful Member of the *Royal Society* above named, but likewiſe to detect the real Principle, by which Water is rais'd and ſuſpended in Capillary Tubes, above the Level.

My Friend's Propoſal was as follows.

Fig. 1. Let *A B C* be a capillary Siphon, compos'd of two Legs *A B*, *B C*, unequal both in length and Diameter; whoſe longer and narrower Leg *A B* having its orifice *A* immerſt in Water, the Water will riſe above the Level, till it fills the whole Tube *A B*, and will then continue ſuſpended. If the wider and ſhorter Leg *B C*, be in like manner immerſt, the Water

will only riſe to ſome height as *FC*, leſs than the entire height of the Tube *B C*.

This Siphon being fill'd with Water, and the Orifice *A* ſunk below the Surface of the Water *D E*, my Friend reaſons thus.

Since the two Columns of Water *A B* and *FC*, by the Suppoſition, will be ſuſpended by ſome Power acting within the Tubes they are contain'd in, they cannot determine the Water to move one way, or the other. But the Column *B F*, having nothing to ſupport it, muſt deſcend, and cauſe the Water to run out at *C*. Then the preſſure of the Atmoſphere driving the Water upward through the Orifice *A*, to ſupply the Vacuity, which would otherwiſe be left in the upper part of the Tube *B C*, this muſt neceſſarily produce a perpetual Motion, ſince the Water runs into the ſame Veſſel, out of which it riſes. But the Fallacy of this reaſoning appears upon making the Experiment.

Exp. 1. For the Water, inſtead of running out at the Orifice *C*, riſes upward towards *F*, and running all out of the Leg *B C*, remains ſuſpended in the other Leg to the height *A B*.

Exp. 2. The ſame thing ſucceeds upon taking the Siphon out of the Water, into which its lower Orifice *A* had been immerſt, the Water then falling in drops out of the Orifice *A*, and ſtanding at laſt at the height *A B*. But in making theſe two Experiments it is neceſſary that *A G* the difference of the Legs exceed *FC*, otherwiſe the Water will not run either way.

Exp. 3. Upon inverting the Siphon full of Water, it continues without Motion either way.

The reaſon of all which will plainly appear, when we come to diſcover the Principle, by which the Water is ſuſpended in Capillary Tubes.

Mr.

Mr. *Hawksbee*'s Obſervation is as follows.

Fig. 2. Let *A B F C* be a capillary Siphon, into the which the Water will riſe above the Level to the height *C F*, and let *B A* be the depth of the Orifice of its longeſt Leg below the Surface of the Water *D E*. Then the Siphon being fill'd with Water, if *B A* be not greater than *C F*, the Water will not run out at *A*, but will remain ſuſpended.

This ſeems indeed very plauſible at firſt ſight. For ſince the Column of Water *F C* will be ſuſpended by ſome power within the Tube, why ſhould not the Column *B A*, being equal to, or leſs than the former, continue ſuſpended by the ſame Power ?

Exp. 4. In fact, if the orifice *C* be lifted up out of the Water *D E*, the Water in the Tube will continue ſuſpended, unleſs *B A* exceed *FC*.

Exp. 5. But when *C* is never ſo little immerſt in the Water, immediately the Water in the Tube runs out in drops at the Orifice *A*, tho' the length *AB* be conſiderably leſs than the height *C F*.

Mr. *Hawksbee* in his Book of Experiments has advanced another Obſervation, namely, that the ſhorter Leg of a Capillary Siphon, as *AB FC*, muſt be immerſt in the Water to the depth *FC*, which is equal to the height of the Column, that would be ſuſpended in it, before the Water will run out at the longer Leg.

Exp. 6. From what miſtake this has proceeded, I cannot imagine; for the Water runs out at the longer Leg, as ſoon as the Orifice of the ſhorter leg comes to touch the Surface of the ſtagnant Water, without being at all immerſt therein.

Having proceeded thus far in obedience to the commands of this illuſtrious Society, I beg leave to go a little farther, and to enquire into the cauſe of the aſcent and ſuſpenſion of Water in capillary Tubes.

That this Phænomenon is no way owing to the preſſure of the Atmoſphere, has been, I think ſufficiently prov'd by Mr. *Hawksbee*'s Experiments.

And that the cauſe aſſign'd by the ſame ingenious and inquiſitive Perſon, namely the attraction of the concave Surface, in which the ſuſpended Liquor is contain'd, is likewiſe inſufficient for producing this effect, I thus demonſtrate.

Since in every capillary Tube the height, to which the Water will ſpontaneouſly aſcend, is reciprocally as the Diameter of the Tube, it follows, that the Surface containing the ſuſpended Water in every Tube is always a given Quantity: but the Column of Water ſuſpended is, as the Diameter of the Tube. Therefore, if the attraction of the containing Surface be the cauſe of the Waters ſuſpenſion; it will follow, that equal cauſes produce unequal effects, which is abſurd.

To this it may perhaps be objected, that, in two Tubes of unequal Diameters, the circumſtances are different, and therefore the two Cauſes, tho' they be equal in themſelves, may produce effects that are unequal. For the leſſer Tube has not only a greater Curvature, but thoſe parts of the Water, which lie in the middle of the Tube, are nearer to the attracting Surface, than in the wider. But from this if any thing follows, it muſt be, that the narrower Tube will ſuſpend the greater quantity of Water, which is contrary to Experiment. For the Columns ſuſpended are as the Diameters of the Tubes.

But as Experiments are generally more ſatisfactory in things of this nature, than Mathematical reaſonings, it may not be amiſs to make uſe of the following, which appear to me to contain an *Experimentum Crucis*.

Fig.

Fig. 3. The Tube *C D* is compoſed of two Parts, in the wider of which the Water will riſe ſpontaneouſly to the height *B F*, but the narrower Part, if it were of a ſufficient length, would raiſe the Water to a height equal to *C D*.

Exp. 7. This Tube being fill'd with Water, and the wider end *C* immerſt in the ſtagnant Water *A B*, the whole continues ſuſpended.

Exp. 8. *Fig.* 4. The narrower end being immerſt, the Water immediately ſubſides, and ſtands at laſt at the height *D G* equal to *B F*.

From which it is manifeſt, that the ſuſpenſion of the Water in the former of theſe Experiments is not owing to the attraction of the containing Surface: ſince, if that were true, this Surface being the ſame, when the Tube is inverted, would ſuſpend the Water at the ſame height.

Having ſhown the inſufficiency of this Hypotheſis, I come now to the real cauſe of that Phænomenon, which is the attraction of the Periphery, or Section of the Surface of the Tube, to which the upper Surface of the Water is contiguous and coheres.

For this is the only part of the Tube, from which the Water muſt recede upon its ſubſiding, and conſequently the only one, which by the force of its coheſion, or attraction, oppoſes the deſcent of the Water.

This likewiſe is a cauſe proportional to the effect, which it produces; ſince that Periphery, and the Column ſuſpended, are both in the ſame proportion as the Diameter of the Tube.

Tho' from either of theſe particulars it were eaſy to draw a juſt Demonſtration, yet to put the matter out of all doubt, it may be proper to confirm this aſſertion, as we have done the former, by actual Experiment.

Fig.

Fig. 5. Let therefore *E D C* be a Tube, like that made uſe of in the 7th and 8th Experiments, except that the narrower Part is of a greater length; and let *A F* and *B G* be the heights, to which the Water would ſpontaneouſly riſe in the two Tubes *E D* and *D C*.

Exp. 9. If this Tube have its wider Orifice *C* immerſt into the Water *A B*, and be fill'd to any height leſs than the length of the wider Part, the Water will immediately ſubſide to a Level with the point *G*; but if the Surface of the contain'd Water enter never ſo little within the ſmaller Tube *E D*, the whole Column *D C* will be ſuſpended, provided the length of that Column do not exceed the height *A F*.

In this Experiment it is plain, that there is nothing to ſuſtain the Water at ſo great a height, except the contact of the Periphery of the leſſer Tube, to which the upper Surface of the Water is contiguous. For the Tube *D C*, by the Suppoſition, is not able to ſupport the Water at a greater height than *B G*.

Exp. 10 *Fig.* 6. When the ſame Tube is inverted, and the Water is rais'd into the lower extremity of the wider Tube *C D*, it immediately ſinks, if the length of the ſuſpended Column *D H* be greater than *G B*; whereas in the Tube *D E* it would be ſuſpended to the height *A F*. From which it manifeſtly appears, that the ſuſpenſion of the Column *D H* does not depend upon the attraction of the Tube *D E*, but upon the Periphery of the wider Tube, with which its upper Surface is in contact.

For the ſake of thoſe who are pleas'd with ſeeing the ſame thing ſucceed in different manners, we ſubjoin the two following Experiments, which are in ſubſtance the ſame with the 9th and 10th.

Fig. 7. *A B C* is a Siphon, in whoſe narrower, and ſhorter Leg *A B*, if it were of a ſufficient length, might be

be ſuſpended a Column of Water of the height *E F*; but the longer and wider Leg *B C* will ſuſpend no more than a Column of the length *G H*.

Exp. 11. This Siphon being fill'd with Water, and held in the ſame Poſition as in the Figure, the Water will not run out at *C* the Orifice of the longer Leg, unleſs *D C*, the difference of the Legs *A B* and *B C*, exceed the length *E F*.

Fig. 8. *Exp.* 12. If the narrower Leg *B C* be longer than *A B*, the Water will run out at *C*, if *D C* the difference of the Legs exceed *E F*; otherwiſe it will remain ſuſpended.

In theſe two Experiments it is plain, that the Columns *D C* are ſuſpended by the attraction of the Peripheries at *A*, ſince their lengths are equal to *E F*, or to the length of the Column, which by the ſuppoſition thoſe Peripheries are able to ſupport; whereas the Tubes *B C* will ſuſtain Columns, whoſe lengths are equal to *G H*.

Tho' theſe Experiments ſeem to be concluſive, yet it may not be improper to prevent an Objection, which naturally preſents it ſelf, and which at firſt view may be thought ſufficient to overturn our Theory.

Fig. 5. For ſince a Periphery of the Tube *E D* is able to ſuſtain no more than a Column of the length *A F*, contain'd in the ſame Tube; how comes it to ſuſtain a Column of the ſame length in the wider Tube *D C*, which is as much greater than the former, as the Section of the wider Tube exceeds that of the narrower?

Fig. 6. Again, if a Periphery of the wider Tube *D C* be able to ſuſtain a Column of Water in the ſame Tube, of the length *B G*; why will it ſupport no more than a Column of the ſame length in the narrower Tube *E D*?

Which

Which Queries may likewiſe be made with regard to the 11th and 12th Experiments.

The anſwer is eaſy, for the Moments of thoſe two Columns of Water are preciſely the ſame, as if the ſuſtaining Tubes *E D* and *C D*, were continued down to the Surface of the ſtagnant Water *A B*; ſince the velocities of the Water, where thoſe Columns grow wider, or narrower, are to the velocities at the attracting Peripheries, reciprocally as the different Sections of the Columns.

Fig. 9. *Exp.* 13. From which conſideration ariſes this remarkable Paradox. That a Veſſel being given of whatſoever Form, as *A B C*, and containing any aſſignable quantity of Water, how great ſoever; that whole quantity of Water may be ſuſpended above the Level, if the upper part of the Veſſel *C* be drawn out into a capillary Tube of a ſufficient fineneſs.

But whether this Experiment will ſucceed, when the height of the Veſſel is greater than that, to which Water will be rais'd by the preſſure of the Atmoſphere, and how far it will be alter'd by a *Vacuum*, I may perhaps have the honour of giving an account to the *Society* ſome other time, not being perfectly ſatisfy'd with thoſe Tryals which I have hitherto had the opportunity of making.

Having diſcover'd the cauſe of the ſuſpenſion of Water in capillary Tubes, it will not be difficult to account for the ſeemingly ſpontaneous aſcent of it. For, ſince the Water, that enters a capillary Tube as ſoon as it's Orifice is dipt therein, has it's gravity taken off by the attraction of the Periphery with which it's upper Surface is in contact, it muſt neceſſarily riſe higher, partly by the preſſure of the ſtagnant Water, and partly by the attraction of the Periphery immediately above that, which is already contiguous to it.

It

It might now be ſhown, how naturally the various, and ſeemingly contrary appearances of the above mention'd Experiments are deducible from this Theory; but this is ſo eaſy, that it is needleſs to inſiſt upon it; and our diſcourſe upon this minute Subject has been already ſo tedious, that we could ſcarce hope for Pardon, unleſs it were directed to thoſe, who are ſenſible to how many of the greater, and more conſiderable, Phænomena of Nature this Doctrine is applicable.

P S. When this Paper was reading before the Society, I found that our incomparable Preſident was already acquainted with the above-mentioned Principle, and I have ſince met with ſeveral Paſſages in the 31ſt Query ſubjoin'd to the late Edition of his *Opticks* which plainly ſhew, that he was Maſter of it, when they were written.

I muſt do the ſame Juſtice to that excellent Mathematician Mr. *John Machin*, Profeſſor of Aſtronomy in *Greſham* College.

To theſe two worthy Perſons I am obliged for the following Obſervation, That, what I call a Periphery, or Section of the concave ſurface of the Tube, is really a ſmall Surface, whoſe Baſe is that Periphery, and whoſe height is the diſtance, to which the attractive power of the Glaſs is extended.

III. *De Motu Aquarum fluentium. Authore eodem* D. Jacobo Jurin, *M. D.*

AQuæ Motum ex imi vaſis foramine defluentis ſæpe videmus, tum in ipſa re Hydraulica, tum in ejus Principiis ad Oeconomiam Animalem applicandis, aliis cum Potentiis comparari. Cujus Motus quantitatem cum hactenus nemo, quod ſciam, recte determinaverit, uſurpare ſolent ejus loco ſcriptores Hydraulici Columnæ aqueæ pondus foramini incumbentis. Quod qui faciunt, id ſane neutiquam animum advertunt fieri omnino non poſſe, ut Motus aliquis cum pondere quieſcente conferatur. Poterit autem Aquæ defluentis Motus facili opera definiri hunc in modum.

Fig. 10. Sit S H A H S Aquæ ſuperficies infinita, *C C* foramen circulare in fundo factum. *A B* recta perpendicularis per foraminis centrum ducta, *S G C C G S* Columna ſive Cataracta Aquæ per foramen *C C* decurrentis, *S G C* Curva cujus rotatione circa Axem *A B* generatur Solidum, ſive Cataracta, *S G C C G S*. Aqua enim cum libere, & motu accelerato deſcendat ad normam corporum omnium gravium, neceſſariò in minorem amplitudinem contrahitur, prout majorem velocitatem acquirit inter cadendum, & profluit ex foramine *C C* ea cum velocitate, quæ cadendo ab altitudine *A B* comparatur.

Velocitas autem corporis gravis cadendo genita, ex *Galilæi* demonſtratis, rationem obtinet ſubduplicatam altitudinis unde cecidit. Quare, ſi ducatur ad Curvam *S G C* Ordinata quævis *D E* atque ipſa *D E* vocetur *y*, & *A D x*, exponetur velocitas Aquæ in ſectione *E E*

per

per $\sqrt{x}$, & Factum ex ea velocitate ducta in ipsam sectionem per $\sqrt{x} \times y^2$.

Quod Factum est ut moles Aquæ dato temporis spatio per eam sectionem transeuntis; cumque eadem Aquæ moles dato tempore per singulas Cataractæ sectiones transeat, proinde Factum istud perpetuo sibi constabit, eritque $\sqrt{x} \times y^2 = 1$, & $x y^4 = 1$.

Quæ est Æquatio Curvæ SGC, cujus partem, intra datum vas comprehensam, delineavit, ejusdemque Æquationem non obscure indicavit Magnus *Newtonus*, *Prop.* 36. *Libr.* 2. *Princip.* qui primus omnium veram Aquæ effluentis velocitatem, ex genuinis Principiis deductam, Orbi Literato exposuit.

Est autem ipsa Curva Hyperboloeides quarti Ordinis, cujus altera Asymptotos est recta AS ad Horizontem parallela, altera AB eidem perpendicularis.

Hujus Potestas est Quadrato-Cubus Ordinatæ FG, ductæ ad punctum G, ubi recta AG, bisecans angulum ab Asymptotis comprehensum, Curvæ occurrit.

Spatium $SADES$, inter Curvam SGE, Ordinatam DE & Asymptotos AD, AS inclusum, æquale est quatuor partibus tertiis Rectanguli HD, sub Abscissa AD & Ordinata DE contenti. Estque proinde Spatium SHE pars tertia ejusdem Rectanguli.

Solidum $SGEEGS$, convolutione spatii $SADES$, circa Axem AD, generatum, duplum est Cylindri incumbentis sectioni EE. Unde Solidum cavum, quod gignit conversio spatii $SHEGS$, circa eundem Axem, Cylindro incumbenti æquale est. Quæ omnia facili calculo inveniuntur per Methodum Fluxionum inversam.

Theorema I.

Aquâ ex vase amplitudinis infinitæ, per foramen circulare in fundo factum, decurrente, Motus totius Cataractæ aqueæ Horizontem versus æqualis est Motui Cy-

lindri aquei, ſub ipſo foramine & altitudine Aquæ, cujus velocitas æquet velocitatem Aquæ per foramen effluentis; vel æqualis eſt Motui molis Aquæ, quæ dato quovis tempore effluit, cujus ea ſit velocitas, qua percurratur eodem dato tempore ſpatium æquale altitudini Aquæ.

Demonſtratio primæ partis.

Ducatur ad Curvam *SGC* alia Ordinata *de*, priori *DE* quam proxima.

Curvâ circa Axem *AB* conversâ, generabunt Ordinatæ *DE*, *de*, Circulos duos, quibus intercipitur Solidum naſcens *EEee*. Id ſolidum æquale eſt Facto ex altitudine *Dd* ducta in ſectionem *EE*, & Motus ejus æquatur Facto ex ipſo ſolido ducto in velocitatem ejuſdem, ſive Facto ex altitudine *Dd*, ſectione *EE*, & velocitate Aquæ in ea Sectione. Cumque ſupra oſtenſum ſit, Factum ex quavis Sectione Cataractæ & velocitate Aquæ in ea Sectione, quantitatem eſſe conſtantem, erit proinde Motus totius Cataractæ æqualis Facto ex quantitate illa conſtante ducta in Summam omnium altitudinum *Dd*, ſive in ipſam *AB*, hoc eſt, Motui Cylindri ſub ipſo foramine & altitudine Aquæ, cujus velocitas æquet velocitatem Aquæ per foramen effluentis. *Q. E. D.*

Corol. 1. Data altitudine Aquæ, erit Motus Cataractæ in ratione foraminis.

2. Dato foramine, erit Motus Cataractæ in ratione ſeſcuplicata altitudinis, ſive in ratione triplicata velocitatis, qua Aqua per foramen exit.

3. Dato Motu Cataractæ, erit foramen reciprocè in ratione ſeſcuplicata altitudinis, vel reciprocè in ratione velocitatis triplicata.

Demonſtratio ſecundæ partis.

Moles Aquæ dato tempore effluentis eſt ad Cylindrum ſub ipſo foramine & altitudine Aquæ, ut longitudo quam Aqua effluens æquabili velocitate dato iſto

tempore

tempore percurſura ſit, ad altitudinem Aquæ. Cumque velocitas, quæ tribuitur moli Aquæ effluentis, ſit ad velocitatem Cylindri reciproce in eadem ratione, erunt Motuum quantitates utrinque æquales. *Q. E. D.*

Corol. 1. Data altitudine Aquæ & mole effluente, Motus Cataractæ eſt in ratione inverſa temporis quo iſta moles effluit.

2. Data altitudine & tempore, Motus Cataractæ eſt ut moles Aquæ tempore iſto effluentis.

3. Dato tempore & mole Aquæ effluentis, erit Motus Cataractæ in ratione altitudinis.

4. Dato Motu Cataractæ & altitudine, moles effluens eſt in ratione temporis.

5. Dato Cataractæ Motu & mole Aquæ effluentis, altitudo eſt ut tempus.

6. Dato tempore & Motu Cataractæ, erit Aquæ effluentis moles reciprocè ut altitudo.

Theorema II.

Fig. 11. Si capiatur BA, quæ ſit ad BD, ut DG^4 ad $DG^4 - BC^4$; Aqua decurrente ex dato vaſe Cylindrico ſemper pleno $GGEE$, per foramen circulare CC in fundo medio factum, Motus Cataractæ aqueæ Horizontem verſus æqualis erit Motui Cylindri ſub foramine & altitudine AB, cujus velocitas æquet velocitatem Aquæ per foramen exeuntis; vel erit æqualis Motui molis Aquæ quæ dato quovis tempore effluit, cujuſque ea ſit velocitas, qua percurratur eodem dato tempore ſpatium æquale altitudini AB.

Demonſtratio primæ partis.

Ducatur AS ipſi DG parallela, & Aſymptotis AS, AB, per puncta G, C deſcripta concipiatur Curva *Newtoniana* SGC.

Ut conſtet Aquæ altitudo, ſupplendus eſt exeuntis locus Cylindro aqueo $ggGG$, deſcendente cum ea velocitate

locitate uniformi, quæ acquiritur cadendo ab *A* ad *D*, quemadmodum docet Vir incomparabilis Propoſitione prædicta.

Motui hujus Cylindri æquatur, per Theorema ſuperius, Motus Cataractæ *S S G G*. Ergo Motus Aquæ deſcendentis, cum ſit compoſitus ex Motu Cylindri aquei *g g G G*, & Motu Cataractæ *G G C C*, æqualis eſt Motui Cataractæ integræ *S G C C G S*, *h. e.* per Theorema primum, Motui Cylindri aquei ſub foramine & altitudine *A B*, cujus velocitas æqualis ſit velocitati Aquæ per foramen decurrentis. *Q E. D.*

Pars ſecunda ſequitur ex priore.

Corol. 1. Oriuntur hinc omnia Propoſitionis præcedentis Corollaria, ſubſtituendo altitudinem *A B*, pro Aquæ altitudine.

2. Si vas alia figura fuerit, atque Cylindrica; aut foraminis figura pro circulari fuerit quadrata, triangularis, vel qualiſcunque; aut ipſum foramen non ſit in medio fundo ſitum, vel etiam in latere vaſis factum; idem erit Motus Cataractæ, ſcilicet æqualis Motui Priſmatis aquei ſub foramine & altitudine *A B*, cujus velocitas par ſit velocitati Aquæ effluentis. Nam eadem Aquæ moles, cum eadem velocitate atque in priori Hypotheſi, tum per ipſum foramen, tum per ſingulas Cataractæ ſectiones tranſibit.

3. Si vaſis Diameter permagnam rationem obtineat ad Diametrum foraminis, negligi poterit altitudo *A D*, & vaſis ipſius altitudo pro altitudine Cylindri, vel Priſmatis aquei, uſurpari.

Hactenus caſum illum particularem, quo Aqua, Gravitatis vi, ex vaſe defluit, ſeorſim conſideravimus. Id eo fecimus lubentius, tum quod illum fere ſolum adhibere ſoleant Mathematici, quoties agitur de Fluidorum impetu, tum quod Curvæ Hyperbolicæ ſupra expoſitam proprietatem, qua Cataractam Aquæ deſcendentis formant,

mat, non indignam cenſeamus contemplatione Geometrarum. Alioqui potuiſſet iſte caſus nullo negotio deduci ex Theoremate generali, quod proximo loco proponemus.

Theorema III.

Fig. 12. Aqua fluente per Canalem plenum quemcunque *A B C D* ſecundum lineam *E F*, cui ſit perpendiculare utrumque Canalis orificium *A B* & *C D*, Motus Aquæ verſus Orificium *C D*, ſive Motus impedimenti, quod in ipſo orificio oppoſitum ſiſtat Motum totius Aquæ, æqualis eſt Motui Priſmatis aquei ſub qualibet Sectione Canalis *C H* & linea directionis, ſive longitudine Canalis *E F*, quod moveatur eadem cum velocitate, qua Aqua fluit per iſtam Sectionem: ſive æqualis Motui molis Aquæ, quæ dato quovis tempore effluit ex Canali, cujuſque ea ſit velocitas, qua percurratur eodem dato tempore ſpatium æquale longitudini Canalis.

Caſ. 1. Sit linea directionis recta quævis *E F*.

Facile demonſtratur pars prima eodem modo, quo Theorema primum. Eſt enim Factum ex quavis ſectione Canalis *C H*, & Aquæ velocitate in ea Sectione, quantitas conſtans.

Pars ſecunda ſequitur ex prima.

Caſ. 2. *Fig.* 13. Si linea directionis *A B C D E*, ex pluribus rectis *A B*, *B C*, *C D*, *E F*, ad ſeſe invicem inclinatis ſit compoſita, idem erit Aquæ Motus. Nam Motus Aquæ in toto Canali compoſito *A B C D E*, conficitur ex Motibus Aquæ in partibus Canalis *A B*, *B C*, *C D*, *D E*, additis ſibi invicem. Statuimus autem Aquam fluentem ſecundum rectam *A B*, mutata iſta directione in aliam, qua feratur ſecundum rectam *B C*, nihil ex Motu deperdere. Leges enim illas, quæ in motu corporum ſolidorum obſervantur, quoties eorundem

directio

directio mutatur, fluida non ſequuntur. Alioqui fi
dum, mutata directione in aliam priori perpendicu
rem, penitus ſiſteretur, quod Experimentis neutiqu
deprehenditur. Aqua porro ex Vaſis foramine exilie
ſive deorſum, ſive ſecundum Horizontis planum, ſ
rectâ ſurſum feratur. eandem obtinet velocitatem. Qu
ſi aliquando vel ratiocinio ſubtiliori, vel Experimen
innoteſcet, aliquam Motus imminutionem ex mutata
rectione proficiſci, erit ejuſdem ratio habenda.

Si Curva fuerit linea directionis *AB*, referetur
hunc Caſum, quippe quæ ex pluribus rectulis co
fecta concipi queat. *Fig.* 14.

Caſ. 3. *Fig.* 15. Si diviſus fuerit Canalis *AB* in pl
res ramos *BC*, *BD*, *BE*, longitudine æquales, eade
ratione invenietur Aquæ Motus, uſurpando pro lin
directionis longitudinem *ABD*, compoſitam ex lo
gitudine Canalis principis *AB*, & longitudine cujuſv
rami *BD*. Perinde autem eſt, ſive Aqua à Canali pri
cipe verſus ramos, ſive à ramis fluxerit verſus princ
pem Canalem. Quod ſi rami fuerint inæquales, inv
niendus eſt Motus Aquæ in ſingulis ramis, adhibend
pro linea directionis longitudinem confectam ex long
tudine cujuſque rami, & longitudine principis Canalis.

Nullo negotio deducitur ex Caſu ſecundo.

Caſ 4. *Fig.* 16. Si rami æquales, in quos diſtributu
eſt Canalis *AB*, iterum in Canalem unicum *FG* coll
gantur, ad Motum Aquæ inveniendum adhibenda e
pro linea directionis longitudo integra *ABDFG*, co
fecta ex longitudine principis Canalis *AB*, rami cuju
vis *BDF*, & Canalis recompoſiti *FG*. Si Rami ſint in
æquales, inveniendus eſt in ſingulis Aquæ Motus,
eorum Motuum Summa Motui Aquæ in Canali recon
poſito addendus. Sequitur ex Caſu 2, & 3.

Corol. 1. Data longitudine Canalis, & qualibet Se
tione ejuſdem, erit Motus Aquæ in ratione velocitat
q

qua Aqua fluit per istam Sectionem.

2. Data quavis Sectione, & velocitate Aquæ Sectionem istam præterfluentis, erit Motus Aquæ ut longitudo Canalis.

3. Data Canalis longitudine, & velocitate Aquæ in quavis Sectione, erit Aquæ Motus in ratione illius Sectionis.

4. Dato Motu Aquæ, & aliqua Sectione, erit longitudo Canalis in ratione inversa velocitatis.

5. Dato Aquæ Motu, & longitudine Canalis, erit Sectio quævis reciprocè ut velocitas.

6. Data velocitate in qualibet Sectione, & Motu Aquæ, erit ista Sectio in ratione reciproca longitudinis.

7. Data longitudine Canalis, & mole Aquæ certo quovis tempore effluentis, erit Aquæ Motus reciprocè ut istud tempus.

8. Data Canalis longitudine, & tempore, erit Aquæ Motus ut moles effluens.

9. Dato tempore, & mole Aquæ effluentis, erit Aquæ Motus ut longitudo Canalis.

10 Dato Motu Aquæ, & longitudine Canalis, moles effluens est in ratione temporis.

11. Dato Aquæ Motu, & mole effluente, erit tempus ut longitudo Canalis.

12. Dato tempore, & Motu Aquæ, erit moles effluens reciproce ut longitudo Canalis.

13. Si binæ moles Aquæ motu contrario in directum occurrant, & pares sint utrinque tum superficies quibus in se invicem impingant, tum velocitates quibus istæ superficies in adversum moveantur, fuerit autem altera moles Aquæ guttulæ uni æqualis, altera Aqua omnis Oceano contenta, vel etiam quantitas Aquæ infinita; fieri potest ut una ista guttula Aquam omnem Oceani, vel quantitatem Aquæ infinitam, non solum sustineat, sed

poſt occurſum, eadem ac prius velocitate, ipſa in plagam eandem moveri pergat, eadem illam in partes contrarias repellat. Quod eſt mirabile Paradoxon in re Hydraulica.

14. Si certa moles Aquæ, per canalem ex tubis duobus cylindricis, Diametro inæqualibus, compoſitum, à tubo ampliore verſus anguſtiorem fluat, & motus Aquæ neque minuatur inter fluendum neque augeatur, ſimul ac prima pars Aquæ tubi minoris initium ingreſſa fuerit, ſtatim tardius fluere incipiet, & continuato efluxu ex tubo latiore in anguſtiorem, gradatim magis retardabitur Aqua in tubo anguſtiore, uſque dum tota in eum tubum pervenerit. Contrario modo res eveniet, fluente Aqua à tubo minore verſus ampliorem. Quod eſt alterum Paradoxon in re Hydraulica. Ponitur autem Aqua ubique ſibi cohærere.

Oriuntur bina iſta Corollaria ex Caſu 1.

15 Ex Caſu ſecundo datur Methodus æſtimandi Motum Sanguinis in qualibet Arteria.

16. Datis quibuſcunque Arteriis binis, æqualem Sanguinis molem tranſmittentibus, major eſt impetus Sanguinis in Arteria à Corde remotiore quam in propiore. Quod eſt Paradoxon notatu dignum in Oeconomia Animali.

17. Ex Caſu tertio oritur alterum Paradoxon in Oeconomia Animali, nempe majorem eſſe Sanguinis motum ſive impetum, in Arteriis omnibus Capillaribus ſimul ſumptis, quam in ipſa Aorta. Item, major eſt in Capillaribus Venis, quam Arteriis.

18 Ex Caſu quarto deducitur Methodus definiendi motum Sanguinis in quavis Vena.

19. Ex eodem deducitur tertium in Oeconomia Animali Paradoxon, nempe majorem eſſe Sanguinis impetum in Vena quavis, quam in Arteria ei Venæ reſpondente,

dente, & proinde majorem esse in Vena Cava, quam in Aorta.

Problema I.

Invenire motum Aeris ex Pulmone effluentis.

Sit l = Longitudo totius ductus aerei, ab Ore & Naribus ad extremos ramos Trachææ.

q = Quantitas Aeris mediocri exspiratione ex Pulmone emissa.

Q = Aeris copia validissima exspiratione expulsi.

t = Tempus mediocris exspirationis.

T = Tempus exspirationis fortissimæ.

Inde, per Theorema 3, *Cas.* 3, Motus Aeris ex Pulmone effluentis, in exspiratione mediocri $= \frac{ql}{t}$.

fortissima $= \frac{Ql}{T}$.

Hoc est, Motus Aeris ex Pulmone exeuntis æqualis est motui molis Aeris, quæ unica exspiratione emittitur, cujus ea sit velocitas, qua percurratur tempore exspirationis longitudo totius Canalis Aerei. *Q. E. I.*

Aeris quantitatem exspiratione mediocri emissam Vir Clarissimus, *Alphonsus Borellus*, facto Experimento 18 circiter, vel 20 unciis cubicis definit. Est autem diversa, non solum in diversis Hominibus, sed etiam temporibus diversis, in Homine eodem. Ipse Experimentum in hunc modum institui.

Vesicæ madefactæ à parte inferiore pondus appendebam, & aptato eidem superius tubo vitreo Diametro circiter unciali, naribus obturatis Aerem vesicæ leniter inspirabam, per spatium trium minutorum secundorum, pondere interim in mensa quiescente. Postea Vesicam cum Aere incluso & pondere appenso, sub Aquam in vase Cylindrico contentam, demergebam, notata diligenter altitudine, ad quam Aqua attollebatur.

Deinde, Aere ex veſica expreſſo, iterum eandem cum pondere in Aquam immittebam. Quod cum eſſet factum, facile inveniebatur Aquæ moles, quæ vaſi infuſa altitudinem prius notatam conficeret. Experimento decies repetito, & additis ſibi invicem quantitatibus ſingulis inventis, earum decima, ſive media moles Aquæ vaſi infuſa, reperiebatur 35 unciis cubicis æqualis. Quæ moles eſt Aeris veſica contentæ; & adjecta circiter parte duodecima, ſeu 3 unciis cubicis, ob Aeris condenſationem à frigore Aquæ factam, cum tempeſtas fuerit hyemalis, efficiuntur 38 unciæ cubicæ. Præterea addendum eſt tantillum, tum propter Aquæ preſſionem in veſicam, tum ob Vaporem qui cum halitu emittitur in humorem coactum; quod fiat neceſſe eſt ex frigore Aquæ, & veſicæ madidæ contactu. Æſtimavi igitur Aeris copiam, leni exſpiratione emiſſam tempore trium minutorum ſecundorum, numero rotundo 40 unciarum cubicarum.

In exſpiratione validiſſima exſpirabam uncias cubicas 125, tempore minuti ſecundi unius.

Hujuſmodi autem exſpiratione, cum vehementi Pulmonis contentione ad ſtrangulatum fere continuata, 220 uncias cubicas ex Pectore emittebam. Unde patet, ut id obiter moneam, multo plus Aeris in Pectore ſupereſſe, quam unica exſpiratione mediocri emitti.

Si ergo ponatur $l = 2$ pedes

$q = 40$ unciæ cubicæ

$\mathcal{Q} = 125$ unciæ cubicæ

$t = 3''$

$T = 1''$

Aeris Gravitas Specifica ad Gravitatem Aquæ, ut 1 ad 1000.

Pes Aquæ cubicus = 1000 unc. *Avoird.*

Erit Motus mediocris Aeris Pulmone exeuntis æqualis motui ponderis Scrupulorum 4 & Granorum 9, quod percurrat unciam unam minuto ſecundo; vel

motui

motui ponderis Grani 1 $\frac{1}{3}$, quod eodem tempore conficiat longitudinem 5 pedum & 7 unciarum. Quæ eſt velocitas Aeris per Laryngem effluentis, poſita Laryngis Sectione $= \frac{1}{5}$ unciæ quadratæ.

Motus maximus Aeris Pectore expulſi æquatur motui ponderis unciæ 1 $\frac{3}{4}$ circiter, percurrentis unciam unam minuto ſecundo ; ſive motui ponderis grani 1 $\frac{1}{3}$ percurrentis eodem tempore 52 pedes. Quæ eſt velocitas Aeris in fortiſſima exſpiratione per Laryngem erumpentis.

Corol. 1. Data Aeris copia & longitudine Canalis aerei, motus Aeris eſt in ratione inverſa temporis exſpirandi.

2. Data mole Aeris & tempore, erit motus in ratione directa longitudinis.

3. Data longitudine & tempore, motus eſt ut Aeris copia.

4. Dato motu & Aeris copia, erit longitudo in ratione directa temporis.

5. Dato motu & longitudine, erit Aeris moles directe ut tempus.

6. Dato motu & tempore, erit Aeris moles reciproce ut longitudo Canalis Aerei.

7. Motus Aeris eſt in ratione compoſita ex ratione quadruplicata Diametri cujuſvis homologæ ipſius Animalis, & ratione inverſa temporis exſpirandi ; vel in ratione compoſita ex ratione ponderis totius Animalis, ratione ejuſdem ponderi, ſubtriplicata, & ratione temporis reciproca.

Nam pondus Animalis, Diametri cujuſvis homologæ Cubus & moles Aeris expulſi ſunt in eadem ratione. Ponitur autem Corpora Animalium Machinas eſſe ſimiliter factas.

Scholium. Longitudinem hic uſurpatam, vel ipſam eſſe concipies Canalis aerei longitudinem, ſi Rami omnes

Tra-

Trachææ longitudine æquales ponantur ; vel mediam inter longitudines diverſas, ſi Rami ſint inæquales.

Problema II.

Determinare impetum, ſive impreſſionem quam excipit interna Pulmonum ſuperficies ab Aere exſpirando.

Cum actioni æqualis & contraria ſit reactio ; neceſſe eſt, ut, quanto motu urgetur ab interna Pulmonum ſuperficie Aer exſpirandus, tanto viciſſim ab Aere repellatur ſuperficies Pulmonum.

Unde, per Problema ſuperius, impetus dictus in exſpiratione mediocri $= \frac{q\,l}{t}$

$$\text{fortiſſima} = \frac{2\,l}{T}. \quad Q.E.I.$$

Hinc poſitis iiſdem quæ in ſuperiore ponuntur, impetus mediocris Aeris in Pulmones æquales eſt motui ponderis drachmæ circiter $1\frac{1}{2}$, quod minuti ſecundi ſpatio percurrat unciam unam ; vel motui ponderis 19 librarum, conficientis eodem tempore $\frac{1}{1642}$ unciæ, quæ eſt velocitas Aeris in contactu ſuperficiei Pulmonis internæ. Ponimus autem cum Viro Doctiſſimo *Jacobo Keilio* ſuperficiem Pulmonis internam 21900 circiter unciis quadratis æqualem.

Impetus vero maximus Aeris in Pulmones æquatur motui ponderis unciæ circiter $1\frac{3}{4}$ moti unciam unam minuto ſecundo ; vel motui ponderis 19 librarum, quod partem $\frac{1}{175}$ unciæ conficiat eodem tempore. Quæ eſt Aeris velocitas ad ſuperficiem Pulmonis in exſpiratione vehementi.

Corol. 1. Sequuntur ex hac Propoſitione Corollaria præcedenti ſubjuncta.

2. Impetus mediocris incumbens in partem ſuperficiei Pulmonis, quæ ſit ipſi Laryngis Sectioni æqualis, eſt motus

motus ponderis $\frac{1}{1312}$ grani, conficientis unciæ ſpatium minuto ſecundo; vel motus grani $1\frac{1}{3}$ quod eodem tempore percurrat unciæ partem $\frac{1}{1642}$. Impetus autem maximus in parem ſuperficiem eſt motus ponderis $\frac{1}{131}$ partis grani quod unciam unam; vel motus ponderis grani $1\frac{1}{3}$ quod $\frac{1}{175}$ unciæ ſingulis minutis ſecundis conficiat.

3. Impetus Aeris in mediocri exſpiratione in Pulmones impreſſus, æquatur motui Columnæ aqueæ percurrentis unciam unam minuto ſecundo, cujus Columnæ baſis eſt ipſa Pulmonum ſuperficies interna. altitudo autem eſt $\frac{1}{6838}$ unciæ. Eſtque Columnæ altitudo pars $\frac{1}{7300}$ unciæ, in exſpiratione omnium vehementiſſima.

4. Impetus incumbens in ſuperficiem parem circulo maximo Globuli Sanguinei. in leni exſpiratione, eſt pars $\frac{1}{14}$ ponderis Globuli Sanguinei; in exſpiratione vehementi $\frac{1}{3}$ ejuſdem ponderis, moti unciam unam minuto ſecundo. Qua autem ratione Diametros Globulorum Sanguinis dimenſus ſim. cum uſui eſſe queat ad aliorum Objectorum minimorum magnitudines definiendas, libet obiter exponere Capillum tenuem, & ſatis longum, aciculæ pluries circumvolvi, ut omnes convolutiones ſeſe invicem accurate contingerent, quod admotum ſubinde Microſcopium luculenter oſtendebat. Deinde cum intercapedinem inter extremas utrinque circumvolutiones Circino cepiſſem, eandem Scalæ, quam vocant, Diagonali applicabam, ſpatiumque in Scala repertum per convolutionum numerum dividebam. Unde inventa eſt unius convolutionis latitudo, ſive ipſa Capilli Diameter. Poſtea Capillum eundem, in Segmenta minutula diviſum, plano Microſcopii, cui Sanguinis parum ita erat illitum ut Globuli conſpicerentur diſtincti, ſuperinſpergebam. Ea cum Microſcopio contuerer, reperiebam aliquibus in locis Capilli Segmenta ita commode diſpoſita, ut numerare liceret, quot Globuli Diametro Segmenti opponerentur. Erant autem Segmenta Diametro inæqualia, quod

quod Capillus tenuior versus extremum fuerit, quam propius à Radice, adeo ut jam 7, vel 8, jam 12, 13ve Globuli tranſverſæ Sectioni Capilli reſponderent. Utroque autem Experimento ſæpius iterato, æſtimavi tandem mediam Capilli Diametrum parte $\frac{1}{324}$ unciæ, & Diametrum Globuli Sanguinei parte decima Diametri Capilli, ſive parte $\frac{1}{3240}$ unciæ.

5. Impetus, quem patitur interna Pulmonum ſuperficies ab Aere exſpirando, minor eſt Motu leniſſimi roris è Cælo decidentis.

Scholium. Neglecta eſt in ſolutione Problematum duorum præcedentium impedimenti conſideratio, quod Aeri ex Pulmone egredienti objicitur ex affrictu laterum Arteriæ Trachææ, ejuſque ramorum; cum id perparvum ſit, neque ullo experimento ſatis accurate æſtimari poſſe videatur. Nec fuimus admodum ſolliciti de rationibus numerorum exquiſitè ſervandis, cum id unum nobis propoſitum fuerit, ut methodum exponeremus æſtimandi, aliquanto certius quam videtur antehac factum, vires eas, quibus agit Aer inter exſpirandum in vaſa ſanguinea ſuperficiem Pulmonis internam perreptantia. Unde dignoſci poteſt, utrum pares ſint hæ vires effectis iſtis producendis, quæ iiſdem à Doctiſſimis quibuſdam Scriptoribus Medicis tribuuntur. Quod liberum eſto Lectoris Scientia Mechanica & Anatomica inſtructi Judicium.

Problema III.

Definire impetum Sanguinis in Vena Cava prope dextram Auriculam Cordis; ſive motum Sanguinis per omnes Arterias & Venas fluentis, præter Pulmonares.

Sit q = Quantitas Sanguinis una Cordis Syſtole in Aortam projecti.

l = Longitudo media ductus integri Arterio-Venoſi, ratione habita ramorum longiorum & breviorum.

t =

t = Temporis ſpatium inter binos Pulſus interceptum.

Inde, per Theorema 3. Caſ. 4. impetus quæſitus $= \frac{q\,l}{t}$.

Hoc eſt, Impetus Sanguinis in Vena Cava æquatur motui molis Sanguineæ, quæ una Syſtole in Aortam projicitur, cujus ea ſit velocitas, qua percurri queat integra Arteriarum & Venarum longitudo, temporis ſpatio inter binos Pulſus intercepto. *Q. E. I.*

Si in Corpore Humano ponantur

q = 2 unciæ *Avoird.*
l = 6 pedes
$t = \frac{3''}{4}$.

Erit impetus Sanguinis in Vena Cava æqualis motui ponderis 12 librarum, quod unciæ unius longitudinem conficiat ſingulis minutis ſecundis; ſeu motui ponderis 2 librarum, quod pari temporis ſpatio percurrat pedem $\frac{1}{12}$. Quæ eſt fere Sanguinis velocitas in Cava fluentis. Ponimus autem, ex dimenſione Viri Doctiſſimi ſupra dicti, Cavæ Sectionem dodrantem eſſe unciæ quadratæ.

Corol. Oriuntur ex hoc Problemate mutatis mutandis omnia Problematis primi Corollaria.

Problema IV.

Determinare motum abſolutum Sanguinis in Vena Cava; ſive motum Sanguinis, per omnes Arterias & Venas fluentis rd æter Pulmonales, ſublata Vaſorum reſiſtentia.

Sit velocitas Sanguinis Naturalis, ad eam velocitatem qua Sanguis flueret, dempta omni reſiſtentia, ut 1 ad x. Cumque per *Corol.* ſuperioris Problematis, & *Corol* 1.

Probl. I. Motus Sanguinis fit in ratione velocitatis, erit inde motus quæsitus $= \frac{x\,q\,l}{t}$. *Q. E. I.*

Quod si proportio per Experimentum à Viro Clarissimo supra laudato institutum inventa, ut veræ propinqua, admittatur, erit $x = 2.5$.

Unde, positis iisdem quæ in superiore ponuntur, motus absolutus Sanguinis in Vena Cava æquatur motui ponderis 30 librarum, quod minuto secundo longitudinem uncialem percurrat; sive motui ponderis 2 librarum percurrentis eodem tempore pedem $1\frac{1}{4}$. Qua fere velocitate Sanguis, omni resistentia liber, per Cavam deferretur.

Problema V.

Motum Sanguinis invenire in Vena Pulmonali prope sinistram Cordis Auriculam; sive motum totius Sanguinis per Pulmonem fluentis.

Præter notulas in *Probl.* 3. usurpatas, sit $\lambda =$ Canalis Arterio-Venosi Pulmonici media longitudo.

Unde, per *Theor.* 3. *Cas.* 4. invenitur motus quæsitus $= \frac{q\,\lambda}{t}$.

Hoc est, motus Sanguinis per Pulmonem fluentis æqualis est motui molis Sanguineæ, quæ una Systole in Arteriam Pulmonalem projicitur, obtinentis eam velocitatem, qua percurratur longitudo Arteriarum ac Venarum Pulmonalium, tempore inter duos Pulsus intercepto. *Q. E. I.*

Si ponatur in Corpore Humano $\lambda = 1\frac{1}{2}$ pes.

Erit motus Sanguinis in Pulmone æqualis motui ponderis 3 librarum, percurrentis unciale spatium minuto secundo.

Problema

Problema VI.

Definire momentum Sanguinis abſolutum in Vena Pulmonali.

Eodem argumento, quod in *Probl.* 4. uſurpatum eſt, invenitur motus quæſitus $= 2.5 \times \frac{q\lambda}{t}$. Q. E. I.

Poſitis vero iiſdem quæ ſupra ponuntur, motus abſolutus Sanguinis Pulmonem præterfluentis æquatur motui ponderis $7\frac{1}{2}$ librarum, quod ſingulis minutis ſecundis unciæ unius ſpatium percurrat.

Scholium. Experimento *Keiliano* definita eſt proportio, quam obtinet Sanguinis per Aortam ejuſque ramos fluentis velocitas naturalis, ad eam velocitatem qua Sanguis per eoſdem flueret, ſublata reſiſtentia Arteriarum & Sanguinis præcedentis. Eam nos proportionem ad Sanguinem per Arteriam Pulmonalem fluentem tranſtulimus. Quia vel ſublata vel imminuta ſecundum quamvis rationem reſiſtentia, quæ Sanguini per utramque Arteriam fluenti objicitur, neceſſario Sanguis pariter acceleratur in utraque Arteria. Id enim niſi fiat, bini Cordis Ventriculi aut eodem tempore non contrahentur, aut eandem Sanguinis quantitatem non ejicient. Quorum utrumvis, abſque ſumma totius Machinæ perturbatione & diſcrimine, fieri omnino non poteſt.

Corol. Ad tria Problemata præcedentia.

Sequuntur hinc Corollaria Problemati quinto ſubjuncta, mutatis mutandis.

Scholium ad quatuor Problemata ſuperiora.

Notandum Sanguinis velocitatem, tum per Pulmonem, tum per reliquum Corpus fluentis, cum reipſa æquabilis non ſit, hic tamen talem fingi, ut motus Sanguinis medius inveniatur.

Scholium generale.

Si qui numeri minus accurati videantur, qui ſparſim Characteribus ſpecioſis apponuntur, poterit ille facili opera, inventis per experimenta numeris qui propius ad verum accedant, motuum exempla ſupra poſita, vel Propoſitionum ipſarum vel Corollariorum ope, corrigere. Ignoſcat autem nobis Lector ingenuus, ſi per viam incedentibus nullis præcedentium veſtigiis tritam, adeoque Erroribus in omnes partes opportunam, Humani aliquid forte acciderit.

Damus hanc veniam, petimuſque viciſſim.

IV. *An Account of the Sinking of three* Oaks *into the Ground, at* Manington *in the County of* Norfolk. *Communicated by* Peter Le Neve, *Eſq;* Norroy *King at Arms, and Fellow of the* Royal Society.

ON *Tueſday July* the 23*d*, of the laſt Year, 1717, in the Grounds, and near the Seat of Sir *Charles Potts*, Baronet, in the County of *Norfolk*, and Pariſh of *Manington*, (which lies about mid-way between the Market Towns of *Holt* and *Aylſham*, and about ſeven Miles from the Coaſt near *Cromer*) in the day time, to the great aſtoniſhment of thoſe that were preſent; firſt, one ſingle Oak, with the Roots and Ground about it, was ſeen to ſubſide and ſink into the Earth and not long after, at about 40 Yards diſtance, two other Oaks that were contiguous, ſunk after the ſame manner. into a much larger Pit; being about 33 Foot Diameter, whereas the former is not fully 18. Theſe, as they ſunk, fell a-croſs, ſo that obſtructing each other, only the

the Roots of one of them reaches the Bottom, whereas the firſt ſtands Perpendicular.

When the firſt Tree ſunk, it was obſerved, that the Water boyl'd up in the Hole; but upon the ſinking of the greater Pit, that Water drain'd off into it, from the former, which now continues dry. The depth thereof to the firm Bottom is nine Foot three Inches; and the Tree that ſtands upright in it, is 3 Foot 8 Inches in Girt, and its Trunk about 18 Foot long, half of which is now within the Pit. In the Bottom of the greater Pit, there is a Pool of Water about 8 Foot Diameter; whoſe Surface is 11 Foot 3 Inches below the Ground, and the Trees that are in this Pit, are much of the ſame length with the other, but ſomewhat ſmaller, the one being in Girt 3 Foot 5 Inches, the other but two Foot 9 Inches.

The Soil on which theſe Trees grew, is Gravelly; but the Bottom is a Quick-ſand over a Clay, upon which there are Springs, which feed large Ponds adjoyning to Sir *Charles Potts*'s Houſe, at about a quarter of a Mile from theſe Holes.

The Nature of the Soil ſeems to afford us a reaſonable conjecture at the Cauſe of this odd accident, which ſome perhaps may be apt to reckon as a Prodigy. The Springs running over the Clay at the bottom of a Bed of very minute Sand, ſuch as your Quickſands uſually are, may reaſonably be ſuppoſed in many Ages to have waſht away the Sand, and to have thereby excavated a kind of Subterraneous Lake, over which theſe Trees grew: And the force of the Winds, on their Leaves and Branches, agitating their Roots, may well have looſened the Sand under them, and occaſioned it to fall in, more frequently than elſewhere; whereby in length of time the thin Bed of Gravel being only left, it might become unable to ſupport its own weight and

that

that of the Trees it bore. That this is not a bare conjecture, may appear from the boyling up of the Water at firſt in the leſſer Hole, and its ſtanding in the bigger and lower. And if it ſhall be found that it was a very windy day whereon this accident happen'd, it will much add to the probability of this Solution.

An accident not unlike this lately happened in *Fleet-ſtreet, London*, by the defect of the arched Roof of a very deep Common-Sewer. The Earth gradually falling into the Sewer, was carried away by it, ſo as not to obſtruct the Water; and the continual tremour of the Ground, occaſioned by the conſtant paſſing of Carts and Coaches, by degrees ſhook down the Earth, ſo as to leave a very great Cavern, the Top whereof at length grew ſo very thin, that one day a weighty Cart having juſt paſt it, a great ſpace of the Pavement ſunk in, in the middle of the Street, not without hazard to a Coach then driving by.

V. *A Rectification of the Motions of the five* Satellites *of* Saturn; *with ſome accurate Obſervations of them, made and Communicated by the Reverend Mr.* James Pound. R.S. Soc.

IT is now above thirty Years ſince that great Aſtronomer Mr. *Caſſini* communicated to the World his diſcovery of two new Satellites of *Saturn*, which made their number Five; and the account he gave of them to the *Royal Society*, (of which he was a Member) is to be ſeen in N°. 187, of theſe Tranſactions. Much about the ſame time the excellent M. *Chriſtian Huygens* of *Zulichem*, made the Society a preſent of the Glaſſes of a Teleſcope

Teleſcope of 125 Foot length, with the *Apparatus* for uſing them without a Tube; by help whereof we might have ſatisfied our ſelves of the reality of theſe Diſcoveries. But thoſe here that firſt tried to make uſe of this Glaſs, finding, for want of Practice, ſome difficulties in the Management thereof, were the occaſion of its being laid aſide for ſome time. Afterwards it was deſigned for making perpendicular Obſervations of the fixt Stars paſſing by our Zenith, to try if the Parallax of the *Earths* annual Orb might not be made ſenſible in ſo great a Radius, according to what Dr. *Hook* had long ſince propoſed: but in this we miſcarried alſo, for want of a place of ſufficient height and firmneſs, whereon to fix the Object Glaſs, ſo that it lay by neglected for many Years.

In the mean time we could not but remark a great reſerve in the *French* Aſtronomers, in relation to theſe Satellites, of which they have given us in their Yearly *Memoirs* no Obſervations till very lately, nor have they ſeemed willing to ſhew them in their Glaſſes to ſuch as requeſted it: ſo that it might poſſibly occaſion in ſome Perſons a ſuſpicion of the reality of this Diſcovery: And the Reverend Mr. *William Derham* having borrowed of the Society their long Glaſs, could not thereby aſſure himſelf that the ſmall Stars he ſometimes found about *Saturn*, were really his Satellites, their ſituation not agreeing with their places derived from the Tables of their Motions exhibited in N°. 187. of *Phil. Tranſact.* beſides that he wanted a ſufficient height to raiſe the Object Glaſs, ſo as to view *Saturn* to advantage, above the Vapour of the Horizon. But in the *Memoirs* for 1714, publiſhed but about a Year ſince, M. *Caſſini*, the worthy Succeſſor of his great Father, has given us ſome Obſervations which clear up the Point, and by ſhewing the errors of thoſe firſt Tables, has enabled us to be aſſured,

that

that we have ſeen the whole *Satellitium* of *Saturn* our ſelves.

The Subſtance of theſe Obſervations is as follows.

Anno 1714 *Maii* 6. *St. N.* about Mid-night, *Saturn* being then Stationary in ♍ 4°. 27′, the Fifth and outermoſt Satellite was in its ſuperiour Conjunction with the Planet, and at the ſame time, the Earth was nearly in the Plain of this Satellit's Orbit, ſo that it appeared to paſs very near the Center of *Saturn*: From hence and from ſome other preceeding Obſervations, Mr *Caſſini* concludes that the Nodes of this Satellit's Orb are in 4 degrees of ♍ and ♓, and that its Inclination to the Ecliptick is not much more than half that of the other Satellites. Hence it ſhould follow that the Ellipſes it deſcribes by its apparent motion about *Saturn*, when in ♊ and ♐ are much flatter and nearer to his Body, than thoſe of the other four, which he allows to move in the plain of the *Ring*, and to have their Node in 11° of ♍ and ♓, with an Inclination to the Ecliptick of 31 degrees. To confirm this diſcovery, he produces another Obſervation of his Fathers, near Thirty Years before, *viz.* that, *Anno* 1685, *Maii* 31. *St. N.* about Noon, the ſame Satellite was obſerved in ſuperiour conjunction with *Saturn*, with leſs than one Diameter of the Ring *North* Latitude, *Saturn* being then in ♍ 11°. 48′. So that the Satellite wanted but 7°. 21′ of compleating 134 Revolutions, in the Interval of time between them. From theſe *Data* it was eaſy to ſettle the Theory of this Satellite.

As to the *Fourth* or the *Hugenian* Satellite; in the *Memoirs* for 1715. but juſt now come to hand, we find a very curious Obſervation of it, and the firſt of its kind, *viz.* that *Mart.* 25°. *S. N.* about 11^{h} *P. M.* this fourth Satellite, then in *Apogeo*, did immerge behind the Body of

Saturn

Jurin. With this Emendation the place of this Satellite may for the future be computed with a ſufficient exactneſs.

The Third Satellite, by an original miſtake in the Letters in N°. 187, is all wrong; its dayly Motion being there printed 2°. 18°. 41′. 50″ inſtead of 2°. 16°. 41. 50″; as may be perceived by the Period thereof being determined, in the aforeſaid *Memoirs* of 1714, to be 4ᵈ. 12ʰ. 25′. 12″. that is, that it makes 400 Revolutions in 1807 days. This Satellite was obſerved by Mr. *Caſſini*, *April* 4ᵗʰ. *St. N.* 10ʰ. *P. M.* to have newly paſt its inferior conjunction with *Saturn*, and a perpendicular from it fell on the extremity of the weſtern *Anſe*, ſo that at about 5ʰ. *P. M.* it was with the center of the Planet then in ♍. 5°. 23′. and conſequently in ♓ 5°. 23′. But *ineunte anno Gregoriano* 1686, the *Epoche* thereof was ♍ 9°. 39′. So that from the Noon of the laſt of *December* 1685, to *April* 4°. 6ʰ. 18′. *anno* 1714, that is, in 10320 Days 6ʰ. 18′, there have been made 2284½ Revolutions of this Satellite to the Equinoctial; from which *Data*, the Tables of its Motion are readily derivable.

The Radix of the penintime or ſecond Satellite, according to the aforeſaid Letter, *ineunte anno Greg.* 1686. was in ♍ 9°. 10′. But by the Obſervations of Mr. *Caſſini* made the Nights before and after, this Satellite was in its ſuperior Conjunction *anno* 1714. *April* 4ᵈ. 21′ ½. *St. N.* that is. in ♍ 5°. 21′, where *Saturn* then was: So that *April* 4ᵈ. 22ʰ. 12′, an entire Number of Revolutions were performed ſince the *Epoche* of 1686, that is, in 10320 Days 22ʰ 12: which Number can be no other than 3771, according to the Period thereof given in this *Memoire*, *viz.* 2ᵈ. 17ʰ. 41′. 22″.

Laſtly the innermoſt or firſt Satellite, at the ſame time, *viz.* 1714, *April* 4°. 21ʰ. 30′. *St. N.* was in its in-

ſeriour conjunction *proximæ*, and conſequently in ♐ 5° 21′. But the *Epoche* thereof for 1686, is ♈ 24°. 50′. which place the Satellite had paſt 40ᵍʳ. 31′ at the time of the Obſervation. This Arch it moves in 5ʰ. 6′. Wherefore from the time of the *Epoche* to *April* 4ᵈ. 16ʰ. 24′, 1714, or in 10320 Days 16ʰ. 24′. the Satellite has performed 5467 Revolutions, its Period being determined to be 1 Day, 21 hours, 18′. 27″, in this *Memoire*.

Having by the help of theſe late Obſervations corrected the motions of the Satellites, which it was not poſſible for their firſt Diſcoverer to ſettle truly, in the ſhort interval before 1687; and having fixed their *Epoches* for the preſent Year. we were enabled to know where to expect them with more certainty, and to diſtinguiſh them one from another, and from the ſmall fixt Stars appearing with them. And the Reverend Mr. *James Pound*, (whoſe indefatigable Induſtry is no way inferiour to his incomparable Skill in Aſtronomical matters) having, by means of his Steeple of *Wanſted*, provided a *Gnomon* high enough for the purpoſe, and having fitted a very commodious *Apparatus* for uſing the Society's aforeſaid long Teleſcope, ſoon diſcovered by it all theſe five Satellites; and lately communicated to them the following very curious Obſervations.

1718. *April* 21ᵈ. 10ʰ. 40′. The third and fourth Satellits of *Saturn* were in *Apogæo*, a little paſt their Conjunction with *Saturn*: A perpendicular from the fourth to the Tranſverſe *Axis* of the Ring (or Line of the *Anſæ*) fell a little without the Eaſtern *Anſa*; and a Line through the fourth and third touched the Eaſtern Limb of *Saturn*. *Fig.* 17.

The firſt was Northward of the Line of the *Anſæ* (and therefore in the *Apogæon* Semicircle alſo) diſtant from the ſaid Line about as far as the end of the Conjugate Axis of the Ring was from the Center of ♄, *viz.* nearly

nearly $\frac{3}{4}$ of *Saturns* Semidiameter; and it was about a Semidiameter of the Ring from the Weſtern *Anſa*.

The ſecond was a very little Southward of the Line of the *Anſæ* (and therefore in the *Perigæon* Semicircle) above a Semidiameter of the Ring (or about the Semidiameter of the Ring + the Semidiam. of ♄) from the Weſtern *Anſa*. And the Third, Firſt and Second were in a ſtrait Line.

At $10^h.50'$. A Perpendicular from the 3*d* to the Line of the *Anſæ* fell almoſt on the middle of the bright part of the Eaſtern *Anſa*, but ſomewhat nearer the Center than the ſaid middle.

April $22^d.11^h.5'$. The four innermoſt Satellits were all Eaſtward of ♄. The 2*d* and 4*th* in the *Apogæon*, and the 1*ſt* and 3*d* in the *Perigæon* Semicircle. A Line through the 2*d* and 4*th* touched the *South Eaſt* Limb of ♄. A Line paſſing through the 3*d* and the end of the Conjugate *Axis* of the Ring, was parallel to the Line of the *Anſæ*.

At $11^h.10'$. A Perpendicular from the firſt to the Line of the *Anſæ* fell on the Eaſtern Extremity of the Ring. *Fig.* 18.

Theſe Diſtances and Directions were taken only by Eſtimation, and not by any actual Meaſurement.

The fifth (or outermoſt) Satellite being at this time near its greateſt Elongation Eaſtward, among ſeveral very ſmall Teleſcopick Stars, he could not determine its Poſition. But by obſerving the Motion of this ſome other Nights before, he was now fully ſatisfied, from the Motions rectified as above, that there are five Satellits of *Saturn*, as Mr. *Caſſini* had long ſince aſſerted.

In the bright part of each *Anſa* was a darkiſh Ellipſe nearer to the out ſide than the in-ſide of the Ring, as if it was compoſed of two Rings near to one another.

On

On the Body of ♄, beside the Ring on the *South*-side, there appeared on the *North*-side a Zone not so far from the Center as the Ring, and not much unlike the smallest of *Jupiter*'s Belts. These appearances were first taken notice of by Mr. *Cassini*, as may be seen in *Phil. Trans.* N°. 128 pag. 690. *Vide Fig.* 19.

We shall in our next give the Publick Tables of these Motions, corrected from the aforesaid Observations, instead of those in N°. 187. But it is not to be expected that these Satellites, exceedingly minute in themselves, and so faintly illuminated, should appear when the Air is but ordinarily Serene, they requiring not only the *Medium* to be *summo modo* defecate and limpid but withal in perfect Darkness. For which reasons it may well be understood why the Gentlemen of the *Parisian* Observatory may have sometimes made a difficulty to undertake to shew them upon demand.

FINIS.

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PHILOSOPHICAL TRANSACTIONS.

For the Months of *May*, and *June*, 1718.

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IV. *Remarks on a Fragment of an old* Roman *Inſcription lately found in the North of* England, *and tranſcribed by the Curious and Learned Dr.* James Jurin, *M. D. and* Reg. Soc. S.

I. *Tabulæ Motuum quinque* Satellitum Saturni *ad fidem nuperarum Obſervationum correctæ, Cœloque conformes redditæ.*

CIrca finem Anni 1686. D. *Jo. Dom. Caſſini*, Reg. Soc. Sodalis, & in Aſtronomicis nemini ſecundus, cum Societate noſtrâ inventa ſua de motibus quinque Satellitum *Saturni* communicavit, Epochaſque ſingulorum ad annum ineuntem 1686. eorumque motus diurnos in Epiſtola N° 187. harum *Tranſact.* editâ exhibuit: E quibus datis motuum Tabulas concinnavimus, dictæque Epiſtolæ ſubjunctas una edidimus. Cum vero deinde per triginta fere annos nullas omnino Oſervationes eorum tradiderint, qui ſoli poterant, Aſtronomi *Galli*; cumque aliunde, ob intervallum temporis nimis breve, non niſi laxè periodos Satellitum, præſertim interiorum, definire potuerit præclariſſimus Inventor; non prius dictarum Tabularum defectus corrigere datum eſt, quam in nuperis Actis Academiæ Regiæ *Pariſienſis* Phyſicis & Mathematicis, obſervata ea, quæ ſub finem præcedentis *Tranſact.* N° 355. protulimus, prodiere.

Horum vero ope facta aliquali Motuum caſtigatione, tum demum Teleſcopio *Hugeniano* omne Saturni Satellitium ipſi agnovimus; adhibitiſque accuratis Rev. D. *Jac. Poundi* obſervationibus, Tabulas ſubſequentes cælo ſatis conſonas obtinuimus. Addendo ſc. motui annuo *Interioris*. 2 $^{gr.}$ 9′; *Penintimi* vero 3°. 25′. retentis Epochis D. *Caſſini* ad Annum 1686. Augendo etiam motum annuum *Extimi* 9 min. ſublatis vero 16 grad. ab *Epocha*, quæ in Epiſtola dicta N° 187. perperam ſcribitur ♐ 16°. 19′. pro ♐ 0°. 16′. *Hugenianum* 6′ annuatim tardiorem invenimus. *Tertii* autem Tabellas, ob motum diurnum falsò in Epiſtola illa traditum, de integro recudere neceſſe habuimus, retenta ſaltem *Epocha*.

Tabulæ

Tabula Mediorum Motuum Intimi Satellitis Saturni à Cassino detecti Anno 1686.

Annis Christi ineunt	Epochæ s ° '	Annis	Mot. Med. s ° '	Dicbus	Mot. Med. s ° '	H M	Mot. Med Sex. ° ' "	Min.	Motus Medi. ° '
1681	♒ 8 48	1	4 4 43	1	6 10 42	1	0 7 57	31	4 6
1686	♉ 13 4	2	8 9 25	2	0 21 24	2	0 15 53	32	4 14
1701	♍ 6 34	3	0 14 8	3	7 2 6	3	0 23 50	33	4 22
1714	♎ 9 57	4	10 29 33	4	1 12 47	4	0 31 47	34	4 30
1715	♒ 14 39	5	3 4 16	5	7 23 29	5	0 39 44	35	4 38
1716	♊ 19 22	6	7 8 59	6	2 4 11	6	0 47 40	36	4 46
1717	♉ 4 47	7	11 13 42	7	8 14 53	7	0 55 37	37	4 54
1718	♍ 9 30	8	9 29 6	8	2 25 35	8	1 3 34	38	5 2
1719	♑ 14 13	9	2 3 49	9	9 6 17	9	1 11 31	39	5 10
1720	♉ 18 55	10	6 8 32	10	3 16 59	10	1 19 28	40	5 18
Mens. Anni.	*Mot. Med.*	11	10 13 15	11	9 27 41	11	1 27 24	41	5 26
		12	8 28 40	12	4 8 22	12	1 35 21	42	5 34
Com.	s ° '	13	1 3 23	13	10 19 4	13	1 43 18	43	5 42
Jan.	0 0 0	14	5 8 5	14	4 29 46	14	1 51 15	44	5 50
Febr.	5 1 38	15	9 12 48	15	11 10 28	15	1 59 11	45	5 58
Mar.	3 1 10	16	7 28 13	16	5 21 10	16	2 7 8	46	6 5
April	8 2 48	17	0 2 56	17	0 1 52	17	2 15 5	47	6 13
Maii	6 23 44	18	4 7 39	18	6 12 34	18	2 23 1	48	6 21
Junii	11 25 22	19	8 12 21	19	0 23 16	19	2 30 58	49	6 29
Julii	10 16 19	20	6 27 46	20	7 3 57	20	2 38 55	50	6 37
Aug.	3 17 57	40	1 25 32	21	1 14 39	21	2 46 52	51	6 45
Sept.	8 19 35	60	8 23 19	22	7 25 21	22	2 54 49	52	6 53
Octob	7 10 31	80	3 21 5	23	2 6 3	23	3 2 45	53	7 1
Nov.	0 12 9	100	10 18 51	24	8 16 45	24	3 10 42	54	7 9
Dec.	11 3 5	120	5 15 37	25	2 27 27	25	3 18 39	55	7 17
				26	9 8 9	26	3 26 35	56	7 25
In Anno Bissextili post Februarium adde unius diei motum.				27	3 18 50	27	3 34 32	57	7 33
				28	9 29 32	28	3 42 28	58	7 41
				29	4 10 14	29	3 50 25	59	7 49
				30	10 20 56	30	3 58 22	60	7 57

Tabula Mediorum Motuum Satellitis Saturni *Penitimi, à* Cassino *detecti Anno* 1686.

Annis Jul. ineunt	*Epochæ* s ° ′	*Annis*	*Med. Mot.* s ° ′	*Diebus*	*Med. Mot.* s ° ′	*H M*	*Mot. Med* sex ° ′ ″	*Min.*	*Motus Medi.* ° ′
1681	♐ 3 23	1	4 10 2	1	4 11 32	1	0 5 29	31	2 50
1686	♉ 5 25	2	8 20 4	2	8 23 4	2	0 10 58	32	2 56
1701	♐ 22 3	3	1 0 6	3	1 4 36	3	0 16 26	33	3 1
1714	♍ 7 5	4	9 21 40	4	5 16 8	4	0 21 55	34	3 7
1715	♉ 17 7	5	2 1 42	5	9 27 40	5	0 27 24	35	3 12
1716	♏ 27 9	6	6 11 44	6	2 9 12	6	0 32 53	36	3 17
1717	♊ 18 43	7	10 21 46	7	6 20 44	7	0 38 22	37	3 23
1718	♋ 28 45	8	7 13 20	8	11 2 16	8	0 43 51	38	3 28
1719	♐ 8 47	9	11 23 22	9	3 13 49	9	0 49 19	39	3 34
1720	♑ 18 49	10	4 3 24	10	7 25 21	10	0 54 48	40	3 40
		11	8 13 26	11	0 6 53	11	1 0 17	4	3 45
Mens. An. Co	*Med. Mot.* s ° ′	12	5 5 0	12	4 18 25	12	1 5 46	42	3 51
		13	9 15 2	13	8 29 57	13	1 11 15	43	3 56
Jan.	0 0 0	14	1 25 4	14	1 11 29	14	1 16 44	44	4 1
Febr.	3 27 34	15	6 5 6	15	5 23 1	15	1 22 13	45	4 7
Mar.	6 20 32	16	2 26 40	16	10 4 33	16	1 27 42	46	4 12
Apr.	10 18 6	17	7 6 42	17	2 16 5	17	1 33 11	47	4 17
Maii	10 4 7	18	11 16 44	18	6 27 37	18	1 38 39	48	4 23
Junii	2 1 41	19	3 26 46	19	11 9 9	19	1 44 8	49	4 28
Julii	1 17 43	20	0 18 20	20	3 20 41	20	1 49 37	50	4 34
Aug.	5 15 17	40	1 6 40	21	8 2 13	21	1 55 6	51	4 39
Sept.	9 12 51	60	1 25 0	22	0 13 45	22	2 0 35	52	4 45
Oct.	8 28 53	80	2 13 20	23	4 25 17	23	2 6 4	53	4 50
Nov.	0 26 27	100	3 1 4	24	9 6 49	24	2 11 32	54	4 56
Dec.	0 12 28	120	3 20 0	25	1 18 22	25	2 17 1	55	5 1
				26	5 29 54	26	2 22 30	56	5 7
In Anno Biſſextili poſt Februarium adde unius diei motum.				27	10 11 26	27	2 27 59	57	5 12
				28	2 22 58	28	2 33 27	58	5 18
				29	7 4 30	29	2 38 56	59	5 23
				30	11 16 2	30	2 44 25	60	5 29

Tabula Mediorum Motuum Satellitis Saturni Medii, à Cassino detecti Anno 1671.

Annis Julia. ineunt	Epochæ s ○ ′	Annis	Med. Motus s ○ ′	Diebus	Med. Motus s ○ ′	H M	Med. Mot Sex. ○ ′ ″	M	Med. Mot. ′ ″
1681	♍ 12 16	1	9 17 2	1	2 19 41	1	○ 3 19	31	[43
1686	♏ 27 6	2	7 4 3	2	5 9 23	2	○ 6 38	32	[46
1701	♓ 1 17	3	4 21 5	3	7 29 4	3	○ 9 58	33	[49
1714	♓ 11 43	4	4 27 48	4	○ 18 46	4	○ 13 17	34	[53
1715	♋ 28 45	5	2 14 50	5	1 8 27	5	○ 16 36	35	[56
1716	♉ 15 47	6	○ 1 52	6	3 28 9	6	○ 19 55	36	2 0
1717	♉ 22 30	7	9 18 53	7	6 17 50	7	○ 23 15	37	2 3
1718	♈ 9 32	8	9 25 36	8	9 7 31	8	○ 26 34	38	2 6
1719	♐ 26 34	9	7 12 38	9	11 27 13	9	○ 29 53	39	2 10
1720	♎ 13 35	10	4 29 40	10	2 16 54	10	○ 33 12	40	2 13
Mens. Anni Com.	Med. Motus s ○ ′	11	2 16 42	11	5 6 36	11	○ 36 31	41	2 16
		12	2 23 25	12	7 26 17	12	○ 39 51	42	2 19
		13	○ 10 26	13	10 15 59	13	○ 43 10	43	2 23
Jan.	0 0 0	14	9 27 28	14	1 5 40	14	○ 46 29	44	2 26
Febr.	10 10 24	15	7 14 30	15	3 25 21	15	○ 49 48	45	2 29
Mar.	○ 21 44	16	7 21 13	16	6 15 3	16	○ 53 8	46	2 33
Apr.	11 2 9	17	5 8 15	17	9 4 44	17	○ 56 27	47	2 36
Maii	6 22 51	18	2 25 16	18	11 24 26	18	○ 59 46	48	2 39
Jun.	5 3 16	19	○ 12 18	19	2 14 7	19	1 3 5	49	2 43
Julii	○ 23 58	20	○ 19 1	20	5 3 49	20	1 6 24	50	2 46
Aug.	11 4 23	40	1 8 2	21	7 23 30	21	1 9 44	51	2 49
Sept.	9 14 47	60	1 27 4	22	10 13 11	22	1 13 3	52	2 53
Oct.	5 5 30	80	2 16 5	23	1 2 53	23	1 16 22	53	2 56
Nov	3 15 54	100	3 5 6	24	3 22 34	24	1 19 41	54	2 59
Dec.	11 6 37	120	3 24 7	25	6 12 16	25	1 23 1	55	3 3
				26	9 1 57	26	1 26 20	56	3 6
In Anno Bissextili post Februarium adde unius Diei motum				27	11 21 39	27	1 29 39	57	3 9
				28	2 11 20	28	1 32 58	58	3 13
				29	5 1 1	29	1 36 17	59	3 16
				30	7 20 43	30	1 39 36	60	3 19

Tabula Mediorum Motuum Penextimi Satellitis Saturni *ab* Hugenio *inventi Anno* 1655.

Annis Jul. ineunt	*Epochæ* s ° ′	*Annis*	*Med. Mot.* s ° ′	*Diebus*	*Med. Mot.* s ° ′	*H M*	*Mo. Me.* ° ′ ″	*Min.*	*Motus Medi.* ′ ″
1641	♍ 2 48	1	10 20 35	1	0 22 35	1	0 56	31	29 10
1661	♐ 13 23	2	9 11 10	2	1 15 9	2	1 53	32	30 6
1681	♈ 27 58	3	8 1 45	3	2 7 44	3	2 49	33	31 3
1686	♏ 3 28	4	7 14 55	4	3 0 18	4	3 46	34	31 59
1701	♊ 12 33	5	6 5 30	5	3 22 53	5	4 42	35	32 55
1714	♑ 17 53	6	4 26 5	6	4 15 28	6	5 39	36	33 52
1715	♒ 8 28	7	3 16 40	7	5 8 2	7	6 35	37	34 48
1716	♄ 29 3	8	2 29 50	8	6 0 37	8	7 31	38	35 45
1717	♄ 12 13	9	1 20 25	9	6 23 12	9	8 28	39	36 41
1718	♏ 2 48	10	0 11 0	10	7 15 46	10	9 24	40	37 38
1719	♍ 23 23	11	11 1 35	11	8 8 21	11	10 21	41	38 34
1720	♌ 13 58	12	10 14 45	12	9 0 55	12	11 17	42	39 31
1721	♋ 27 8	13	9 5 20	13	9 23 30	13	12 14	43	40 27
Mens. An. Co	*Med. Mot.* s ° ′	14	7 25 55	14	10 16 5	14	13 10	44	41 24
		15	6 16 30	15	11 8 39	15	14 7	45	42 20
Jan.	0 0 0	16	5 29 40	16	0 1 14	16	15 3	46	43 17
Febr.	11 9 54	17	4 20 15	17	0 23 48	17	16 0	47	44 13
Mar.	8 12 2	18	3 10 50	18	1 16 23	18	16 56	48	45 10
Apr.	7 21 55	19	2 1 25	19	2 8 58	19	17 52	49	46 6
Maii	6 9 14	20	1 14 35	20	3 1 32	20	18 49	50	47 3
Junii	5 19 7	40	2 29 10	21	3 24 7	21	19 45	51	47 59
Julii	4 6 26	60	4 13 45	22	4 16 42	22	20 42	52	48 56
Aug.	3 16 18	80	5 28 20	23	5 9 16	23	21 38	53	49 52
Sept.	2 26 12	100	7 12 55	24	6 1 51	24	22 35	54	50 49
Oct.	1 13 30	120	8 27 30	25	6 24 25	25	23 31	55	51 45
Nov.	0 23 24	140	10 12 5	26	7 17 0	26	24 27	56	52 42
Dec.	11 10 42	160	11 26 40	27	8 9 35	27	25 24	57	53 38
				28	9 2 9	28	26 20	58	54 35
				29	9 24 44	29	27 17	59	55 31
				30	10 17 18	30	28 13	60	56 27

In Anno Bissextili post Februarium adde unius diei motum.

Tabula Mediorum Motuum Satellitis Saturni *Extimi, à* Cassino *detecti Anno* 1671.

Annis Julia. ineunt	*Epochæ* s	o	'	*Annis*	*Med. Motus* s	o	'	*Diebus*	*Med. Motus* s	o	'	*H M*	*Mo. Me.* °	'	"	*M*	*Med. Mot.* '	"
1681	♈	8	40	1	7	6	32	1	0	4	32	1	0	11		31	5	51
1686	♈	15	50	2	2	13	3	2	0	9	5	2	0	23		32	6	3
1701	♉	11	53	3	9	19	35	3	0	13	37	3	0	34		33	6	14
1714	♓	20	20	4	5	0	39	4	0	18	9	4	0	45		34	6	25
1715	♎	26	52	5	0	7	10	5	0	22	42	5	0	57		35	6	37
1716	♊	3	23	6	7	13	42	6	0	27	14	6	1	8		36	6	48
1717	♍	14	27	7	2	20	13	7	1	1	46	7	1	19		37	7	0
1718	♌	20	58	8	10	1	17	8	1	6	18	8	1	31		38	7	11
1719	♓	27	30	9	5	7	49	9	1	10	51	9	1	42		39	7	22
1720	♏	4	2	10	0	14	20	10	1	15	23	10	1	53		40	7	34
Mens. Anni Com.	*Med. Motus* s	o	'	11	7	20	52	11	1	19	55	11	2	5		41	7	45
				12	3	1	56	12	1	24	28	12	2	16		42	7	56
				13	10	8	27	13	1	29	0	13	2	27		43	8	8
Jan.	0	0	0	14	5	14	59	14	2	3	32	14	2	39		44	8	19
Febr.	4	20	41	15	0	21	30	15	2	8	5	15	2	50		45	8	30
Mar.	8	27	46	16	8	2	34	16	2	12	37	16	3	1		46	8	42
Apr.	1	18	27	17	3	9	6	17	2	17	9	17	3	13		47	8	53
Maii	6	4	37	18	10	15	37	18	2	21	42	18	3	24		48	9	4
Jun.	10	25	18	19	5	22	9	19	2	26	14	19	3	35		49	9	16
Julii	3	11	27	20	1	3	13	20	3	0	46	20	3	47		50	9	27
Aug.	8	2	9	40	2	6	26	21	3	5	18	21	3	58		51	9	38
Sept.	0	22	50	60	3	9	38	22	3	9	51	22	4	9		52	9	50
Oct.	5	8	59	80	4	12	51	23	3	14	23	23	4	21		53	10	1
Nov	9	29	41	100	5	16	4	24	3	18	55	24	4	32		54	10	12
Dec.	2	15	50	120	6	19	17	25	3	23	28	25	4	43		55	10	24
In Anno Bissextili post Februarium adde unius Diei motum								26	3	28	0	26	4	55		56	10	35
								27	4	2	32	27	5	6		57	10	46
								28	4	7	5	28	5	17		58	10	58
								29	4	11	37	29	5	29		59	11	9
								30	4	16	9	30	5	40		60	11	21

Motibus mediis Satellitum ad hunc modum conſtitutis, proveniunt Revolutiones eorum jam veris proximæ, ſcilicet

	D	h	′	″
Primi ſive Intimi	1.	21.	18.	26½
Secundi Penintimi	2.	17.	41.	10½
Tertii ſive *Medii*	4:	12:	25.	10
Quarti Hugeniani	15.	22.	41.	28
Quinti ſive *Extimi*	79.	7.	46.	00

Poſito autem, juxta regulam Naturæ (ſaltem in hoc noſtro Syſtemate) univerſalem, quæque tam in Jovialium ac Lunæ motibus, quam Planetarum Primariorum circa Solem, obtinet, Vires centrum Saturni petentes eſſe in duplicata ratione diſtantiarum reciprocè, ac proinde Cubos diſtantiarum à centro eſſe ut quadrata Temporum periodicorum; ex data diſtantia & periodo *Hugeniani*, fiunt reliquorum diſtantiæ ut ſequitur.

	Semidiam. Annuli	*Semidiam. Globi* ♄
Diſt. Primi	1.9289	4,3400
Secundi	2.4708	5,5593
Tertii	3.4508	7,7643
Quarti	8.0000	18.0000
Quinti	23.3146	52,4578

Quæ quidem diſtantiæ cum D. *Caſſini* obſervatis ſatis quadrant. Quatuor autem interiores Satellites juxta planum Annuli Saturni orbitas ſuas deſcribunt proximè; in plano, *ſc.* Æquatoris noſtri plano quoad ſenſum parallelo, quicquid in contrarium proferant nonnulli. Quintum vero orbem ſitu paulo diverſum à cæteris deſcribere, nuper deprehendit D *Jac. Caſſinus* prioris filius & virtutum hæres, ut videre eſt in Actis Academiæ Scientiarum *Pariſienſis* Anni 1714. Sed hæc autem omnia propriis oculis contemplari, atque penitius introſpicere jam ipſi accingimur.

II. *The reſt of the Treatiſe of that Learned Antiquary* Dr. John Tabor *of* Lewes (*whereof the Firſt Part is publiſh'd in* N° 351. *of theſe* Tranſactions) *concerning the Site of the ancient City of* Anderida, *and other Remains of Antiquity in the County of* Suſſex.

THE *former* Part *of this* curious Treatiſe *having found a juſt Eſteem among ſeveral worthy Members of the* Royal Society, *who are Lovers of Antiquity, at Their inſtance we have adventur'd to inſert here the Remainder thereof; entreating our Pholoſophical and Mathematical Reader, to indulge the Liberty we now take, of breaking in upon the uſual Subject of theſe Papers.*

Where *Tacitus* ſpeaks of *Britain* and its Affairs, his Deſcriptions are ſo lively deliver'd, that one would think himſelf had been here, with his Wife's Father *Agricola*; and where he mentions the *Iriſh*[1] Prince, the Expreſſion by him us'd ſeems to give Strength to ſuch a Suppoſition.

The gaining the Southern part of this Iſland, was the greateſt, if not the only Acquiſition, made to the *Roman* Empire, from the Death of *Tiberius* to the Sixth Year of *Claudius*; which we may well ſuppoſe was not paſs'd over in ſilence by that excellent Hiſtorian *Tacitus*: But his Four Books of Annals, which contain'd the Tranſactions of thoſe Nine Years, we have reaſon enough to fear, are irretrievably loſt. From the mention *Suetonius* makes of *Claudius* his Expedition hither; 'tis

 commonly

[1] Tac. Agric. cap. XXIV.

commonly inſinuated his Conqueſt here [2] coſt no Blood. Our Countryman *Bede*, we may ſee, was of that opinion; becauſe, in the Account given by him of *Claudius*, the Words of *Suetonius* [3] are copied. But *Dio Caſſius*, from whom we have the moſt particular Information of that War, gives a quite different Relation of the Matter: He takes notice of at leaſt Four Battels, fought with the *Britons* (before *Claudius* came over) by *Aulus Plautius*; who had *Flavius Veſpaſianus*, *Flavius Sabinus*, and *Hoſidius Gæta*, that commanded under him: In the firſt Conflict, *Cataratacus* was defeated; in the ſecond, *Togodumnus*, and, as may be inferr'd from his Words afterwards, ſlain. From the manner of his delivering the Story, all thoſe Battels ſeem to have been fought, South of the River *Thames*, and North of the *Sylva Anderida*, except the laſt; and that in the firſt Campagne the Conqueſts of *Flautius* could not have extended beyond *Kent* and *Surry*: For it's likely [4] that the Two firſt Actions happen'd about the Skirts of the *Sylva Anderida*, Eaſtward of the River *Medway*; and the Third, which held Two Days, on the Banks of that River; becauſe, from the River, where they were routed Two Days ſucceſſively, the *Britons* retiring, aſſembled [5] their Strength again before their Fourth Overthrow,

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[2] Suet. Claud. cap. 17. *Ac ſine ullo prælio aut ſanguine, intra pauciſſimos dies parte inſulæ in deditionem recepta, ſexto quam profectus erat menſe Romam rediit.* [3] Beda Eccleſ. Hiſt. Gent. Angl. Lib. I. cap. 3.

[4] Dionis Caſſii Hiſt. Rom. Lib. LX. Claud. V p 768. A. Οἱ γὰρ Βρεττανοὶ καὶ προσδοκήσαντες αὐτὸὺς δι' ἅπερ ἐπωνδάρουν ἥξειν, ὑπεχώρησαν, ὑπὸ μὲν οὖν τότε ἐς χεῖρας αὐτοῖς ἦλθον, ἀλλ' ἔς τε τὰ ἕλη καὶ ἐς τὰς ὕλας κατέφυγον, ἐλπίσαντες σφᾶς ἄλλως κρατεῖν. ἔπειτα δὲ ὅπερ ὑπὸ τοῦ Καίσαρος τοῦ Ἰουλίου ἐγένετο, διακενῆς αὐτὸὺς ἀνάπλεσαν. Ὁ δὲ Πλαύτιος πολλὰ μὲν πρόγυμνα ἀναλαβὼν σφᾶς, ἔσχεν, ἐπεὶ δὲ εὗρε ποτὲ πρῶτον μὲν Καταρακτακον ἔπειτα Τογόδυμνον Κυιντιλλίνου παῖδας ἐνίκησεν.——συχνῶν δὲ ἐκείνων——πρόσω ἤδη ὡς δ' ὑπὸ ποταμῷ τινὶ ἐγένοντο. [5] Pag. 678. D. Ἀναχωρησάντων δ' ἐντεῦθεν τῶν Βρεττανῶν, ὑπὸ τὸν Ταμέσαν ποταμὸν, καθ' ὃ ἐς τε τὸν ὠκεανὸν ἐκβάλλει, πλημμύρηντός τε αὐτῷ λιμνάζει.

throw, in that part of *Kent* which borders on the *Thames*, not far from its entrance into the Sea; and having pass'd it, were follow'd by *Plautius* his *Germans*, and on the other side put to flight; which was the Fourth Action mention'd by *Dio*. *Claudius* having been sent for, comes the Second Year with powerful Succours to the Assistance of *Plautius*; who with his Forces waited his Arrival near the *Thames*, not unlikely still where he quarter'd in the Winter; which perhaps was in that large strong Camp, as yet to be seen [6] not far from *Bromly* in *Kent*, on the River *Ravensbourn*. The Emperour joining him [7], immediately cross'd the *Thames*; overthrew the *Britons* posted on the other side to resist him; advanced to *Cynobelin*'s chief Residence *Camalodunum*, and took it: Then receiving Homage of some States, return'd to *Rome*.

Considering therefore that *Claudius* staid but Sixteen Days [8] in this Island, we must conclude his Dispatch was great; and that his Progress could not have been through more Parts than *Kent*, *Essex*, *Hertfordshire*, *Middlesex*, and *Surry*. As to what else relates to the *British* War in the time of *Claudius*, save that Three Years after *Titus* rescued his Father *Vespasian* when in great danger, we have no Account from *Dio*. But where *Suetonius* [9] treats of *Vespasian*'s Life; we are told, when that Emperour commanded in *Britain* for *Claudius*; that he fought Thirty Battels, subdu'd Two of the

[6] Camden Brit. Edit. 1695. Col. 213. c.

[7] Dion. Cassii. Hist. Rom. Lib. LX. pag. 679. B. Καὶ ντεῦθεν τὰ μὲν πεζῇ, τὰ δὲ καὶ διὰ τῆς ποταμοῦ πορθμόμενος, ἐς τε τὸν Ὠκεανὸν ἀφίκετο, καὶ περιωθεὶς ἐς τὴν Βρεττανίαν, συνέμιξε τοῖς στρατοπέδοις ἐς τὸν Ταμέσα ἀναμένουσιν αὐτὸν καὶ ἐξαπιναίως σφᾶς, ἐπεννόητε, καὶ τοῖς βαρβάροις ἐς τὴν ἔφοδον αὐτῶν συνεσκευασμένοις ἐς χεῖρας ἐλθὼν, μάχῃ τε ἐνίκησε

[8] Dion. Lib LX. pag. 680. B. Ἦλθέ τε ἐς τὴν Ῥώμην ὁ Κλαύδιος, ἐξ ἡμερῶν ἀποδημήσας (ἀφ' ὧν ἐπεραίωσε μόνας ἐν τῇ Βρεττανίᾳ ἡμέρας ἐποίησε.)

[9] Suet. Vespasian. cap. 4.

moſt powerful Nations, won Twenty Towns, and brought the Iſle of *Wight* under the *Roman* Obedience. Of which Actions, beſides what might have been ſaid in the loſt Books of Annals; *Tacitus*, in other Pieces of his, largely [1] hints, that when *Claudius* rul'd, *Veſpaſian*'s Behaviour and Succeſs in this Iſland, ſhew'd to the World his Conduct and Courage in the Affairs of War: The ſame is alſo taken notice of [2] by *Dio*. From his Conqueſt of the Iſle of *Wight*, it may be imply'd, the Stage of his Actions here, was in thoſe Countries which border on the South Channel rather than in the North: Since therefore the Clime, the Soil, and the more ready Conveniencies for foreign Trade and Correſpondence, might entitle this Part of the Land, to ſuſtain as numerous, as ſtout, and as experienc'd a People as any other (becauſe *Cæſar* [3] takes notice they not only lent Aids to the *Veneti* in their Revolt, but were wont to aſſiſt the *Gauls* in moſt of their Wars againſt [4] the *Romans*:) And whereas no Hiſtorian afterwards mentions any Diſturbance given to the *Romans* from the Southern Parts; we may conclude, *Veſpaſian* entirely ſubdu'd them; and that before he left the Iſland, the Methods he eſtabliſh'd for ſecuring Peace, were no way inferior to thoſe he had ſhewn in making War.

The

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[1] Tacit. Agricol. cap. xiii. *Divus Claudius auctor operis tranſvectis legionibus auxiliiſque, & aſſumpto in partem rerum Veſpaſiano; quod initium venturæ mox fortunæ fuit, domitæ gentes, capti Reges, & monſtratus fatis Veſpaſianus.* Tacit. Hiſt. Lib. III. cap. xliv. *Et Britanniam inclytus erga Veſpaſianum favor, quod illic ſecundæ Legioni à Claudio præpoſitus, & bello clarus egerat, non ſine motu adjunxit cæterarum.* [2] Dion Caſſ. Hiſt. Rom. Lib. LXV. p. 736. C. ἥτε γὰρ τῆς ἀνθρώπων εὐνοίας πολλὴν ὑπ' αὐτοῦ——· γὰρ ἐκ τῆς Βρεττανίας δόξα, καὶ ἡ ἐκ τοῦ ἐν χερσὶ πολέμου εὐκλεια. [3] De Bello Gal. Lib. III. *ſocios ſibi ad id Bellum Oſiſmios, Lexobios, &c. auxilia ex Britannia, quæ contra eas Regiones poſita eſt, acceſſant.* [4] Idem Lib. IV. *Tamen in Britanniam proficiſci contendit, quod, omnibus ferè Gallicis Bellis, hoſtibus noſtris inde ſubminiſtrata auxilia intelligebat.*

The *Romans* well knew, that thoſe who were Strangers to Civility, could not without great Difficulty be kept in Obedience: As ſoon therefore as the Countries they had conquer'd, were reduced to ſome degree of Quiet; they endeavour'd to make the People in love with their Government, by introducing their Arts and Cuſtoms among them: From that inconſiderable Inſtance recorded [5] by *Pliny*, we may ſee, how ready the *Romans* were, to oblige the People under their Power, with any Curioſity that might entertain their Sences, in order to endear them to the Authority they had over them. (He tells us, Cherries were not known in *Italy*, till the 680th Year of *Rome*, when *L. Lucullus* firſt brought them thither from *Pontus*; and that in a Hundred and Twenty Years, they were ſo increas'd, that not only many other Countries, but *Britain* alſo was ſupply'd with them; which muſt have been about Three Years after *Claudius* himſelf had been here. The uſual Landing from *Rome* being then in the County of *Kent*; that Fruit without queſtion was there firſt planted; and the Soil well agreeing with it, may be the reaſon that the beſt and greateſt Quantity of it is yet there to be had.)

Agricola, in the Second Year of his Lieutenancy here, when in Winter-Quarters, purſu'd the ſame Maxims (which *Tacitus* terms *Saluberrima Conſilia*; and, as it may be inferr'd from an Expreſſion of [6] *Cæſar* conducive to the ſame End) to gain the *Britons*, by making them acquainted with the *Roman* Manners: He not only in private perſuaded, but publickly help'd and

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[5] Plin. Lib. XV. cap xxv.

[6] De Bell. Gallic. Lib. I. *Horum omnium fortiſſimi ſunt Belgæ: Propterea quod à cultu atque humanitate Provinciæ longiſſimè abſunt, minimeque ad eos Mercatores ſæpe commeant, atque ea quæ ad effeminandos animos pertinent, important.*

and incourag'd them to build Temples, Places for common Aſſemblies, and private Houſes after the *Roman* Mode: He took care to have the principal Youth inſtructed in the Liberal Arts: He allur'd them to affect the Habit of the *Romans*: And laſt of all, to engage them the more firmly, help'd them to a Taſt of the *Roman* Luxury and Goodfellowſhip, by introducing the Uſe of ſhady Piazzas and Baths [7], and their way of Banqueting. But here, *Tacitus* may be underſtood to ſpeak of what was done in order to civilize the Northern Parts of this Nation, where *Agricola*'s Preſence was required: The Southern was, we may ſuppoſe, ſoftned and quieted by the ſame Methods near Forty Years before, when reduced by *Veſpaſian*.

From hence it may be inferr'd; that ſhould never any other Tokens of the Antiquity of theſe Works be found; yet would the Bath denote the Age of the Pavement, and ſet it near as high as the moſt early Time, that the *Romans* had any real Authority in this Iſland.

As by the Loſs of ſome of the Annals of *Tacitus*, we may have been depriv'd of the moſt early Hiſtory of this County; ſo likewiſe, for want of antient Religious Houſes; there has been little or no Accounts left of its Circumſtances, in the Times next after the *Roman* Authority expired here. *Malmsbury* [8] ſays, that in his Time, there were here only the Abbies of *Battell* and *Lewes*, and thoſe not long erected. The earlieſt Mention made of it, is by [9] *Bede*, who informs us, that Biſhop *Wilfrid*, in the Year 678. being thruſt out of his Province of *Northumbria* by King *Ecgfrid*, ſetled at *Selſey* in 680. and ſtaid Five Years, labouring in

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[7] Tacit. Agricol. cap. xxi.
[8] Gul. Malmsb. de geſtis Pontific. Angl. Lib. II.
[9] Bedæ Hiſt. Eccleſ. Lib. IV. cap. xiii.

in the Converſion of the neighbouring Parts; but of what elſe relates to the County, ſave the miſerable Ignorance of the Inhabitants, and the Number of Families, he has left no Account. *Bede* ſpent moſt of his Time in the Monaſteries of *Wiremouth* and *Jarrow*, and travel'd little; ſo, that conſidering the Diſtance from thence to this County, and the different Governments and Intereſts that lay between, he may well be excus'd for the few Particulars he has left us of it.

The next Records we have to view are thoſe of *Ethelwerd*, the *Chronicon Saxonicum*, and *Henry* Archdeacon of *Huntingdon*. But that you may the more clearly apprehend the antient State of this County; look into the beſt Map of it you can get. At the Weſt End, you will find *Weſt-Harting* and *Stanſted*, diſtant from each other Six or Seven Miles; imagin a ſtreight Line to be drawn from *Harting* to *Bourne* near *Pevenſey*, and another to be drawn from a Point which muſt be little South of *Stanſtead* to *Brighthelmſtone*; What lies North of theſe Lines is the Weald or Lowlands, formerly the *Sylva Anderida*; that which is comprehended between theſe Lines, and bounded by the Sea, from *Brighthelmſtone* to *Bourne*, is the *Downs*, ſo famous for their pleaſant Situation and Fruitfulneſs. The Part South of theſe Lines, is a flat champain Ground, ending like a Wedge at *Brighthelmſtone*. Theſe two laſt Parts were thoſe only that were inhabited in *Bede*'s Time; they contain not more than Two Fifths of the whole County; which muſt be the reaſon why *Bede* ſaid, *Suſſex* [1] conſiſted not of more than 7000 Families or Farms; whereas in another place he computes *Kent* to have 15000 Families.

In

[1] Bedæ Hiſt. Eccleſ. Lib. IV. cap. xiii.

In the three Accounts [1] above-mention'd 'tis agreed, that in the Year 477. *Ella*, with his Three Sons *Cymen*, *Wlencing*, and *Ciſſa*, landed his Forces at *Cymenes-Ora* (which from a Charter of King *Cedwallas* to the Church of *Selſey* the learned [2] *Cambden* proves to be about *Wittering* near *Selſey*;) not far from which he routed the *Britons*, and drove them into the Weald (**Andredesleige**): Their farther Progreſs is moſt diſtinctly and naturally deliver'd by the Archdeacon of *Huntingdon*, in theſe Words; *Saxones autem occuparunt littora Maris in* **Sudsexe**, *magis magiſque ſibi regionis ſpatia capeſſentes, uſque ad nonum annum adventus eorum. Tunc verò cum audaciùs regionem in longinquum capeſſerent; convenerunt Reges & Tyranni Brittonum apud* **Vercredesburne**, *& pugnaverunt contra* Elle *& filios ſuos, & fere dubia fuit victoria. Uterque enim Exercitus valdè læſus & minoratus, alterius congreſſum devovens, ad propria remearunt. Miſit igitur* Elle *ad compatriotas ſuos auxilium flagitans.*

This County having been invaded in the moſt Weſtern part of it by the *Saxons*; if what they did afterwards, was to poſſeſs themſelves of it; their Progreſs muſt have been from Weſt to Eaſt. And ſo much *Henr. Huntingdon*'s Words plainly imply. He ſays farther, they were Eight Years about it; which, if we conſider the Circumſtances of the Country, 'twill be no great wonder it ſhould take up ſo much Time; unleſs their Forces had been very great, which we have no warrant from any Hiſtory to ſuppoſe: For the *Weald* then uncultivated, muſt have been moſt difficult to paſs, even in the drieſt Summers. The *Downs*, like a Wall (with a Terras-Walk on the top) have a very ſteep Deſcent

[1] Ethelward Hiſt. Lib. I Cap. 5. Chronic. Saxon. Ann. CCCCLXXVII. Hen. Hunt. Hiſt. Lib. II.

[2] Camden Brit. Suſſex.

Deſcent into it, their whole Length; excepting, that every Ten Miles, or thereabouts, they have deep Channels through them to afford Paſſage for the Rivers into the Sea: Therefore, what was then habitable, being thus canton'd out into ſo many Parcels by the Rivers; nothing could be more difficult to gain, than thoſe Cantonments; were there any Forces to defend the Paſſes that ſhould have been attempted; the Rivers being deep and muddy, and the Moraſſes on each ſide broad and boggy: Hence we may conceive, 'twas no very difficult Task for the *Britons* to defend, nor an eaſy one, for the *Saxons* to gain the Country. And indeed, the many old Camps, ſtill to be ſeen on the *Downs*, are an Evidence that ſcarce any part eſcaped being a Scene of War. Mr. *Camden* mentions but two, *Ciſsbury* and *Chenkbury*. In the new Edition of his Works Dr. *Harris* has added Three more; a *Roman* Camp at the *Brile* near *Chicheſter*, St. *Rooks-hill*, and *Gons-hill* near the Weſt Limits of the County. It may not be improper here to inſert an Account of the reſt; in which, I ſhall firſt take notice of thoſe that are on the North Edge of the *Downs*, and overlook the *Weald*.

Firſt, *Chenkbury*, mention'd by Mr. *Cambden*, Two Miles Weſt of *Steyning*, and about Three Miles North of *Ciſsbury*; 'tis circular; its Circumference about two Furlongs. From *Chenkbury* Eight Miles Eaſt, over *Poynings*, is a very large one, an Oval, not leſs than a Mile round; acceſſible at one narrow Neck only, and that fortify'd, with a deep broad Ditch, and a very high Bank: I could never learn any other Name it has gone by, than *Poor-Mans Wall*; perhaps from its having been a Security to the diſtreſſed *Britons*. About Three Miles Eaſt from thence, is *Wolſenbury*, on a Hill, projected beyond the reſt of the *Downs*, like a Baſtion; it comes near a Circle in ſhape; its Diameter a little

more than a Furlong. Near Three Miles East of *Wolſonbury*, on the higheſt part of the *Downs* in that Quarter, is a Camp, near ſquare, about 60 Rods long, and 50 broad; much like a *Roman* Camp; the ſide next the North is ſecur'd by the Precipice of the Hill, which is both very deep and ſteep; the other Three Sides have each their *Portæ* after the *Roman* manner ſtill very viſible; the Ditch ſeems to have been not leſs than Eleven Foot broad; but the Ground having been plough'd, the Bank is but low: This is call'd *Ditchling*, as is the old Town under it. Near Seven Miles farther East, and a Mile and half East of *Lewes*, is the laſt on the North Edge of the *Downs*; it goes by the Name of *Caburn*; which perhaps is but a Corruption of the *Britiſh* Word *Cadir*; the Pariſh below it ſtill retains its *Britiſh* Name *Glynd*: This is a round Camp, ſcarce Three Furlongs in Circuit; its Ditch very broad and deep, and the Rampart within very high; the Places where the Tents were pitch'd are yet viſible; which, from the Strength of the Out-Works, intimates that thoſe within held it no ſmall time. Near a Quarter of a Mile Weſt of it, there is a ſtrong Work much larger, but not ſo perfect; yet enough to ſhew, it was made to ſecure a Power, that might lie there to bridle thoſe in the ſtrong Camp, and prevent their making Excurſions towards *Lewes*.

The Camps on the Southern Limits of the *Downs*, are St. *Rooks* near *Chicheſter*. *High-Down*, a ſmall Square, Four Miles East of *Arundell*, and in the Pariſh of *Goring*. *Ciſsbury*, Four Miles South-Weſt of *Steyning*. *Hollingbury* is the only one in the middle of the *Downs*, Two Miles North of *Brighthelmſtone*, and Three Miles South of *Ditchling*; 'tis a Square; the *Portæ* ſtill remaining; it contains about Five Acres. A Mile East of *Brighthelmſtone* on the top of a Hill, half a Mile from the Sea,

Sea, is a Camp, which has a triple Ditch and Bank; this alſo is a Square, only the Corners are rounding; the outmoſt Trench meaſures about three Quarters of a Mile. In the Pariſh of *Telſcomb*, about Five Miles Eaſt of the laſt, are two, but both imperfect; the Cliff is a South-Fence to One; the Other is a Mile diſtant from it; their Weſt Sides are both finiſh'd with very able Works; they were deſign'd for Squares, and to contain 12 or 15 Acres. At *Meeching* or *Newhaven*, on the Point of the Hill, which overlooks the Harbour's Mouth from the Weſt, is a Fortification which they call *the Caſtle*; its Banks are very high, the Shape near half oval, containing about Six Acres; formerly it might be much more, becauſe the Cliff, which forms the Diameter, every Year more or leſs moulders away, and falls into the Sea. Near a Mile Eaſt of *Seaford* is another call'd alſo *the Caſtle*, bounded by the Cliff on the South; its Figure almoſt ſemicircular, the Trench and Rampart large, incloſing Twelve Acres. Three Miles Eaſt of *Cukmere* Haven is the laſt, near a narrow Paſs coming up from the Sea call'd *Burling-gap*; it incloſeth a Hill nam'd *Belltout* of a half oval Shape; the Works have the ſame Figure, and meaſure about three Quarters of a Mile; the Cliff here alſo makes the Diameter.

Though neither Hiſtory nor Tradition, has handed to us any Relation, when either of theſe Works were made or by whom us'd (except *Ciſsbury* by *Ciſſa*) yet from this View we may conceive, the Calamity of War once rag'd in all theſe Parts: that the Ground was diſputed inch by inch: that in the Attack, as well as Defence of it, the Pick-Axe and Spade, were as much made uſe of, as the Sword: and laſtly, that, unleſs the Aggreſſors were very numerous, eight Years was no long time taken up, in diſpoſſeſſing the Inhabitants of this faſt Country.

Some may imagine, many of these Camps were made by the *Danes*; but by what may be observ'd from the History of those Times, that People seem'd not to be so formal an Enemy, as to prolong War by Encampments: Their Refuge was in their Fleets that always attended them; so that, when likely to be vigorously oppos'd, they betook themselves to their Ships, and suddenly invaded another Part where was less Opposition: and what they could not carry with them, consum'd with Fire and Sword. Thus continually harassing the Nation by their hasty and rapacious Visits, they exhausted it of its Riches and Strength, and as it were imitating the Quality of the Faulcon their Ensign, they flew the Prey to a Stand, and then seiz'd it.

The Archdeacon of *Huntingdon*, in the Prologue or Dedication of his Annals, to *Alexander* Bishop of *Lincoln*, assures his Diocesan, that he compil'd his History from Chronicles reserv'd in ancient Libraries; no question therefore, when speaking of the *Saxons* here, he had good Authority to say (as above cited), *magis magisque sibi Regionis spatia capessentes*; and that no other Meaning could belong to it; than that they carried their Conquest from West to East, *in longinquum* lengthways. Had they entirely made themselves Masters of the Country, 'twould have been too late: But before they had wholly gain'd it, the *Britons* assembled against them; the *Saxon* Chronicle says neah, i. e. *propè*; *Ethelwerd*, *juxta*; or, as *Huntingdon* has it, *apud* Wercredesburne; where a Battle was so hard fought, that each Side had enough on't, and retir'd. The *Saxons* were so diminish'd, that *Ella* was oblig'd to send for more Forces. This Action was in the Ninth Year after *Ella*'s first footing here, Three Years before *Hengist*'s Death, *Ann. Dom.* 485. It so weaken'd *Ella*, that we hear no more of him till he receiv'd his Supplies from

from *Germany*; which came not, according to *H. Huntingdon*, till the first Year of the Emperour *Anastasius*. Three Years after *Hengist*'s Death, and Six Years after the hard Battel, *viz. An. Dom.* 491.

Being thus strengthned, *Ella* mov'd again, besieg'd *Anderida* (in *Huntingdon*'s Words, *Urbem munitissimam*) at last forced the Place; and by reason of the stout Resistance the Defendants made, Savage like, left not a Soul alive, and raz'd the City, which in *Huntingdon*'s Time remain'd desolate.

As to the Field where the Battel was fought; the *Saxons* extending their Power Eastward, the Check that was given them, in all probability must have been where they push'd on their Victories; and it being near **Wercredesburn**, this *Bourne* near *Pevensey* may be the Place meant, since it sounds like the latter part of that Name (for there not being a West *Bourne* that it relates to, the Name of it may rather be *Esbourne* than East-*Bourne*;) and likewise that *Anderida*, the *Britons* last Stake and Support, was not far from it. 'Tis probable therefore the Battel was fought on the *Downs*, between the Camp last mention'd at *Burling-Gap* and East *Bourne*; for there are no where on the *Downs*, that I have seen (and there are few Parts of them that I have not often view'd), Marks of a greater Battel than there; because, from the top of that very high Cliff, by the Inhabitants call'd *The Three Charles* (and by Mariners *Beachy-Head*) to *Willington Hill*, which is four Miles, the Ground is full of large *Tumuli* or Places of Burial; and in many parts within that Tract, where the Position of the Ground seems to offer, there are deep Trenches and Banks, which one would imagin were Breast-Works made to defend the Front of an Army; and the *Tumuli* on each side of them seem to shew, there was no small Struggle, in forcing as well as defending them. The

The Learned and Judicious Mr. *Somner* [1] diſlikes, that the Site of *Anderida* ſhould be fix'd at *Newenden*, and is inclin'd to aſſign ſome Place in *Suſſex* for it: But from a modeſt Deference to the Opinions of the Learned *Camden* and *Selden*, he drops the matter.

But let us ſee, what our more elder Hiſtorians ſay of it; *Henry* of *Huntindon*'s Words are, *Et quia tot ibi damna toleraverant Extranei, ita Urbem deſtruxerunt, quòd nunquam poſtea reædificata eſt. Locus tantum, quaſi nobiliſſimæ urbis, tranſeuntibus oſtenditur deſolatus.* *Mathew* of *Weſtminſter* ſays, *Locus autem Civitatis uſque hodie tranſeuntibus oſtenditur deſolatus. Manſit ergo ibidem* Ella *cum tribus Filiis ſuis, & Regionem illam, quæ uſque hodie Anglicè* **Suthſex**, *Latinè autem Regio Auſtralium Saxonum dicitur, colere cœpit.* From the Expreſſions above-cited, it may be ſuppos'd the Ground where that City ſtood was not quite forgot, in either of thoſe Hiſtorian's Days. *Henry* of *Huntindon* being the elder by 200 Years (had *Newenden* been the Place), his Words might have been true, in ſaying it was deſolate: But 'tis very improbable *Mathew* of *Weſtminſter* ſhould have ſaid ſo likewiſe; or at leaſt, not taken notice of the Act of Piety and Charity of Sir *Thomas Albuger*, who, in his Time, had newly erected a Monaſtery at *Newenden* [2] for the *Carmelites* who came from *Paleſtine*: But let that paſs: what Authority Mr. *Camden* had for ſaying [2] *Hengiſt* ſent for *Ella* out of *Germany*, to help him reduce *Anderida*, is not to be found. From the Accounts above ſtated, and others that might be produced, it is clear, that *Hengiſt* was dead Three Years before the Siege was laid to *Anderida*. In the Time of *Hengiſt*'s Life, we find, for Eight Years

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[1] Somner's *Roman Ports and Forts in* Kent, p. 106. [2] *Camd. Brit. Kent Edit.* 1695. *Col.* 211.

Years *Ella* had enough to do in *Suſſex*; and the Blow he had given him the Ninth Year at **Wercredesburne**, oblig'd him to be quiet the other two Years of *Hengiſt*, and till his Succours (as above-mention'd) came to him from *Germany*. Beſides, we have not the leaſt Hint from any of our Hiſtorians, that *Anderida* was an Eye-ſore, either to *Hengiſt* or his Son *Esk* after him; or that *Ella* aſſiſted the *Kentiſh Saxons*, or the *Kentiſh Saxons Ella* in reducing it: Therefore this muſt be a Suppoſition only of Mr. *Camden*, in order to give Strength to the Notion of *Anderida*'s being at *Newenden*. Taking no notice therefore of that Suppoſition, we may conſider *Newenden* is on the *Kent* ſide of the *Limen* (for ſo is the River *Rother* call'd [3] in the *Saxon* Annals, and by *Mathew Weſtminſter*; and the Mouth of it nam'd *Portus Limeneus*, and **Limene** by *Ethelwerd* [4] and *Henr. Huntindon*;) and that *Kent* having been ſubdued by *Hengiſt* and his *Saxons*, near Forty Years before; the Town at the Mouth of the *Limen*, and the reſt, if any, up the Stream on the ſide of *Kent*, were alſo part of their Conqueſt.

Furthermore, after it had coſt *Ella* ſo much Time, and no doubt Pains too, in reducing the plain Ground of *Suſſex*, 'tis not likely he ſhould call more Forces out of *Germany*, that he might lead them Thirty Miles, through the Difficulties of the great Wood (which he muſt have done if *Newenden* were the Place,) to beſiege a City, ſo far from his own, and within the *Kentiſh-Saxon* Limits, eſpecially if there's any heed to be given to the Words of *Math. Weſtminſter* before cited; who, after relating the ſad Fate of the Inhabitants and

City

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[3] Chron. Sax. A. Dom DCCCXCIII. Mat. Weſtm. Fl. Hiſt. A. Dom. DCCCXCII. [4] Ethelwerd Lib. III. cap. iii. A. D. DCCCXCIII. Hen. Hunt. Hiſt. Lib. V. Alfr. Reg. an. 19.

City of *Anderida*, immediately ſubjoins, *Manſit ergo, &c.* Ella *and his Sons reſided there* (*i. e.* in that part of *Suſſex* where *Anderida* was), *and began to cultivate and improve the Country.*

In the laſt place, from the Uſe made of *Anderida* by the *Romans*, 'tis not likely (as Mr. *Somner* [5] judiciouſly hints) its Place was at *Newenden* ; for being one of the Stations, under the *Præfectus littoris Saxonici*, where Forces were quarter'd, to have a watchful Eye on the Sea, when ever the *Saxon* Pyrats came to infeſt the Coaſt : We may ſuppoſe it, like the reſt of the Gariſons under that Officer, conveniently ſituated for the ſame purpoſe ; as were *Branodunum* [6] *Brancaſter* at the North Point of *Northfolk* ; *Gariannonum*, North-*Yarmouth*. or very near it ; *Othona*, *Ithancheſter* in *Dengy* Hundred, in *Eſſex*, ſome Ages ſince ſwallowed up by the Sea ; *Regulbium*, *Reculver* in *Kent* ; *Rutupis*, *Richborow* ; *Dubris Dover* ; *Lemannis* (which from the *Saxon* Chronicle [7] we muſt look for, Four Miles Eaſt of *Appledore*) probably New *Romney*, all ſituate near the Sea, on Ground which had a full Proſpect of the Sea : whereas *Newenden* lies low, at leaſt Eight Miles within *Appledore*, on a turning of the River, where the Land Eaſtward muſt have cut off any Proſpect of the Sea. To all this may be added, that the *Romans* having a *Numerus*, Cohort, or Battalion of the *Turnacenſes*, in Gariſon at the *Portus Lemanis* on the Mouth of the Haven, we may ſuppoſe they knew how to husband their Strength to better purpoſe, than to place another

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[5] Somner Rom. *Ports and Forts*, pag. 103. [6] Not. Imperii à Pancirol. cap. lxxiii. pag. 162. [7] Chron. Sax. A. Dom. DCCCXCIII. *Tum appulerunt* (ſc. *Dani*) *in Limeni oſtium cum* CCL. *Navibus. Super eum Fluvium traxerunt ſuas Naves uſque ad ſylvam, quatuor millariis ab exteriore parte æſtuarii ; ibique expugnarunt quoddam munimentum* (ſc. *Apuldre.*)

another Gariſon to watch the Motions of the *Saxon* Rovers, Twelve Miles up the little River, quite out of ſight of the Sea, where they could be of no Service.

Thoſe who would have the Seat of *Anderida* to have been at *Haſtings*; let them look on theſe Words of *Henr. Huntindon* [1] (*Haraldus rex Anglorum, eadem die reverſus ad* Eouirwic *cum ſumma lætitia, dum pranderet, audivit nuntium dicentem ſibi, Willielmus dux Normanniæ littora Auſtralia occupavit, & caſtellum conſtruxit apud* Paſſings,) and they will conclude *Haſtings* was not a deſolate place, in the Ages of the Hiſtorians, who affirm *Anderida* was: If at *Pevenſey*; that Place was ſo far from being raz'd by *Ella*, that even after the *Norman* Conqueſt it remain'd a ſtrong Caſtle, where *Odo*, Biſhop of *Bayon* and his Forces ſuſtain'd a Six Weeks Siege; and for want of Proviſion were oblig'd to ſurrender to K. *William* II. At this time there is ſo much of *Pevenſey* ſtanding, that perhaps 'tis the greateſt and moſt entire Remain of *Roman* Building, any where to be ſeen in *Great Britain*.

From the Arguments on the foregoing Authorities, *Anderida* muſt have been ſomewhere in *Suſſex*, not in the Weſt but Eaſt part of it, and not far from the Eaſt End of the *Downs*, near the Sea. From the Bath, Pavement, Coins, and Bricks, 'tis ſure the *Romans* had once an Abode, and not a ſhort one, at this Place near *Eaſt-Bourne*: From the large Extent of Foundations about the Place where theſe were diſcover'd; that there was a large Town or City there: From the common Height thoſe Foundations bare under the Surface of the Ground; that the Buildings they ſuſtain'd were effectually levell'd or raz'd: And from the Coals dug up amongſt the Rubbiſh, 'tis evident that Part was burnt; all which Cir-

[1] *Henr. Huntindon*, Hiſt. Lib. VI.

cumſtances well enough agree with the Account given us of *Anderida*.

The Situation likewiſe of a Town here, gives reaſon enough to ſuppoſe, it was a Place of Importance, and whence it had its Name; no Part hereabouts being any way ſo convenient, for a ſecure Settlement; or for ſuch a uſe as the *Romans* might have occaſion to make of it. We are inform'd by *Cæſar*, that the Maritime Parts of *Britain* (ſpeaking of what he ſaw, which was the South-Eaſt) were inhabited by People from *Belgium*; and that they call'd their Settlements by the Name of the Places from whence they came. It was the Opinion of *Tacitus* alſo, that [2] thoſe who inhabited next to *Gaule*, came from *Gaule*. And *Bede* ſays, the Tradition in his Time was, that the Southern Part of the Iſle was peopled [3] from *Bretaign*. In the Third and Seventh Books of *Cæſar*'s Commentaries, mention is made of the *Andes*, a City and a People belonging to it among the *Celtæ*, inhabiting on the Sea-Coaſt. Time varying the Names of Things, near Two Hundred Years after *Cæſar*, *Ptolemy* calls the City *Anderidum*: And near 250 Years after him, when the *Notitia Imperii*, now extant, was in uſe, the *Claſſis Anderetianorum* [4] is regiſter'd; and the Reſidence of their Admiral fix'd at *Paris*. From whence 'tis to be inferr'd, that tho' the Capital of the *Andes* might have been *Angers* near the *Loyre*, yet their Country had on the North the *Britiſh* Chamel; and on the Eaſt the *Seine* for its Bounds. The *Britiſh* Coaſt about *Eaſt Bourne* is the neareſt of any to the Mouth of the *Seine*: Therefore, according

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[1] De Bell. Gal. Lib. V. *Qui omnes, ferè iis nominibus civitatum appellantur, quibus orti ex civitatibus eo pervenerunt.* [2] Tac. Agric. cap. xi. *In univerſum tamen æſtimanti, Gallos vicinum ſolum occupaſſe credibile eſt.* [3] Bedæ Hiſt. Eccleſ. Gent. Angl. Lib. I. cap. 1. *In primis hæc Inſula Britones ſolum, à quibus nomen accepit, incolas habuit; qui de tractu Armoricano (ut fertur) Britanniam advecti, Auſtrales ſibi partes illius vindicarunt.* [4] Pancirol. Comm. in Notit. Imp. Cap. XC. pag. 179, 180.

according to the Uſage before *Cæſar*'s Time, the Name of *Anderida* there, is readily accounted for. Moreover, this Place ſeems moſt naturally ſeated, for giving an Appellation to the great Wood, to which it adjoin'd: For, as it ſelf is on the Shoar, ſo alſo the *Sylva Anderida* here, came very near the Shore; and a large part of it might be ſeen from the Sea before it: Indeed, on the Sea off of *Romney*, it might be diſcover'd; but then the Diſtance was great: At all other parts of the Coaſt, the Sight of it from Sea, is hinder'd by Hills, or high Cliffs.

Setting aſide the want of a navigable River, the Spot of Ground where this old Town ſtood, yields to none in the County for Importance and Pleaſure: For here, like a Wedge, ends the firm Soil of the *Downs*; Nature has ſhap'd it like an Equilateral Triangle, having each ſide half a Mile in Length: Towards the Sea, on the Southern ſide, 'tis fenc'd by a low Cliff, of 12, 15, and in ſome Places 20 Foot high (in which Cliff is now to be ſeen a ſtrong Foundation, that has acute Angles, which ſhews it to have been for a Fort rather than a Dwelling-Houſe) On the Northern ſide is a Moraſs, with a large Rivulet of very good Water. Between the Weſt ſide and the *Downs* lies a ſmall Valley, by which Advantage, there was formerly a Harbour, capable of a ſmall Fleet; the Banks on each ſide of it are an Evidence it was ſunk by Induſtry; but by Weeds and Gravel from the Sea, and by Mould annually add[1], as is obſervable in Valleys, it is now ſo rais'd, that 'tis never flow'd but at high Spring-Tydes, when a ſtrong Wind forceth the Waves into it. This Harbour muſt have been a good Security to part of the Weſt ſide; what other Works might have been to guard it, from

[1] Philoſ. Tranſact. An. 1701. N°. 274. Pag. 926.

the end of the Harbour to the Moraſs, cannot be ſaid; by reaſon the Ground between has for many Ages been in Tillage. It is eaſy to imagin of what Importance a Town fortified at this Place muſt have been in thoſe Ages, when the only Paſs by Land from the Weſt to the Eaſt End of the County was through it; for other there could not be, in many Miles North; unleſs the Lands in that Tract, which are ſtill very owzy and tender, had been well drain'd.

As the Situation deſcrib'd, render'd this Place ſtrong; it is very pleaſant withal; for the Ground is high enough for a good Proſpect of the Low Lands adjoining, and the Country towards *Battell*; beſides, it has a commanding View over that Bay, which is between *Beachy-Head* and *Haſtings*. If the Uſe made of it by the *Romans*, was to guard the Coaſt, there was this Advantage belonging to it; that a Centinel on the top of *Beachy*, not Two Miles from it, in a clear Day, without turning his Body, might ſee the *Iſle of Wight*, the Hills in *France* near *Bologn*, and the *Neſs* in *Kent*; ſo that from the *Neſs* to *Selſey* it muſt have been a ſmall Sail that could eſcape his Eye. It was my purpoſe to have added a Deſcription of *Pevenſey*-Caſtle; together with an Account of ſome Remains of Antiquity, diſcover'd laſt Summer towards the Weſt End of the County: But having been too tedious already, muſt defer that for the preſent, and ſubſcribe my ſelf

Your moſt humble Servant,

Lewes, Jan. 26. $17\frac{16}{17}$.

JOHN TABOR.

III. *Tractatus*

III. *Tractatus de* Curvarum *Conſtructione & Menſura; ubi plurimæ ſeries Curvarum Infinitæ vel rectis menſurantur vel ad ſimpliciores Curvas reducuntur. Autore* Colin Maclaurin, *in Collegio novo* Abredonenſi *Matheſeos Profeſſore.*

EXimiæ Matheſeos Theoriæ, ob infinitam Propoſitinum Univerſalitatem, æternam ac neceſſariam Veritatem, Evidentiam omni dubitatione majorem, Idearum claritatem luculentiſſimam, Demonſtrationum elegantiam, Theorematum nexus & mutuas dependentias, pulcherrimis certè ac ſummis humani intellectus repertis ſunt annumerandæ; inter eas vero eminent ſummorum hujus ſæculi Philoſophorum de Curvarum Longitudinibus & areis menſurandis ardua Theoremata. Ad hos diffuſos cognitionis campos diu altè latentes tandem eruendos infinitæ ſcientiæ portiunculam mutuari, vix ſibi temperare poſſet quin pronuntiaret, qui Arithmeticæ Infinitorum vires in immenſo elegantiſſimarum Veritatum abyſſo eruendo, & humani intellectus Horizontem infinite ferè extendendo, paucis præteritis annorum decadibus, amplè ſatis comprobatas, animo perpenderit; Hujus vero methodi (ſicut nunc aucta & exculta eſt) ope, incidi in rationem menſurandi infinitas Curvarum ſeries, quam pauciſſimis explicabo.

Cum in omni linea curva ſit aliqua curvaturæ regularitas licet fortè implicata, ſecundum quam figura determinatur; ideo Geometræ varias Curvarum characteres ex Æquatione Ordinatarum relationem ad abſciſſas axis alicujus exprimente definirunt. Cum verò idem fieri poſſit ex conſideratione Curvarum reſpectu unius dati centri,

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imo ſimpliciſſima Naturæ uniformitas in ejus indagine id fieri ſæpe poſtulet, ideo hanc Curvas conſiderandi Methodum impræſentiarum uſurpabimus, & imprimis oſtendemus qua facillima ratione (ſecundum hanc Methodum Curvas determinandi) ex ſimplicibus complexiores conſtrui poſſint.

§ I. Sint L & *l* puncta quamproxima in Curva B *l* L; ſit *l o* arcus centro S deſcriptus perpendicularis in S L; & erit L *l* ut momentum Curvæ & L *o* momentum Radii S L: Ac ſi detur ratio L *l* ad L *o*, vel ad *l o* in diſtantia S L, dabitur æquatio Curvæ ad centrum S. Sint L P, *l p* Tangentes Curvæ in punctis L & *l*, in quas ex S demittantur normales S P, S *p* iis occurrentes in punctis P & *p*; ſimiliter in omnes Curvæ Tangentes demittantur perpendiculares ex dato puncto S, & conſtruetur Curva tranſiens per omnes Tangentium & perpendiculorum interſectiones. Hujus triangulum elementare P *n p* ſimile erit triangulo L *o l*, quæ proinde dabitur ex data Curva B *l* L. Quippe ob æquales S *n p*, P *n* L, & rectos S *p n*, S P L æquiangula erunt triangula S *p n*, P *n* L, & proinde P *n* : *pn* :: L *n*: S *n* :: L *o* : *l o*, adeoq; ob angulos P *n p*, S *n* L, L *o l* æquales, erunt triangula P *n p*, S *n* L, L *o l* ſimilia. Cum igitur eadem ſit ratio L *l* ad *l o* quæ P *p* ad *p n*, & S L ad S P, manifeſtum eſt, datâ ratione L *l* ad *l o*, & rectâ S L, dari rationem P *p* ad *p n* & rectam S P, adeoque Curvam D P *p*. Eadem ratione ex D P conſtrui poteſt Tertia, & ex ea dein Quarta, & progrediendo prodibit ſeries Curvarum infinita, quæ omnes ex uno dato innoteſcunt. Quod ſi erigantur L N &

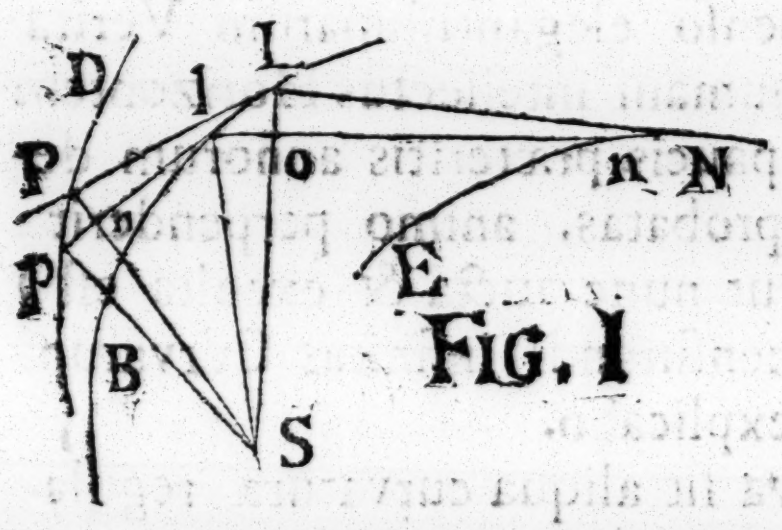


FIG. I

& *l n* perpendiculares in radios S L, S *l*, ſibi mutuo occurrentes in *n*; & per omnia ſimiliter definita perpendicularium concurſuum puncta deſcribatur Curva E N: ea ipſa erit Curva ex qua deduci poteſt B L, eadem ratione qua conſtruximus D P ex B L. Ex E N ſimiliter conſtrui poteſt alia Curva, atque ex hac quoque parte Series infinita Curvarum conſtrui poterit.

§ II. Curvarum verò hac ratione conſideratarum ſimpliciſſimæ ſunt quarum L *l* eſt ad L *o* in ratione poteſtatis alicujus Radii, ita ut, ſi *a* ſit data quantitas, *r* denotet Radium Curvæ, *n* numerum quemcunque, ſit L *l* ad *l o* ut a^n ad r^n æquatio earum generalis. Omnes verò hæ Apſidem habent cum $r = a$, quoniam in eo caſu L *l* = *l o*. Ut inveſtigem æquationem Curvæ D P, cum in B L eſt ut L *l* ad *l o* ita a^n ad r^n, ita *r* ad $SP = \frac{r^{n+1}}{a^n}$, ita $\frac{n}{a^{n+1}} \times SP^{\frac{1}{n+1}}$ ad S P, ita $a^{\frac{n}{n+1}}$ ad $SP^{\frac{n}{n+1}}$, ita P *p* ad *p n*. Proinde ſi *s* repreſentet momentum Curvæ, *y* arcum circularem radio deſcriptum à centro S, & *r* radium correſpondentem, quæcunque ſit Curva cujus Æquatio inveſtigatur, erit Æquatio Curvæ B L, $s : y :: a^n : r^n$; Æquatio verò Curvæ D P, $s : y :: a^{\frac{n}{n+1}} : r^{\frac{n}{n+1}}$. Angulus autem P S *p* erit ad Angulum L S *l* ut $\frac{pn}{SP}$ ad $\frac{l\,o}{SL}$, ſive ut $\frac{Pn}{SP}$ ad $\frac{L\,o}{SL}$, vel (ſi S P dicatur *x* & S L, *r*) ut $\frac{\dot{x}}{x}$ ad $\frac{\dot{r}}{r}$, hoc eſt, (ob $x = \frac{r^{n+1}}{a^n}$) ut $\frac{n+1}{r}\dot{r}$ ad $\frac{\dot{r}}{r}$, ſive ut $n+1$ ad 1. Hinc (*vid. Fig.* II.) B S P eſt ad B S L ut $n+1$ ad 1; unde facilius abſque Tangentium ope duci poteſt Curva B P. Si ſumatur angulus B S P ad B S L in ratione $n+1$ ad 1, & in S P demittatur perpendicularis ex L, erit occurſus perpendiculi cum S P, in Curva B P prius Tangentium ope deſcripta.

§ III. Oſten-

§ III. Oſtendimus quo pacto ex unâ ſeries Curvarum infinita deducitur; quo verò pacto ſingularum longitudines ex illius & unius alterius longitudinibus datis innoteſcant pergo demonſtrare. Cum angulus S P *p* = S L *l*,

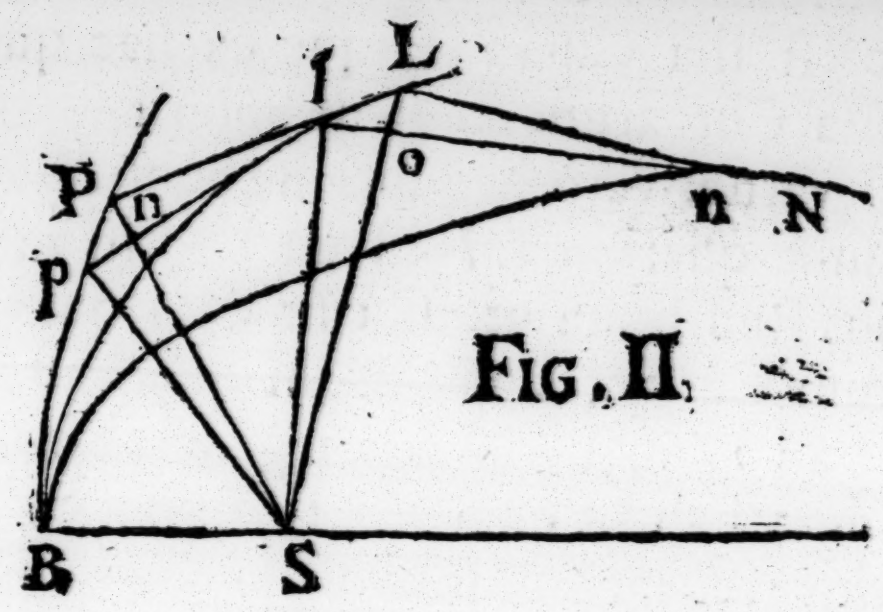


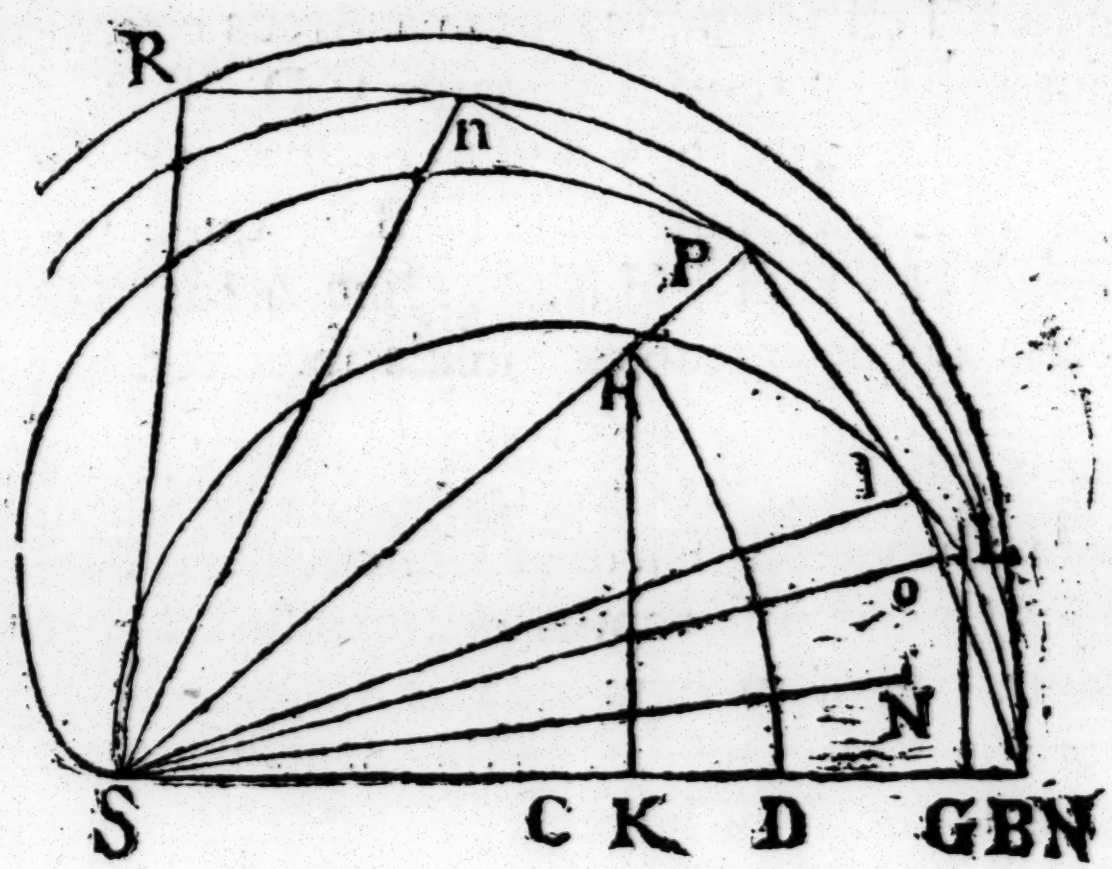
FIG. II.

atque L S *l* ſit ad P S *p* ut 1 ad *n* + 1, erit L *l* ad P *p* ut S L ad *n* + 1 S P, ſive (ob S L : S P :: L *l* : *l o*) ut L *l* ad *n* + 1 *l o*, ac proinde P *p* = *n* + 1 *l o* : ſed *l o* = *l n* — *o n* = *l n* — L N + N *n* ; ergo P *p* = *n* + 1 × *l n* — L N + N *n*. Sed *l n* — L N eſt momentum rectæ L N normalis in S L, P *p* momentum Curvæ B P, & N *n* momentum Curvæ B N : Cumque B P, B N, B L ſimul evaneſcant in B, erunt in ratione momentorum, adeoque B P = *n* + 1 × B N + vel — L N. Unde Curva B P eſt ad ſummam vel differentiam Curvæ penultimæ in Serie ejuſque Tangentis ab intermedia interceptæ, ut *n* + 1 ad 1 ; ſive, ſi *m* ſit Index æquationis Curvæ B P (quoniam $m = \frac{n}{n+1}$) ut 1 ad 1 — *m*.

Hinc 1^mo^ in ſerie Curvarum infinitâ ſupra deſcriptâ, ſi dentur Longitudines duarum proximarum, dabuntur longitudines omnium; quippe menſura cujuſvis pendet à menſura penultimæ ſemper in ſerie, & proinde unum par

par omnibus menſurandis ſufficiet : Si una Curva ſit rectis commenſurabilis vel incommenſurabilis, erit integræ ſeriei pars dimidia rectis commenſurabilis vel incommenſurabilis. Hinc 2^do. Licet Curvæ B P & B N eſſent rectis incommenſurabiles, differentia tamen Curvæ B P ab $n+1$ Curvæ B N eſſet æqualis aſſignabili rectæ. 3^io. Si Curva tranſit per S, rectâ L N evaneſcente in S, erit $BPS = \frac{BNS}{1-m}$.

§ IV. Curvarum de quibus egimus, quarum nimirum $s : y :: a^n : r^n$, maxime inſignis eſt Circulus, exiſtente S in circumferentia, cujus æquatio eſt $s : y :: a : r$, ut ex ſimilitudine triangulorum *L o l*, B L S (*Fig.* III.)



manifeſtum eſt, adeoque $n = 1$; & proinde $m = \frac{n}{n+1} = \frac{1}{2}$, & æquatio Curvæ Б P erit $s : y :: a^{\frac{1}{2}} : r^{\frac{1}{2}}$, quæ ipſa eſt æquatio *Epicycloidis* revolutione Circuli ſuper baſim ſibi æqualem revolventis deſcripti, ad punctum ubi punctum deſcribens tangit baſim, quæ D^no *Paſchal* dicitur *la Li-*

maſon de M. Roberval, quamque *M. De la Hire* conſiderat ut *Conshoidem* Baſis Circularis, in Actis Academiæ Pariſienſis Anni 1708. Perpendiculares omnes L N, *ln* concurrunt in puncto B, adeoque B N = c: unde B P = $\frac{BN+NL}{1-m}$ = 2 B L: Hinc Curva tota B P S = 2 B S, ac longitudo *Epicycloidis* ſemper dupla eſt chordæ arcus in circulo correſpondentis. 2^do. Ex *Epicycloide* deſcribatur Curva B Π S, eadem ratione qua *Epicycloidem* ex Circulo deſcripſimus: In hoc caſu $n = \frac{1}{2}$, & $m = \frac{n}{n+1} = \frac{\frac{1}{2}}{\frac{1}{2}+1}$ $= \frac{1}{3}$, ac proinde æquatio Curvæ B Π S erit $\dot{s} : \dot{y} :: a^{\frac{1}{3}} : r^{\frac{1}{3}}$. Longitudo Curvæ erit $\frac{BL+LP}{1-m} = \frac{3}{2}\overline{BL+LP} = \frac{3}{2}\overline{BL+LG}$, & proinde BΠ eſt ſeſquiplus ſummæ Arcus circularis ejuſque Sinus recti. Quod ſi ſumatur C D = B D, & radio S D centro S deſcribatur Circulus occurrens rectæ S P in H, & ſit H K perpendicularis in B S; quoniam D H = $\frac{3}{2}$ B L, erit B Π = D H + H K. Hinc arcus B Π neque ſunt rectis neque arcubus circularibus commenſurabiles, differentia tamen arcuum B Π & D H eſt recta H K. In puncto S evaneſcit L G, adeoque B Π S = $\frac{3}{2}$ B L S, unde tota Curva eſt ſeſcupla ſemicirculi. Nulla vero pars hujus Curvæ aſſignabilis commenſurari poteſt toti, nec integra Curva in data quavis ratione ſecabilis eſt, ita ut portiones rationem aſſignabilem habeant ad ſe mutuo aut ad totam. Si hæc curva in data aliqua ratione Geometricè ſecari poſſet, conſtaret Quadratura Circuli, nam ſi *c.gr.* eſſet B Π ad B Π S ut 1 ad *m*, & B L ad B L S ut 1 ad *n*, eſſet BΠ = $\frac{B\Pi S}{m} = \frac{3\,BLS}{2m} = \frac{3n\,BL}{2m} = \frac{3}{2}BL+LG$, unde eſſet B L = $\frac{m\,LG}{n-m}$ & B L S = $\frac{n\,m}{n-m}$ L G. 3^tio Ex BΠS conſtruatur explicata methodo Curva B R, & quoniam

$n = \frac{1}{3}$ erit $m = \frac{n}{n+1} = \frac{1}{4}$, atque æquatio Curvæ BR erit $\dot{s} : \dot{y} :: a^{\frac{1}{4}} : r^{\frac{1}{4}}$. Hinc longitudo Curvæ fiet $\frac{4}{3}\sqrt{2BL + PI}$, totalis verò Longitudo Curvæ BRS $= \frac{8}{3}$ diametri SB. Si harum Curvàrum Conſtructiones continuentur, prodibit hujuſmodi ſeries Æquationum quæ facile produci-tur ad libitum.

Æquatio Circuli	1.	$\dot{s} : \dot{y} :: a : r$
Epicycloidis	2.	$\dot{s} : \dot{y} :: a^{\frac{1}{2}} : r^{\frac{1}{2}}$
Secundi	3.	$\dot{s} : \dot{y} :: a^{\frac{1}{3}} : r^{\frac{1}{3}}$
Tertii	4.	$\dot{s} : \dot{y} :: a^{\frac{1}{4}} : r^{\frac{1}{4}}$
Cujuſvis	n.	$\dot{s} : \dot{y} :: a^{\frac{1}{n}} : r^{\frac{1}{n}}$, &c.

Obſervare licet in genere, omnes quarum Indicum denominatores ſunt Numeri pares, perfectæ rectificationis eſſe capaces; cumque quævis ſit ad penultimam ut 1 ad $1-m$, perpendenti manifeſtum erit Curvæ cujuſvis longitudinem fore $= \frac{1}{1-m} \times \frac{1-2m}{1-3m} \times \frac{1-4m}{1-5m} \times \frac{1-6m}{1-7m}$, &c. $\times$ SB continuando ſeriem donec ad nihilum reducatur Fractio. Quod ſi Indicis denominator ſit Numerus impar, Curvæ erunt perfectæ rectificationis incapaces, & earum arcus quicunque erunt ſibi mutuo, ipſis totis, rectis quibuſvis & arcubus Circularibus incommenſurabiles: exprimi verò poſſunt omnes arcubus circularibus & rectis: At Curvæ cujuſvis totalis Longitudo erit ad Semicirculum ut $\frac{1}{1-m} \times \frac{1-2m}{1-3m} \times \frac{1-4m}{1-5m}$, &c. ad unitatem. Denique ſi Areola à Corpore in harum quavis revolvente ſumatur conſtans, hoc eſt ſi $r\dot{y} = 1$, ſubtenſa anguli contactus, cui ſemper (ob datum datâ areâ tempus) proportionalis eſt *Vis Centripeta* tendens ad S, erit reciproce ut poteſtas diſtantiæ cujus Index eſt $2m+3$; atque hoc eſt non con-

temnendum harum Curvarum privilegium, quod in iis omnibus Vis centripeta tendens ad S ſit ut aliqua reciproca diſtantiæ dignitas, quæ ſimpliciſſima eſt, & utiliſſima in Naturæ indagine, Virium Centripetarum lex.

§. V. Curvarum quarum $s : y :: a'' : r''$ proxime conſideranda venit (quæ Curva quidem improprie dicitur) ipſa Linea recta, exiſtente S extra rectam. In hac linea, ob ſimilia triangula P p n, P B S erit (ſi B S $=$ a & S P $=$ r) $s : y :: r : a$. Ex linea recta methodo directâ

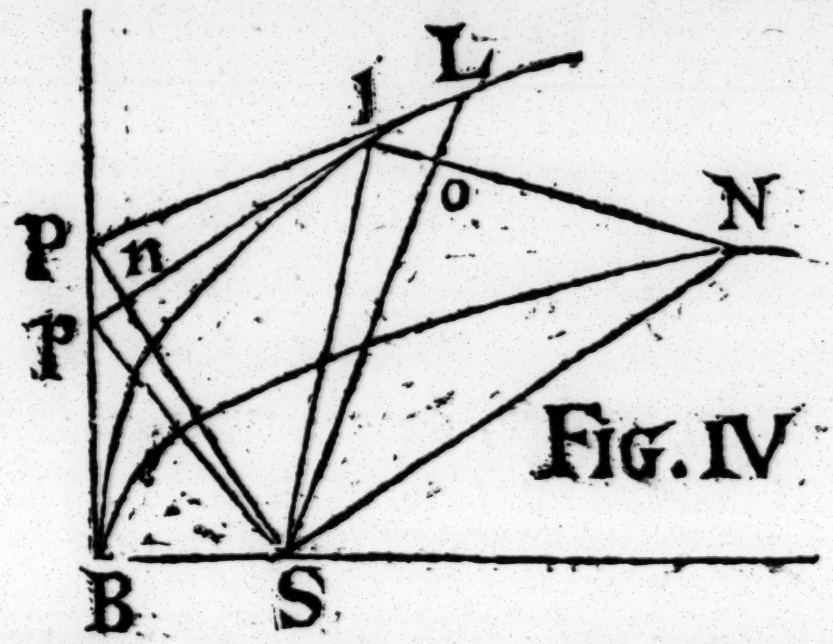


FIG. IV

nihil niſi punctum B conſtrui poteſt, Methodo vero inversâ perpendicularium nimirum P L, $p\,l$ concurſu, conſtrui poteſt Curva, cujus Index (ſi m ſit Index Curvæ B P) æqualis erit $\frac{m}{1-m}$; nam ſi Index Curvæ B L ſit n, erit $m = \frac{n}{n+1}$, ac proinde $n = \frac{m}{1-m}$. Unde in hoc caſu, cum $m = -1$ erit $n = \frac{-1}{2}$, & æquatio Curvæ B L erit $s : y :: r^{\frac{1}{2}} : a^{\frac{1}{2}}$, quæ æquatio eſt Parabolæ ad Focum. Ex hac conſtrue aliam, conſtituendo angulum L S N $=$ L S B & erigendo L N normalem in S L occurrentem ipſi S N in N. Quoniam vero $m = \frac{-1}{2}$ erit $n = \frac{-1}{3}$, & æquatio Curvæ $s : y :: r^{\frac{1}{3}} : a^{\frac{1}{3}}$ & B P $= \frac{BN - LN}{1-m} = \frac{1}{2} BN - LN$, ergo B N $=$

$BN = 2BP + LN$; proinde hæc Curva eſt rectificabilis. Si Series continuetur, prodibunt ut prius æquationes in hoc ordine.

Æquatio Rectæ	$\dot{s} : \dot{y} :: r : a$
Parabolæ	$\dot{s} : \dot{y} :: r^{\frac{1}{2}} : a^{\frac{1}{2}}$
Secundæ	$\dot{s} : \dot{y} :: r^{\frac{1}{3}} : a^{\frac{1}{3}}$
Tertiæ	$\dot{s} : \dot{y} :: r^{\frac{1}{4}} : a^{\frac{1}{4}}$
Cujuſvis	$\dot{s} : \dot{y} :: r^{\frac{1}{n}} : a^{\frac{1}{n}}$

In hac Serie primæ ſunt Recta & Parabola, unde patet dimidiam hujus ſimiliter ac prioris Seriei eſſe rectis menſurabilem: alia vero dimidia pars in rectis & arcubus Parabolicis exhiberi poteſt. In his omnibus Vis centripeta ad S eſt reciproce ut poteſtas diſtantiæ cujus Index $3 - 2m$, ac proinde ſemper inter duplicatam & triplicatam rationem diſtantiæ reciproce.

§ VI. Æquatio-Hyperbolæ æquilateræ ad centrum eſt $\dot{s} : \dot{y} :: r^2 : a^2$, ex qua deducitur methodo directa Series hujuſmodi,

1.	$\dot{s} : \dot{y} :: r^2 : a^2$
2.	$\dot{s} : \dot{y} :: a^2 : r^2$
3.	$\dot{s} : \dot{y} :: a^{\frac{2}{3}} : r^{\frac{2}{3}}$
4.	$\dot{s} : \dot{y} :: a^{\frac{2}{5}} : r^{\frac{2}{5}}$
5.	$\dot{s} : \dot{y} :: a^{\frac{2}{2n-1}} : r^{\frac{2}{2n-1}}$

Ex his Curvæ, quarum Indicum denominatores ſunt in progreſſione $-1, 3, 7, 11$, &c. exhiberi poſſunt in rectis & arcubus Hyperbolicis; reliquæ verò in rectis & arcubus Curvæ, cujus æquatio ad axem SB (ſi x ſit abſciſſa, y verò Ordinata) eſt $\overline{xx+yy}|^2 = a^2x^2 - a^2y^2$, quæque conſtruitur (*vid. Fig.* III.) biſecando angulum BSL & ſumendo

ſumendo S N mediam proportionalem inter S B & S L.

Curvæ quæ ex Hyperbola methodo inverſa conſtrui poſſunt progrediuntur in hac Serie,

Hyperbolæ 1. $\dot{s} : \dot{y} :: r^2 : a^2$

2. $\dot{s} : \dot{y} :: r^{\frac{2}{3}} : a^{\frac{2}{3}}$

3. $\dot{s} : \dot{y} :: r^{\frac{2}{5}} : a^{\frac{2}{5}}$, &c.

Ubi Curvæ quarum Indicum denominatores ſunt in progreſſione 1, 5, 9, 13, &c. exprimi poſſunt in rectis & arcubus Hyperbolicis; reliquæ verò in rectis & arcubus Curvæ modo explicatæ.

Si aliæ Curvæ deſiderentur quæ alias exhiberent Series, id facillime fieri poteſt ope vel Circuli vel Rectæ: quippe ex earum aliqua omnes, in quibus $\dot{s} : \dot{y} :: \dot{a}^n : r^n$, conſtrui poſſunt, ſumendo, ſi ope Circuli Problema ſit ſolvendum, B S R ad B S L ut 1 ad n,

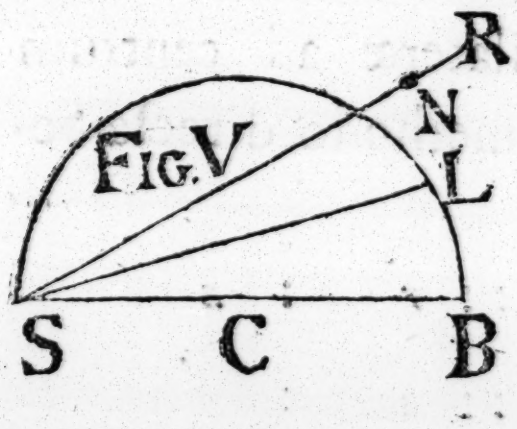


Fig. V

& S N in ipſa S R $= a^{\frac{n-1}{n}} \times S L^{\frac{1}{m}}$; quippe Curvæ per omnia puncta N ductæ æquatio erit $\dot{s} : \dot{y} :: \dot{a}^n : r^n$. Similiter ope Rectæ conſtrui poſſunt quarum æquatio eſt $\dot{s} : \dot{y} :: r^n : a^n$.

Duas exhibuimus Series infinitas Curvarum rectis commenſurabilium; aliam arcubus circularibus, aliam Parabolicis, aliam Hyperbolicis una cum rectis menſurabiles demonſtravimus: eæ vero ad rectarum menſuram arte ſola infinita reduci poſſe videntur, ſicut æquatione ſola infinita in rectis exprimuntur.

Hæc Cl. Author brevitati ſtudens paucis tradit, illum autem plenius rem pro dignitate ejus illuſtraturum ſperamus.

IV. *Remarks*

IV. *Remarks on a Fragment of an old* Roman *Inſcription lately found in the North of* England, *and tranſcribed by the Curious and Learned Dr.* James Jurin, *M. D. and* Reg. Soc. S.

OUR worthy Member, Dr. *Jurin*, having reſided for ſome time at *Newcaſtle* upon *Tine*, had the Curioſity to travel the Country between that and *Carliſle*, in order to obſerve what might occur worth notice in the Remains of the Ruins of the famous *Picts-Wall*, built by the *Romans* to ſecure themſelves, againſt the Incurſions of the Natives of that part of *Britain* they cared not to conquer. In this Perambulation, beſides many other valuable Obſervations which in time he may be prevail'd with to beſtow on the Publick, Dr. *Jurin* ſaw and tranſcrib'd no leſs than Twenty *Roman Inſcriptions*, ſome of which we had formerly receiv'd from others, but many of them wholly new; among them the following, which, tho' broken and in great part illegible, ſuffices to fix the Name of one of the Ancient Nations of *Britain*, that has hitherto been greatly miſcall'd. 'Tis thus,

CIVITATE CAT
VVILLAVA'
ORVM· L· OL S
C D I O'

and is to be ſeen on the *Wall*, about two Miles Weſt from *Lenercroſs-Abby*, near the Confines of our two Northermoſt Counties.

Here

Here 'tis obſervable, that the laſt A of the ſecond Line has a Mark that follows it, not unlike to the laſt Stroak of an *N*; and if inſtead of A′ we put *N*, we ſhall read it CIVITATE CATVVILLAVNORVM, which we cannot doubt to have been the true Name of that People which *Dion. Caſſius*, *Lib.* LX. calls Καλετελλανοὶ, and *Ptolomy*, in his Geography, *Lib.* II. *cap.* 3. more falſly, Καλευχλανοὶ; the firſt λ by producing the tranſverſe Stroak having been miſtaken for χ. This Nation appears by *Dion* to have been more potent than their Neighbours the *Dobuni* (whom he calls *Boduni*) and had, according to *Ptolomy*, *Verolamium* for their Capital, which 'tis moſt probable, was the *Caſſivellauni oppidum* of *Cæſar*. So that it ſhould ſeem *Caſſivellaunus* King of theſe *Catuvillauni* when *Cæſar* invaded *Britain*, either gave his Name to his People, or took theirs. But he was no doubt the moſt potent Prince at that time in *Britain*, ſince by common Conſent of the reſt, he was made General of their united Forces, in defence of their Country's Cauſe againſt the *Romans*.

F I N I S.

ERRATUM, N° 355.

Page 770. lin. 22. for *Maii* 31. lege *Martii* 31.

Printed for W. and J. INNYS, Printers to the *Royal Society*, at the *Princes-Arms* at the Weſt-End of St. *Paul's* Church Yard. 1718.

PHILOSOPHICAL TRANSACTIONS.

For the Months *July*, *Auguſt*, and *Septemb.* 1718.

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II. *A Diſcourſe occaſion'd by an Inſcription found, about Three Years ago, at* Langcheſter *in the Biſhoprick of* Durham, *and communicated to the* Royal Society *from Dr.* Hunter *by Dr.* Woodward, *as it is printed in the Philoſophical Tranſactions,* N° 345. *By* Roger Gale, *Eſquire,* R. S. S,

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V. *Accuratarum Obſervationum Aſtronomicarum, anno ſuperiore & currente cum Reg. Societate communicatarum, Sylloge.*

Jos: Banks

I. *Cometæ* Berolini *nuper visi observationes, ut & Eclipseos Solaris* Feb. 19^no^ *mane,* Noribergæ & Berolini *habitæ, è* Novis Litterariis Berolinensibus, *hoc anno primum edi cœptis, desumptæ.*

CL. *Christfridus Kirchius*, motus Corporum cœlestium, ut munere sibi à Societate Scientiarum Regia (*Berolini*) demandato recte fungeretur, sedulò observans, a. d. XV. Kal. Febr. (*Jan* 18. *st. n.*) anni præsentis, vesperi dimidiâ Septimâ, versus Septentriones fortuitò Cometam conspexit. Vicinus erat ad dextram (stellarum) γ & β *Bayeri* in *Ursa minore*, nudoque oculo longè distinctius apparebat, quam β *Ursæ minoris*, licet ea insignis sit Stella secundæ magnitudinis, cum longè pallidior quidem majore tamen diametro, atque satis clara luce maxime circa centrum, conspiceretur. Per Tubum visus lucidam rotundamque referebat nubeculam; Caudæ autem nullum observari potuit vestigium, neque Nucleus dignosci. Motu celerrimo ab hora VII. ad XI. processit, graduſque quatuor cum dimidio absolvit, ut ex observationibus colligitur.

Die 19^no^ & 20^mo^ *Januarii* cœlum nubibus fuit obductum. Die vero 21^mo^ Cometa longè recesserat à loco suo nupero, atque in *Cassiopea* deprehendebatur, ubi cum stellis ε & δ triangulum conficiebat (*an æquicrure?*) scil. Hora 5^h^ 45′ in 17° 34′ ♉, sub latitudine Boreali 49° 54′ hærebat: Deinde 9^h^ 15, in 16° 38′ ♉ sub Lat. Bor. 49° 2′ conspiciebatur. Cæterum multum decreverat, atque à celeritate sua remiserat; præterquam enim quod pallidior quam ante apparebat, stellas etiam quartæ dignitatis magnitudine haud superare nudo oc

lo conſpectus videbatur, inque orbita ſua, quatuor cum dimidia horis, non ultra ſeſquigradum proceſſerat: Tubi autem beneficio diameter ejus 7 min. inveniebatur.

Jan. 23. hora IV. mat. Cometa cum ♂ & φ *Caſſiopeæ* triangulum æquicrurum efficiebat, cum ab utraque 2° 41 ½ abeſſet. Hoc mane duarum horarum ſpatio vix dimidium gradum abſolvit; hora decima veſpertina cum ♂ *Caſſiopeæ* & φ *Perſei* in linea recta cernebatur, atque à priori 3° 38′, à poſteriori 3° 9′ diſtabat. Diameter ejus erat 5 min. nudoque oculo conſpectus ſtellam quintæ magnitudinis referebat.

Die 24^to^ *Jan.* hora VI. mat. nondum attigerat φ *Perſei*, ſed cum υ & *g* ejuſdem Aſteriſmi triangulum æquicrurum ſiſtebat, & ab utraque non planè 3 ½ grad. aberat. Plura ex obſervationibus docebit Aſtrophilos Vir accuratiſſimus, in pleniori quam parat hujus Cometæ Hiſtoria.

Hactenus Nova Litteraria *dicta* pag. 43, 44. *ubi deſiderantur Obſervationes Diei* 18^væ^, *cum Cometa velociſſime motus terræ proximus erat, unde certius de Via ejus tam vera quam apparente judicium ferre poſſemus. Manifeſtum autem eſt eum Polo Æquatoris Boreo viciniſſimum die* Januarii 19^no^ *tranſiiſſe. Quod ſi cui libeat has Obſervationes ad examen revocare, calculoque accurato ſubjicere; in illius gratiam, loca Stellarum fixarum, quarum hic fit mentio, ex Catalogo* Britannico *excerpta, ſubnectuntur: Unde etiam patebit nonnulla in hac motus Cometæ deſcriptione haud ritè ſe habere; quæ tamen à Cl.* Kirchio *corrigi, in pleniori ejus quam promiſit hiſtoria, ſpes eſt.*

Stellarum

Stellarum fixarum Loca ineunte Anno 1718.

BAYERO		Long. °	'	"	Lat. Bor. °	'	"
Urſæ minoris	β	☊ 9	18	0	72	58	10
	γ	☊ 17	35	15	75	13	15
Caſſiopeæ	δ	♉ 14	00	35	46	23	25
	ε	♉ 20	50	8	47	31	50
	φ	♉ 11	36	35	45	4	5
Perſei	υ	♉ 8	32	0	35	23	45
	φ	♉ 10	41	35	36	49	15
	ζ	♉ 12	15	20	36	18	37

Ex iiſdem Novis *etiam obtinuimus duplicem obſervationem Eclipſeos exiguæ Solaris, currentis anni* Feb. 19^no^ ſt. vet. *mane celebratæ; alteram* Noribergæ *à Cl. D.* Wurtzelbau, *alteram à præfato D.* Kirchio Berolini *habitam.*

Noribergæ *autem Sol ortus eſt aliquantulum deficiens in limbo ſuperiore, qui quidem defectus ad tres plene digitos accrevit; Deſiit Eclipſis* 8^h $8'$ $48''$ *circa* 60 *grad. à Vertice Solis ad Siniſtras.* Berolini *vero Sol ſtatim ab ortu cœpit deficere, Hora* ſcil. 6 $49'$ *vel* $49'\frac{1}{2}$. *Circa medium Eclipſeos, nempe* 7^h $35'$, *erant Partes lucidæ in Sole reſiduæ* $24'$ $40''$, *unde digiti obſcurati* $2^{dig.}$ $50'$. *Finis autem incidit in* 8^h $28'$ $10''$. *Qui plura de his cupit, adeat* Nova *ipſa* Berolini *edita.*

II. *A Diſcourſe occaſion'd by an Inſcription found, about Three Years ago, at* Langcheſter *in the Biſhoprick of* Durham, *and communicated to the* Royal Society *from Dr.* Hunter *by Dr.* Woodward, *as it is printed in the Philoſophical Tranſactions,* N° 354. *By* Roger Gale, *Eſq*; R. S. S.

DR. *Hunter,* who communicated this Inſcription, having only given us his Conjectures as to the firſt fortifying the Place where it was found, and the Time of its Repair after it had been deſtroy'd, but ſaid nothing relating to the Explanation of the Inſcription itſelf, tho' extremely curious; it will not, I hope, be taken amiſs, if I offer ſome Thoughts that occurr'd to me at firſt ſight of it, and afterwards induced me to put together what follows upon that Subject. I ſhall not in the leaſt diſpute or call in queſtion the Time of its Foundation, as fix'd by the Doctor, but begin with the Place where it was diſcover'd, namely *Langcheſter* or *Lancaſter,* in the Biſhoprick of *Durham,* which I am, [1] with him, fully perſuaded was the *Longovicus,* where the *Notitia Imperii* places the [2] *Numerus Longovicariorum.*

This place is ſeated upon a great Military Way, about 12 Miles diſtance from *Bincheſter,* and 7 from *Ebcheſter,* the one the *Vinovia,* and the other the *Vindomora* of *Antoninus,* as the Correſpondence of the Numbers may evince; *Bincheſter* being 19 *Roman* Miles from *Ebcheſter,* as that is 9 from *Corbridge,* the exact Numbers the *Itinerary* gives us between *Vinovia, Vindomora,* and *Corſtopitum.* What is very ſtrange is, that the *Itinerary,* which muſt go upon the great Road directly thro' this

[1] *Philoſoph. Tranſ.* N° 266. p. 657. [2] *Not. Imp.* fol. 176.

Town of *Longovicus* betwixt *Vindomora* and *Vinovia*, takes not the leaſt Notice of it, but meaſures the Way at the whole Length and Number of Miles from the firſt to the latter of thoſe Stations. If *Longovicus* was founded, as Dr. *Hunter* ſuppoſes [3], ſo early as the Time of *Julius Agricola*, and if that *Itinerary* was compoſed by any of the Emperors that bore the Name of *Antoninus*, this Station might have been deſtroy'd or deſerted during the Wars with the *Britains*, and not being repair'd till the Reign of *Gordian* III. was paſs'd over by the Author of the Itinerary, as a Camp not then in being, or of no uſe to the *Roman* Armies; and this would be no weak Argument for the Antiquity of that Work: And perhaps ſome Parts of it may have been deſcribed as early as the Reigns of thoſe Emperors, or earlier, and ſuch Names of more modern Places as are found in it, may have been afterwards added as Occaſion requir'd. As a farther Confirmation of this Conjecture, I beg leave to obſerve, that this Place, after it was repair'd by *Gordian*, ſubſiſted even to the Ruine of the *Roman* Empire in *Britain*, as is evident by the Mention of it in the *Notitia Imperii*; ſo that had this [4] Journey which carries us from *Vindomora* to *Vinovia* been compos'd after the Reign of *Gordian*, it would be very hard to account for the Omiſſion of this remarkable Station and Town, as it appears to have been from this, and many other Inſcriptions found there.

Having this Opportunity of doing it, I am unwilling to let it ſlip without rectifying a Miſtake in the *Eſſay towards the Recovery of the* Roman *Highways thro'* Britain, printed *in the 6th Volume of Mr.* Hearne's Itinerary *of* Leland [5], which having brought the *Ermingſtreet* (not the *Watlingſtreet*, as Dr *Hunter* and the Country call it) a little beyond *Cattarick* in *Yorkſhire*, divides it there into two Branches,

[3] *Ph. Tranſ.* N° 354. p. 702. [4] *Iter.* I. *à Limite Prætor. uſq;* [5] P. 111, 114.

Branches, tracing one of them to *Tinmouth*, and the other to *Carliſle*, but omits the main Stemm of it, that runs almoſt directly Northward to *Piercebridge*, ſo to *Denton*, *Houghton*, *Binchester*, *Langchester*, *Ebchester*, *Corbridge*, and through the Heart of *Northumberland* into *Scotland*, about a Mile and a half to the Weſt of *Berwick*. It is in ſeveral places very intire and fair, eſpecially between *Corbridge* and *Binchester*, the Ridge of it there being for the moſt part two Yards in Height above the Level of the Soil, no leſs than Eight Yards broad, and all pav'd with Stones, that are as even as if new laid: as I am inform'd by the ingenious Mr. *Warburton*, who has often view'd it, and to whom we are obliged for the moſt accurate and uſeful Map of the County of *Northumberland* that was ever yet publiſh'd. This Digreſſion, if it may be ſo call'd, I hope will be excus'd, ſince it not only ſets right an Error, but acquaints you with a noble *Roman* Way, ſcarcely yet known or obſerv'd by any body.

Having fix'd the Seat of this *Longovicus*, where the Inſcription was found, let us conſider next what ſort of a Place it was; and upon due Enquiry it will appear to have been one of the moſt ancient and eminent Stations the *Romans* were poſſeſs'd of in theſe Parts. As to its Antiquity, Dr. *Hunter* has made it probable, that we ought to look for it as high as *Julius Agricola*'s commanding under *Domitian*, in this Iſland: As to its Eminency, the Inſcription that came laſt from him to the Society, as well as ſeveral others found there, is an undeniable Evidence of its being a Place of great Conſideration; but nothing can put that more out of Diſpute than the firſt which was ſome Years ago tranſmitted by the ſame Hand [6], which therefore I beg leave to inſert here with that which came laſt from him, and the rather becauſe little or nothing has ever been ſaid upon it, and that they will give great Light one to the other.

[6] *Phil. Tranſ.* N° 266.

IMP·CAES·M·NT·GoRDIA
NvS·P·F·AVG·BALNEvM·CvM
BASILICA·A LO·INsTRVXIT
PRENLCIANvM·LEG·AVG
PR·PR·CvRANTE·M·AVR
QVIRINO PRECoHILGR

II.

IMP·CÆSAR·M·ANToNIVS
GoRDIANVS·P·F·AVG·
PRINCIPIA·ET·ARMAMEN
TARIA·CONLAPSA·REsTITv
IT·PER·MAECILVM·FVSCvM·LEG
AVG·PR·PR·CvRANTE·M·AVR
QVIRINO·PR·CoH·I·L·GOR·

The Stone whereon the firſt is cut has been broke in two, whereby ſome of the Letters are defaced, however, it may be very well read as follows; the Letters PRE in the fourth Line I take to be a Miſtake of the Workman, having ſeen ſeveral Copies, where they are ſo tranſcribed; that they ſhould be PER is evident from the fifth Line of the ſecond Inſcription.

I. *Imperator Cæſar Marcus Antonius Gordianus Pius Felix Auguſtus Balneum cum Baſilica à ſolo inſtruxit Per Cneium Lucilianum Legatum Auguſtalem Proprætorem Curante Marco Aurelio Quirino Præfecto cohortis primæ Longovicariorum*; or rather, *Legionis Gordianæ.*

The ſecond can be read only after the following manner.

II. *Imperator Cæſar Marcus Antonius Gordianus Pius Felix Auguſtus Principia & Armamentaria Conlapſa reſtituit Per Mæcilium Fuſcum Legatum Auguſtalem Proprætorem curante Marco Aurelio Quirino Præfecto Cohortis primæ Legionis Gordianæ.*

From theſe Two Inſcriptions compar'd together, it will be apparent that they were not only erected under the ſame Emperor, but by the Care of the very ſame Perſon *Aurelius Quirinus*, tho' not in the ſame Year. The Emperor can be no other than *Gordianus* the youngeſt, or third of that Name: the two former having been ſlain ſo very ſoon after they had aſſumed the Purple, that it is improbable they ſhould have given

ven any Orders or Commands for the erecting of new, and repairing of antient Buildings, in ſo remote a Province as *Britain* was from *Africa*, where they were murder d after a ſhort joint Reign of ſcarce ſeven Weeks.

[1] Dr. *Hunter* tells us, that that which was firſt diſcover'd (and which I ſhall therefore always diſtinguiſh by the Name of the firſt) was dug up about a Hundred Yards Eaſt from a great Square, which had been fortified with a thick, ſtrong Wall, faced with hewen Stone, within which, and without, eſpecially towards the Eaſt, are nothing but ruinous Heaps of Stone, and thinks the Lodging of the Gariſon only to have been included within thoſe Walls. His Conjecture is very much confirmed by the [2] Account he gives us of the finding the laſt Inſcription within that ſquare Incloſure; ſo that there ſeems to have been at this *Longovicus* a large Town, and one of thoſe Camps call'd *Caſtra ſtativa*, where the Legions lay in Quarters during the time of Peace and Quiet.

The firſt Inſcription tells us, that the Emperor *Gordian* built the *Balneum* and *Baſilica* from the Ground, *à Solo*; whereas, by the ſecond he appears to have been only the Repairer of the *Principia* and *Armamentaria*. Perhaps therefore here might be no Town, till the *Romans* thought fit to repair their old deſerted Camp at this Place, and then the Emperor might alſo build the *Bath* and Palace for the Reſidence of the *Proprætor*, when in theſe Parts of *Britain*; the Word *Baſilica* importing both a Palace, and an Edifice for hearing of Cauſes, and tranſacting all publick Affairs. As this eminent Building was erected by the Emperor's Command, it is an undeniable Argument of the Splendor of this Town, as are the great Heaps of Rubbiſh, and Ruines, where this

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[1] *Phil. Tranſ.* N° 266. p. 658. [2] *Phil. Tranſ.* N° 354.

this Inſcription was found, of its Largeneſs and Extent

The ſecond equally puts the being of the *Caſtrum ſtativum* out of diſpute, when it acquaints us with the Rebuilding of the *Armamentaria* and *Principia* there, that is the *Arcenal* and *Quarters* either of the Legionary Soldier, that were call'd the *Principes*, or the place where the Eagles and other military Enſigns were kept. It is probable they did not belong to one particular Legion, but to ſeveral, as they had occaſion to be employ'd here; [1] tho the *Legio ſexta Victrix* ſeems to have the beſt Title to them, as being conſtantly quarter'd in the North; whereas, the [2] *Legio Secunda*, and [3] *Viceſima* were generally garriſon'd, the firſt at *Caerleon* in *Wales* and *Richburrow* in *Kent*, and the other at and about *Cheſter*; ſo that the [4] Monuments they have left in the North were erected by them when the Wars, and other Works, as particularly the *Wall* carry'd croſs the Iſland, call'd them thither; which being finiſh'd, they returned home to their more Southern Quarters, and continu'd in them till commanded Abroad upon new Services. I will not pretend to determin when theſe *Armamentaria* and *Principia* firſt fell to ruin; perhaps it might be when *Hadrian*, *Lollius Urbicus* and *Severus* had carried their Conqueſts farther into the Enemy's Country, and having built thoſe famous *Wall*, the Relicks of which we ſtill ſee in the Shire of *Sterling* in *Scotland*, and in *Northumberland* and *Cumberland* in *England*, that this Camp might be thought uſeleſs, the *Roman* Forces being drawn nearer to, and quarter'd upon the Frontiers; and ſo this Fortreſs abandoned and ſuffer'd to fall into decay, as the Word *conlapſa* implies: and not that it was

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[1] *Ptol. Leg.* VI. *Niceph. Ebor locat.* [2] *Anton. Itin.* XII. *Not. Imp.* p. 161. [3] *Anton. Itin.* II. [4] *Camd.* p. 835, 920. *Phil. Tranſ.* N° 269.

was deſtroy'd by any Fire, War, or other Enemy than Age and Neglect.

Tho' the Word *conlapſa* is wrote here with an *N*, there can be no doubt but the Pronunciation of it was as we uſually find it ſpelt, *collapſa*; a certain Argument of the Letter *N*'s being ſilent in the middle of a Word, before two Conſonants, eſpecially NS, and NT, when the T was pronounced like an S. To omit what [1] *Quintilian* ſays to this purpoſe, it is confirm'd by the Abſence of that Letter in numberleſs Inſcriptions in *Gruter*, *Reinesius*, *&c.* and no wonder, ſince the Workmen in thoſe Days, as well as ours, uſually wrote as they ſpoke their Words. I ſhall not trouble you with Quotations of any of them to this end, but as a Proof of what I ſay, only aſſure you from ocular Inſpection and a moſt accurate Examination, that there is no tranſverſe Line over the Letters I S belonging to the Word FABRICIESIS in the Inſcription of IVL. VITALI at *Bath*, whatever has been affirm'd to the contrary, but that the Letter N is totaly omitted there. You will alſo pardon my Endeavours, before I leave this Subject, to explain a ſhort Inſcription belonging to ſome of our Countrymen, tho' found at *Amerbach* in *Germany* [2], ſince it will be a new Proof of the foregoing Aſſertion.

III.

NYMPHIS ⊕	*Nymphis.*
N⊕BRITTON	*Numerus Brittonum*
TRIPVTIEN	*Triputienſis*, or ——*enus*
SVB CVRA ⊕	*Sub cura*
M⊕ VLPI	*Marci Ulpii*
MALCHI⊕	*Malchi*
⊃LEG XXII	*Centurionis Leg.* 22.
PR⊕P⊕F⊕	*Primigeniæ, Piæ, Felicis.*

There

[1] *Quintil. Inſtit.* Lib. I. c. 7.

[2] *Gruter.* p. XCIII.

There is no Difficulty but in the Word TRIPVTIEN, and that will preſently vaniſh if you inſert the Letter N, and read it TRIPVNT, *i. e. Tripontienus* or *Tripontienſis*, the Mutation of the O and V being ſo frequent, that no body is ignorant of it. This will bring you to *Tripontium* [1] or *Dowbridge* in *Northamptonſhire*; tho' that excellent Antiquary Dr. *Battely* [2], in his *Antiquitates Rutupinæ*, would read it RIPVTIEN, and fix'd the Place whence this *Numerus* took its Appellation at *Richburrow* in *Kent*.

But to return where we left the Camp at *Longovicus*, it will be as difficult to aſſign a Reaſon for its being repaired, as it was for its being deſerted; unleſs that the *Proprætors* might judge it adviſable about the Time of *Gordian* III. to fix their Reſidence there, and conſequently refortify the old Camp for their State and Security. And that it was not refortify'd upon any ſudden Emergency, but for Time and Duration, is evident both from the ſtrong Stone-Works that encompaſs'd it, and a Body of Forces lying here, even at the Expiration of the *Roman* Empire and Authority in this Iſland, which from its Continuance in the ſame Station, had got the Name of the *Longovicarii* [3].

The Perſon that under the Emperor gave Direction for theſe Repairs, was *Mæcilius Fuſcus*: As *Mæcilius* is a Diminutive of *Mæcius*, it is not unlikely that he was the Son of *Mæcius Fuſcus*, who was Conſul with *Turpilius Dexter*, *A. D.* 225. in the Reign of *Alexander Severus*: By this Inſcription it appears that this *Mæcilius* was the Emperor *Gordian*'s *Lieutenant* here and *Proprætor*; For tho' in *Phil. Tranſact.* N° 354, by the Inadvertency of the *Engraver*, we read only PR. inſtead of PR. PR; it is right in the Original, and in the Tran-

[1] *Antonin. Itin.* VI. [2] p. 21. [3] *Notit. Imp.* fol. 176. b.

ſcript ſent up by Dr. *Hunter*, and accordingly in *pag.* 826. the Fault is amended. And as the Name of *Fuſcus* ſtands in the ſame Place in the ſecond as that of *Lucilianus* does in the firſt, and with the ſame Adjuncts both before and after, we may fairly conclude he was either his Predeceſſor or Succeſſor, but which, it is impoſſible to determin.

And here, perhaps, it may not be amiſs to remark, we never meet with a *Legatus Auguſtalis* in any Inſcription in this Iſland, without the joint Title of *Proprætor*; and [1] *Tacitus* himſelf either makes them the ſame Office, or at leaſt unites them in the ſame Perſon, when he tells us, *In Britanniâ P. Oſtorium Proprætorem turbidæ res excepere*; and having preſently after related the manner of the Fight with the *Iceni*, ſtiles him *Legatus*, *Quâ pugnâ filius Legati*, *M. Oſtorius*, *ſervati civis decus meruit* [2]; and a little after he gives both the ſame Titles to *A. Didius* the Succeſſor of *Oſtorius*.

We are indebted therefore to theſe two Monuments, not only for the Account they have preſerved of the *Roman* Arms and Magnificence at *Longovicus*, but for the indiſputable Records of the Names of two *Legates* and *Proprætors* of *Britain*, that would otherwiſe have been buried in Oblivion, *viz.* *Cneius Lucilianus* and *Macilius Fuſcus*: For from *Virius Lupus* (who was *Propretor* and *Legate* here about the Year 208, under *Severus*, and juſt before that Emperor's coming into this Iſland repaired a Bath burnt down at *Lavatræ*, or *Bowes* [3], in *Yorkſhire*) we have no where extant the Name of one of thoſe Officers, till we come to *Nonnius Philippus*, whom I take to have ſucceeded the laſt of theſe; the [4] Stone which was found at *Old Carliſle* in

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[1] *Tac. Lib.* Ann. XII. c. 32. [2] Ibid. c. 39. [3] *Camd.* p. 762. Edit. 1695. [4] *Camd. Britan.* p. 830.

in *Cumberland*, and has preſerv'd his Memory, ſetting forth that he was *Legate* and *Proprætor* when *Atticus* and *Prætextatus* were *Conſuls*, which was *A. D.* 242. the very Year that our *Gordian* went upon his *Perſian* Expedition, from which he never return'd. And as that Emperor left *Nonnius Philippus* in that Poſt, when he march'd into the Eaſt, where he was murder'd about two Years after, it is highly probable that he was the laſt *Proprætor* of his appointing, and conſequently, that *Mæcilius Fuſcus* was his Predeceſſor, and the Repairs begun at *Longovicus* before the Year 243 [1]. I would not have troubled you with this Inſcription, but that it is faultily tranſcribed in *Camden*, and that I ſhall have occaſion by-and-by to refer to it again, upon a material Point, which therefore I hope will plead my Pardon.

IV.

I. O. M.

PRO. SALVTE. IMPERATORIS
M. ANTONI. GORDIANI. P. F.
INVICTI. AVG. ET. SABINAE. FvR
IE. TRANQVILE. CONIVGI. EIVS. TO
TAQVE. DOMV. DIVIN. EORVM. A
LA. AVG. GORDIA. OB VIRTVTEM
APPELLAT. POSVIT. CVI. PRAEEST
AIMILIVS. CRISPINVS. PREF
EQQ NATVS. IN. PRO. AFRICA DE
TVSDRO. SVB. CVR. NONNII. PHI
LIPPI. LEG. AVG. PROPRETO.
ATTICO. ET. PRETEXTATO COSS.

[1] *Philoſ. Tranſact.* N° 354. p. 702.

The Perſon who had the Care of theſe Repairs both in Town and Camp, was *Marcus Aurelius Quirinus, Præfect* or *Commander* of a Company of Foot; another Argument for the Proprætors *Lucilianus* and *Fuſcus* ſucceeding immediately one the other, he ſerving in the ſame Poſt under both. I muſt obſerve however, that altho' the two firſt Inſcriptions have been cut very near the ſame time, and by the ſame Hand, as appears by the Form of the Letters, and Manner of the Abbreviations in each of them, yet the Office that this *Quirinus* bore is ſomething differently expreſs'd in the firſt from what it is in the ſecond, if they have been accurately tranſcribed; the Firſt ſhewing, after *QVIRINO* the Letters PRE. COH. I. LG. R, which, before I had ſeen the Latter, I was induced to read *Præfecto Cohortis primæ Longovicariorum*, the [1] *Notitia Imperii* placing the *Præfectus numeri Longovicariorum Longovico*. That *Numerus* and *Cohors* were the ſame thing, [2] *Pancirollus*, in his Notes upon that Book, quotes St. *Chryſoſtome* to prove, and ſome others, *Cohors erat qui vocatur Numerus*; but I rather take it to be an indefinite Number of Men, which might comprize ſeveral Companies, independent of any Legion. [3] *Vegetius*, ſpeaking of the *Legati Imperatoris*, ſays, *in quorum locum nunc illuſtres viros conſtat Magiſtros Militum ſubſtitutos, à quibus non tantum binæ Legiones, ſed plures Numeri gubernantur*; by which it is plain, the *Numeri* were no *Legionary Cohorts*. Neither was the Name ſo modern as from the *Notitia Imperii* and *Chryſoſtome* it might appear to be; for we meet with a [4] *Numerus Britonum* upon an Altar found in *Tranſylvania*, dedicated to the *Nymphs*, when the Emperor *Commodus* and *Glabrio* were Conſuls, *A. D.* 186. And [5] another *Numerus* upon an Altar erected to *Hercules* for the Proſperity of

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[1] fol. 166. b [2] fol. 161, b [3] Lib. II. c. 9. [4] *Gruter.* p. 94. 2. [5] id. 469.

of *Septimius Severus*, when *Lateranus* and *Rufinus* were Conſuls, *A. D.* 197. But after I had review'd the Letters at the end of the ſecond Inſcription, which are plainly tranſcribed PR. CoH. I. L. GOR. I could read them no otherwiſe than *Præfecto Cohortis primæ Legionis Gordianæ*. *Gordian* III. was ſo beloved of the Soldiery, that ſeveral Legions complimented him by honouring themſelves with his Name, as the [1] *Legio tertia Italica*, which took the Addition of *Gordiana*; and the [2] *Legio decima gemina*, and [3] *Decima tertia gemina* did both give themſelves the ſame Appellation. But which of the Legions quartered in this Iſland ſo ſtiled itſelf is not determin'd by this Inſcription or any other that I know of. However, as the *Legio ſexta Victrix* was all along quarter'd in the Northern parts of this Kingdom (as I obſerved before) where theſe Inſcriptions were erected, I make no doubt but it was that which call'd it ſelf *Gordiana*, tho' the numeral Diſtinction of VI is omitted, only perhaps for want of Room on the Stone. We find by ſeveral Inſcriptions in *Camden*, that there was an *Ala* in thoſe Parts which prided it ſelf upon its Valour, and was therefore call'd the *Ala Auguſta*; of the many Memorials it has left us of its Title, I ſhall only mention [4] one found at *Old Carliſle*, and which is the ancienteſt of them all, by any certainty of Date.

I. O. M.
AL. AVG. OB
...RTVT. APPEL. CVI
PRAEEST. TIB. CL. TIB. F. P.
LING..N. IVSTINVS.
PRAEF. FVSCIANO.
II. SILANO. II. COS.

that

[1] *Velſ. Monum. Auguſtæ Vindel.* p. 431. [2] *Grut.* p. 80. [3] *Gruter.* p. 433. 1. [4] *Camd.* p. 827.

that is,

Jovi Optimo Maximo, Ala Auguſta ob Virtutem appellata, cui præeſt Tiberius Claudius Tiberii filius, provinciâ Lingonenſi, Juſtinus præfectus, Fuſciano ſecundò, Silano ſecundò Conſulibus.

This Altar was dedicated when *Fuſcianus* and *Silanus* were the ſecond time *Conſuls*, that is, in the Year 188. under the Reign of *Commodus*, and Fifty Years before our *Gordian* came to the Empire. At the ſame place was alſo diſcover'd the Fourth Inſcription by me quoted, where we find this ſame *Ala Auguſta* ſtiling itſelf alſo *Gordiana*; from whence I think it is not a little probable that the Legion to which this Wing appertain'd was the *Legio Gordiana* mention'd in the Inſcriptions found at *Langcheſter*; and that Legion to have been the *Legio ſexta Victrix*, from the long Continuation of this *Ala Auguſta* in theſe Northern parts of the Nation, the conſtant Quarters of that Legion.

July 10.
1718.

III. *A Letter of that curious* Naturaliſt *Mr.* Henry Barham, R. S. S. *to the Publiſher, giving a Relation of a fiery* Meteor *ſeen by him, in* Jamaica, *to ſtrike into the Earth; with Remarks on the Weather, Earthquakes, &c. of that Iſland.*

SIR,

ACcording to your Requeſt I have collected what I can remember, relating to a *Meteor* I ſaw in *Jamaica* about the Year 1700, as I was riding one Morning from my Habitation ſituated about Three Miles North-Weſt from *St. Jago de la Vega*: I ſaw a Ball of Fire, appearing to me of the Bigneſs of a Bomb, ſwiftly falling down with a great Blaze. As I thought it fell into the Town; but when I came within a quarter of a Mile of the Town, I ſaw many People gather'd together a little to the Southward in the *Savanna*, to whom I rode up, where they were admiring at the Ground's being ſtrangely broke and plough'd in by a Ball of Fire, which, as they ſaid, fell down there. I obſerved there were many Holes in the Ground, one in the middle of the Bigneſs of a Man's Skull, and five or ſix ſmaller Holes round about it, of the Bigneſs of a Man's Fiſt, and ſo deep (eſpecially the biggeſt) as not to be fathom'd by what long Switches or Sticks they had at hand. I did not hear that any was ſo curious as to make any farther Search: It was obſerv'd, that the green Graſs was perfectly burnt near the Holes, and a ſtrong Smell of Sulphur remain'd thereabouts for a good while after.

Note that we had a terrible rainy Night before, with much Lightning and great Thunder-Claps, which we

have

have very frequently in *Jamaica*, often killing Cattle in the Fields. Mr. *Henry Lord*, who lives at *Dry-River*, had two Sons (big Boys) ſtruck dead with Lightning, in 1716, without any Wounds or Appearance of Hurt found about them. And as theſe Claps are much louder and ſtronger than any I ever heard in *Europe*, ſo are our Showers of Rain, pouring down in a moſt violent manner. We have Lightning all the Year round, but our great Rains are in the Months of *May*, *Auguſt*, and *October*. I knew *May* for two or three Years without Rain, which was lookt upon as a great Wonder; and we paid dear for it in our *Indigo*; for a Catterpillar appear'd and wove a fine Silk about the *Indigo-Plant*, and deſtroy'd it all, hurting nothing elſe. *May*-Rains us'd to deſtroy theſe Worms. *Auguſt* and *October* never go out without a Flood, we having then univerſal Rains, all over the Iſland, coming from the Sea: For we have often Rains in the Mountains from the Clouds lodging there, when we have none in the Lowlands.

Our Iſland is full of Mines, and, if ſearch'd into, I queſtion not but very rich. We are very ſubject to Earthquakes, ſeveral happening every Year, eſpecially after great Rains, which fill up all our great Cracks in the Surface of the Earth: For in a very dry Time, we have them ſo very large, deep, and gaping ſo open and wide, that it is dangerous to ride over ſome Parts of the *Savannaes*, for fear a Horſe ſhould get his Legs into them. Our Earthquakes make a Noiſe or Rumbling in the Earth, before we feel the Shake; and ſeem to run ſwiftly to the Weſtward. This is all I can inform you of this kind at preſent, relating to the Iſland of *Jamaica*, being ever ready to ſhew how much I am, *&c.*

Decemb. 19.
1717.

Henry Barham.

IV. *An Attempt to prove the Antiquity of the* Venereal Diſeaſe, *long before the Diſcovery of the* Weſt-Indies; *in a Letter from Mr.* William Beckett, *Surgeon, to Dr.* James Douglaſs, *M.D. and* R. Soc. Soc. *and by him communicated to the* Royal Society.

SIR,

THE Undertaking I am at preſent engaged in, has unavoidably obliged me to conſult, among others, a great Number of ancient Phyſical and Chyrurgical Books, written by my own Countrymen: From theſe I took the Hint, that the *Venereal Diſeaſe* was known among us, much earlier than the *Æra*, which has been generally aſſign'd for its Riſe by modern Authors; for it's believed it was not known, at leaſt in *Europe*, till about the Year 1494. Notwithſtanding which, I determin, in the following Papers, to make it evident, it was frequent among us ſome Hundreds of Years before that Date. I could mention ſeveral Phyſicians and Surgeons of Eminence, who have been of the ſame Sentiments with me, particularly, the Learned Dr. *Charles Patin*, who has written a curious Diſſertation to prove the Antiquity of this Diſeaſe, which is ſufficient to excuſe me from the Imputation of having ſtarted a Novelty, or being at the trouble of quoting ancient Authorities before taken notice of, from the moſt ancient Writers of Medicine; as the Great *Hippocrates*, *Galen*, *Avicen*, *Celſus*, &c. and even the *Holy Scriptures*. I ſhall in theſe and ſome following Papers, lay aſide all thoſe foreign Aids and Aſ-

 ſiſtances,

ſiſtances, and trace out the Symptoms of the Diſeaſe, as they naturally ariſe, from the firſt Infection to the laſt deſtructive Period, and ſhew that, by ſearching into our own Antiquities, we may be furniſhed with Inſtances of the Frequency of the Diſtemper among us, in all its reſpective Stages, before ever our Modern Authors dream it had its Appearance in *Europe*.

I ſhall begin with the firſt Degree of this Diſeaſe, and prove from authentick Evidences, it was anciently call'd the **Brenning** or **Burning**; and that this Word has been ſucceſſively continu'd for many Hundreds of Years, to ſignify the ſame Diſeaſe we now call a *Clap*; and that it was not diſcontinu'd till that Appellation firſt began to have its Riſe. The moſt likely Method to accompliſh my Deſign, will be firſt to examine thoſe Records that relate to the *Stews*, which were by Authority allowed to be kept on the *Bank-Side* in *Southwark*, under the Juriſdiction of the Bp. of *Wincheſter*, and which were ſuppreſſed the 37*th* of *Hen* VIII. For it's impoſſible but, if there were any ſuch Diſtemper in being at that Time, it muſt be pretty common among thoſe lewd Women who had a Licence for entertaining their Paramours, notwithſtanding any Rules or Orders which might be eſtabliſh'd to prevent its Increaſe: But if we ſhall find that there were Orders eſtabliſh'd to prevent the Spreading of ſuch a Diſeaſe, that Perſons might be ſecure from any contagious Malady after their Entertainment at thoſe Houſes (which were anciently Eighteen in Number, but in the Reign of *Hen.* VII. reduced to Twelve) we may then ſecurely depend upon it, that it was the Frequency of the Diſeaſe that put thoſe that had the Authority, under a neceſſity of making ſuch Rules and Orders. For the ſame Powers that granted a Liberty for keeping open ſuch lewd Houſes, muſt find it their Intereſt to ſecure,

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as much as poſſible, all Perſons from receiving any Injury there; leſt the Frequency of ſuch Misfortunes ſhould deter others from frequenting them, and ſo the original Deſign of their Inſtitution ceaſe; from the entire ſinking of the Revenues. Now I find that, as early as the Year 1162. divers Conſtitutions relating to the Lordſhip of *Wincheſter*, (being alſo confirmed by the King.) were to be kept for ever, according to the old Cuſtoms that had been time out of mind. Among which theſe were ſome, *viz.* No *Stew-holder* to take more for a Woman's Chamber in the Week than 14*d*. Not to keep open his Doors upon Holy Days. No ſingle Woman to be kept againſt her Will, that would leave her Sin. No ſingle Woman to take Money to lie with any Man, except ſhe lie with him all Night till the Morning. No Stew holder to keep any Woman that hath the perilous Infirmity of **Burning**. Theſe and many more Orders were to be ſtrictly obſerved, or the Offenders to be ſeverely puniſhed. Now we are aſſured there is no other Diſeaſe that can be communicated by carnal Converſation with Women, but that which is Venereal, by reaſon that only is contagious; and it's evident the **Burning** was certainly ſo: For, had it been nothing elſe but ſome ſimple Ulceration, Heat, or Inflammation, there would have been no Contagion; and that affecting only the Woman, could not be communicated by any Venereal Congreſs, and ſo not inferr a Neceſſity of her being comprehended under the reſtraining Article. Theſe Orders likewiſe prove the Diſeaſe was much more ancient than the Date above-mentioned; becauſe they were only a Renewal of ſuch as had been before eſtabliſhed time out of mind.

But to confirm this farther, I find that in the Cuſtody of the Biſhop of *Wincheſter*, whoſe Palace was

ſituated on the *Bank ſide*, near the *Stews*, was a Book written upon Vellum, the Title of which runs thus; **Here begynne the Ordinances, Rules, and Cuſtumes, as well for the Salvation of Mannes Lif, as for to aſchewe many Myſchiefs and Inconvenients that dayley be lik there for to fall owte to be rightfully kept, and due Execution of them to be don unto any Perſonne wythin the ſame.** One of the Articles begins thus; *De his qui cuſtodiunt Mulieres habentes Nephandam infirmitatem.* It goes on, *Item*, **That no Stew-holder keep noo Woman wythin his Hous that hath any Syckneſſe of** BRENNING, **but that ſhe be putte out upon the peyne of makeit a fyne unto the Lord of a hundred Shylyngs.** This is taken from the Original Manuſcript which was preſerv'd in the Biſhop's Court, ſuppos'd to be written about the Year 1430. From theſe Orders we may obſerve the Frequency of the Diſtemper at that Time; which, with other Inconveniencies, was **dayley lik there for to fall owte:** and the Greatneſs of the Penalty, as the Value of Money then was, that is laid on it, proves it was no trifling or inſignificant thing.

But the bare Proof of there having been anciently ſuch a Diſeaſe as was called the **Burning**, may be thought to be inſufficient, unleſs we were perfectly aſſured what it was, and how it was in thoſe Times deſcribed: I ſhall therefore do it from an unqueſtionable Authority, which is that of *John Arden*, Eſq; who was one of the Surgeons to our King *Richard* II. and likewiſe to King *Henry* IV. In a curious Manuſcript of his upon Vellum, he defines it to be, a certain inward Heat and Excoriation of the *Urethra*; which Deſcription gives us a perfect Idea of what we now call a *Clap*; for frequent Diſſections of thoſe that laboured under

under that Diſeaſe, have made it evident, that their *Urethra* is excoriated by the Virulency of the Matter they receive from the infected Woman; and this Excoriation or Ulceration is not confined to the *Oſtiola* or Mouths of the *Glandulæ Muſcoſæ*, as has been lately thought, but may equally alike attack any part of the *Urethra* not beyond the reach of the impelled malignant Matter. The Heat before deſcribed, which theſe Perſons are ſenſible of, as well now as formerly, is a Conſequent of the excoriated *Urethra*; for the Salts contained in the Urine muſt neceſſarily prick and irritate the nervous *Fibrillæ*, and excite a Heat in thoſe Parts of the *Urethra* which are diveſted of its natural Membrane; which Heat will always be obſerved to be more or leſs, as the Salts are diluted with a greater or leſs Quantity of Urine; a thing I have often obſerved in Perſons that have laboured under this Infirmity in hot Weather, when the perſpirable Matter being thrown off in greater Quantities, the Salts bear a greater Proportion to the Quantity of Urine, and thereby make its Diſcharge at that Time ſo much the more painful and troubleſom.

Thus we ſee this very early and plain Deſcription of this Diſeaſe among us, to be entirely conformable to the lateſt and moſt exact Anatomical Diſcoveries. Here is no Tone of the *Teſticles* depraved, according to *Trajanus Petronius*; no Exulceration of the *Paraſtatæ*, according to *Rondeletius*; no Ulceration of the *Seminal Veſſels*, according to *Platerus*; no Seat of the Diſeaſe in the *Veſiculæ Seminales* or *Proſtatæ*, according to *Bartholin*; nor in thoſe Parts and the Teſticles at the ſame Time, according to our Countryman *Wharton* and others, who have falſly fixed the Seat of this Diſeaſe, and whoſe Notions, in this reſpect, are now juſtly exploded; but a ſingle and true Deſcription of it, and

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its Situation, about an Hundred and Fifty Years before any of thoſe Gentlemen obliged the World with their learned Labours.

Having, I hope, ſufficiently made it appear, the **Burning** was a Diſeaſe very early among us, and given the Deſcription of it, I ſhall proceed to ſay ſomething of the ancient Method that was made uſe of to cure it. We are not to expect the Meaſures our Predeceſſors, in thoſe early Times, made uſe of, ſhould be calculated for the removing any Malignity in the Maſs of Blood, or other Juices, according to the Practice in Venerial Caſes at this Time; becauſe they looked upon the Diſeaſe to be entirely local, and the Whole of the Cure to depend upon the Removal of the Symptoms: Hence 'twas they recommended ſuch Remedies as were accommodated to the taking off the inward Heat of the Part, and cure the Excoriations or Ulcerations of the *Urethra*. The Proceſs for the accompliſhing of this, I ſhall ſet down from the beforementioned *John Arden*, who wrote about the Year 1380. his Words are as follow, *Contra Incendium. Item contra incendium Virgæ Virilis interius ex calore & excoriatione, fiat talis Syringa* (i. e. *injectio*) *lenitiva. Accipe Lac mulieris maſculum nutrientis, & parum zucarium, Oleum violæ & ptiſanæ, quibus commixtis per Syringam infundatur, & ſi prædictis admiſcueris lac Amigdalarum melior erit medicina.* There is no doubt but this Remedy, being uſed to our Patients at this Time, would infallibly take off the inward Heat of the Part, and cure the Excoriations or Ulcerations of the *Urethra*, by which means what iſſued from thence would be entirely ſtopt; and this was all they expected from their Medicines, foraſmuch as they were entirely unacquainted with the Nature of the Diſtemper; and did not in the leaſt imagine, but if the Symptoms that firſt attack'd the Part were removed, the Patient was entirely cured.

I ſhall now, as a farther Confirmation of what I have advanced, proceed to prove, that by this **Brenning** or **Burning** is meant the Venereal Diſeaſe, by demonſtrating that ſucceeding Hiſtorians, Phyſical and Chirurgical Writers, and others, have all along with us in *England* uſed the very ſame Word to ſignify the Venereal Malady. In an old Manuſcript I have by me, written about the Year 1390. is a Receipt for **Brenning of the Pyntyl, yat men clepe ye Apegalle**; **Galle** being an old Engliſh Word for a running Sore. They who know the *Etymologie* of the Word *Apron*, cannot be ignorant of this. And in another Manuſcript, written about 50 Years after, is a Receipt for **Burning** in that Part by a Woman. *Simon Fiſh*, a zealous Promoter of the Reformation in the Reign of *Hen.* VIII. in his Supplication of Beggars, preſented to the King in 1530. ſays as follows, *Theſe be they* (ſpeaking of the *Romiſh Prieſts*) *that corrupt the whole Generation of Mankind in your Realm, that catch the Pockes of one Woman and bear them to another; that be* **Burnt** *with one Woman and bare it to another; that catch the Lepry of one Woman and bare it unto another.* But to make this Matter ſtill more evident, I am to obſerve, that *Andrew Boord*, a Doctor in Phyſick, and Romiſh Prieſt, in the Reign of *Henry* VIII. in a Book he wrote, entitul'd *The Breviary of Health*, printed in 1546. ſpeaks very particularly of this ſort of Burning; one of his Chapters beginneth thus, **The 19th Chapiter doth ſhew of BURNING of an Harlotte**; where his Notion of communicating the *Burning* is very particular. The ſame Author adds, that if a Man be **Burnt** with an *Harlot*, and do meddle with another Woman within a Day, he ſhall **Burn** the Woman that he ſhall meddle withal; and as an immediate Remedy againſt the **Burning**, he recommends the waſhing the *Pudenda* two or three

three times with White Wine, or elſe with Sack and Water; but if the matter have continued long, to go to an expert Chirurgeon to have Help. In his 82d Chapter, he ſpeaks of two ſorts of Burning, the One by Fire, and the Other by a Woman through carnal Copulation, and referrs the Perſon that is Burnt of a *Harlot* to another Chapter of his for Advice, what to do, yf he get a Doxter or two, ſo called from its Protuberancy or bunching out: For I find about that Time the Word *Bubo* was moſtly made uſe of, to ſignify that ſort of Swelling which uſually happens in peſtilential Diſeaſes.

From hence it appears, the *Burning*, by its Conſequents, was *venerial*, ſince every Day's Experience makes it evident, that the ill Treatment of the firſt Symptoms of the Diſeaſe, either by aſtringent Medicines, or the removing them by cooling and healing the excoriated Parts, will generally be attended with ſuch Swellings in the Groin, which we rarely obſerve to happen from any other Cauſe whatſoever.

I ſhall give a few more Inſtances of this Diſeaſe being call'd the *Burning*, and conclude. In a Manuſcript I have of the Vocation of *John Bale* to the Biſhoprick of *Oſſory* in *Ireland*, written by himſelf, he ſpeaks of Dr. *Hugh Weſton* (who was Dean of *Windſor* in 1556. but deprived by Cardinal *Pole* for Adultery) as follows, " At this Day is lecherous *Weſton*, who is more " practiſed in the Art of Brech Burning than all " the *Whores* of the *Stews*. And again, ſpeaking of the ſame Perſon, he ſays, " He not long ago brent a *Beg-* " *gar* in St. *Botolph*'s Pariſh. The ſame Author ſays of him elſewhere, " He had been ſore Bitten with a *Win-* " *cheſter Gooſe*, and was not yet healed thereof; which was a common Phraſe for the Pox at that Time, becauſe the *Stews* were under the Juriſdiction of the Biſhop of

of *Winchester*. *Mich. Wood*, in his *Epiſtle* before *Steph. Gardiner*'s Oration *de vera Obedientia*, printed at *Rhoan*, 1553. gives another Evidence of the **Burning**. And *William Bullein*, a Phyſician in the Reign of Queen *Eliz.* in a Book he publiſh'd, call'd *The Bulwark of Defence, &c.* printed in 1562. bringing in *Sickneſs* demanding of *Health* what he ſhould do with a Diſeaſe call'd the *French Pockes*, *Health* anſwers, " He would not that a-" ny ſhould fiſhe for this Diſeaſe, or to be bold when " he is bitten to thynke thereby to be helped, but ra-" ther to eſchewe the Cauſe of thys Infyrmity, and " filthy rotten **Burning** of *Harlots*.

I believe, by this time, I have ſufficiently prov'd what I propoſed, that the firſt Degree of the Venereal Diſeaſe was very anciently known among us, under the Title of **Burning**; and that you may loſe no more Time at preſent upon this Subject, I ſhall reſerve my Collections, which ſhew that the Diſeaſe, when it came to be confirmed, was no Novelty here in thoſe early Times, for a further Opportunity, and detain you no longer than to expreſs my Pleaſure in profeſſing my ſelf, *Yours*, &c.

London, Feb. 4. 1717-18.

Will. Beckett.

V. *Accuratarum Obſervationum Aſtronomicarum, anno ſuperiore & currente, cum Reg. Societate communicatarum* **Sylloge.**

INtereſt ſanè Scientiæ ne pereant Obſervata Aſtronomica, debita curâ fidiſque Inſtrumentis ab Artificibus idoneis cælitus deprompta: Hoc enim ſolo fundamento nititur Urania practica. Itaque in his *Tranſ-*

actionibus, per pluſquam quinquaginta annorum curriculum, paſſim ſparguntur hujus generis Notæ. Auſim tamen ſpondere vix ullas unquam reperiri poſſe Obſervationes quæ certitudine eas quas nunc damus vincant, ne dicam quæ pares ſint, utpote Tubis prælongis ac Micrometris præter ſolitum affabre factis menſuratæ. Cape igitur primo.

Planetarum Obſervationes.

Anno 1717. *Aprilis* 15°. 9^h 49′ T. æq. obſervavit D. *Pound* apud *Wanſted*, *Jovem* jam reverſum ad ſtellam illam, quam *Novemb.* 22°. 1716. manè corpore ſuo texerat, de qua vide N° 350. *Phil. Tranſact.* pag. 508. *Jovis* autem centrum tum temporis diſtabat ab ea Stella (quæ tertia eſt Geminorum in *Catalogo Britannico*) 23′ 22″ boream verſus; ſimulque ab alia vicinâ, quæ quarta eſt Geminorum in dicto Catalogo, 27′ 11″. atque huic fere conjunctus erat planeta.

Aprilis 25^to^ ſequente, eodem obſervatore ac loco, 10^h 3′ T. æq. *Jupiter* apud quatuor Fixas exiguas viſus eſt, eas omnes præcedens, & in ipſo quaſi principio Cancri. Centrum autem planetæ diſtabat ab *e* 13′ 00″, ab *h* 13′ 50″, ab *f* 19′ 53″, & à *g* 9′ 27″.

Poſtridie vero *Apr.* 26°. 9^h 7′ *Jovis* centrum diſtabat ab *e* 8′ 35″, ab *f* 9′ 00″, à *g* 4′ 5″, & ab *h* 13′ 50″. Jamque præterierat omnes præter *f* ad quam tendebat, quamque parum admodum die craſtino infra ſe relinquere debuit.

Eodem fere momento, horâ ſcil. nonâ, *Londini* viſa eſt ſtella *g* in vertice Trianguli Iſoſcelis ac fere Iſopleuri cum *Jovis* centro ac tertio Satellite, tum ſex *Jovis* diametris ad occaſum diſtante, niſi quod parum admodum baſe longiora erant crura; ac intra quadrantem horæ,

horæ, angulus ad *Jovis* centrum, qui prius major erat angulo ad Satellitem, factus eſt eo ſenſibiliter minor.

Tres autem Stellæ *h*, *g*, *e*, ſunt 10^ma^, 11^ma^, & 12^ma^. Geminorum in *Catal. Britan.* juxta quem tum temporis ſitum habuere, *h* in ♋ 0° 22′ 55″, cum Latit. Borea 0° 11′ 25″. Et *g* in ♋ 0° 28′ 25″. Lat. Bor. 0° 3′ 40″: *e* vero in ♋ 0° 29′ 20″ cum Lat. Auſt. 0° 8′ 05″. Diſtat autem quarta *f* à Stella *g* 11′ 40″, ab *e* 12′ 50″, ac denique ab *h* 20′ 36″, unde conſtabit locus ejus. Ex his manifeſtum eſt *Jovem* Latitudinem habuiſſe parvam admodum Borealem, nec ſemiminuto majorem, ſaltem ſi dictis ſtellarum locis habenda fides. Hæc poſteris uſui eſſe poſſunt definiendo Nodorum *Jovis* motu, ſi quem habeant.

Ejuſdem anni *Junii* 18^vo^ 10^h^ *Londini*, in ædibus Societatis Regiæ, viſus eſt *Saturnus* Stellæ fixæ Teleſcopicæ admodum propinquus, à qua vix diſtabat ad Auſtrum una Annuli diametro, & normalis in lineam Anſarum à Stella demiſſa incidebat in medium Anſæ orientalis. Fixa hæc parvula nullique Catalogo adſcripta tunc habuit ♎ 12° 58′ ½ cum Lat. Bor. 2° 33′ proximè; comitemque habet ſibi adjunctam ac luce æqualem, quatuor minutis ad ortum diſtantem, ac paulò auſtraliorem, unde facile dignoſci poterit, locuſque ejus ſi cui libeat verificari.

Eadem nocte 10^h^ 30′ *Mars* viſus eſt prope Stellam quæ præcedit 35. *Scorpii*, à qua diſtabat Tubo 24 pedum menſurata 7′ 16″; idque in recta per claram in pede Ophiuchi θ & dictam Stellam producta. Hæc autem Stella præcedit 35. *Scorpii* 30′ 27″ Aſc. Rect. eâque Auſtralior eſt 2′ 28″, unde fit locus ejus tum temporis *Sagitt.* 15° 24′ 20″ Lat. Auſt. 3° 59′ 25″. Sed θ Ophiuchi tunc habuit *Sagitt.* 17° 28′, & Lat. Auſt. 1° 47′ 38″. *Mars* itaque Stellam præcedebat Longitudine 4′ 58″, auſtralior ea 5′ 30″.

Deinde *Sept.* 13° 8ʰ 5′. T. æq. *Mars* visus est à Dom. *Pound* præcedere claram in humero *Sagittarii* σ 11′ 54″ Asc. Rect. simulque borealior erat Stellâ 22′ 56″. Hora 8ʰ 25′ erat distantia Planetæ à Stella 25′ 00″ accuratè.

Decemb. 5. 18ʰ 30′. T. æq. consensu sæpius repetitarum observationum, invenit D. *Pound Saturnum* præcedere Telescopicam claram sibi vicinam 27′ 19″ Asc. Rect. Stellaque australiorem esse 1′ 59″. Simul *Saturnus* præcedebat ϰ in Syrmate *Virginis* 1° 25′ 21″, eâque australior erat 4′ 05″. Hinc *Saturni* locus *Libra* 29°. 16′ 21″. Lat. Bor. 2° 22′ 21″. Telescopica autem tunc habuit *Libr.* 29° 40′ 56″. Lat. Bor. 2° 33′ 43″.

Anno 1718. *Jan.* 7. 5ʰ 30′. T. æq *Venus* apud duas Stellas in *Catal. Britan.* omissas observata est. Erat autem Planeta utrâque Fixâ Borealior, distans à præcedente 32′ 30″, à sequente 17′ 30″. Stella præcedens tunc habuit *Pisc.* 14° 42′ 20″, cum Lat. Aust. 0° 40′ 10″; altera vero sequens *Pisc.* 15° 21′ 55″, Lat. Austral. 0° 27′ 15″, prout ex observationibus D. *Flamstedii* colligere licet.

Jan. 15. 8ʰ 00′, T. æq: *Jupiter* præcedebat ν in pectore *Cancri* 3° 30′ 50″ Asc. Rect, fixâque Australior erat 14′ 15″. Hinc provenit *Jovis* locus *Canc.* 28° 20′ cum Latitudine Borea 0° 36′ 45″.

Martii 11. 10ʰ 36′, T. æq *Saturnus* præcedebat ϰ in Syrmate *Virginis* 18′ 51″, eâque Fixâ australior erat 5′ 23. Hinc fit Locus *Saturni Scorp.* 0 18′ 34″ cum Lat. Bor. 2. 44′ 8″. Posito scilicet, juxta *Catal. Britan.* ϰ *Virginis* occupare ♏ 0. 34′ 10″, cum Lat. 2. 55′ 40″. Eadem nocte 1ʰ 00′ *Westmonasterii* observarunt DD. *Desaguliers* & *Gray Saturnum* præcedere Stellam 19′ 00″ cum declinatione majore in Austrum 4′ 45″.

April 8. 11ʰ 30′ *Londini* visus est *Saturnus* nuper Acronychus parum admodum occidentalior Telescopicâ clarâ, eâdemque 5 minutis borealior. Unde Fixæ locus

Libræ

Libræ 28. 18′ 30″ Lat. Bor. 2. 41′. Circulus autem magnus per hanc Stellam & *Saturnum* ductus dirigi videbatur ad Stellam 5^tæ magnitudinis in *Catal. Brit.* omissam, sed quæ *Hevelio* est *in cuspide Alæ Boreæ Virginis*, cuique locum assignat *Libr.* 26. 10′, cum Lat. 14. 43′ Bor.

Eadem nocte 13^h 20′, apud *Wansted*, perpendiculum à dictâ Stella Telescopicâ in lineam Ansarum *Saturni* demissum præcedebat centrum planetæ quasi sesquialtera diametro annuli; aberat autem Stella ad Austrum ab Ansarum axe 4′ 30″. Simul Ansæ orientalis extremitas deprehensa est in linea recta inter hanc Stellam & aliam eidem quasi longitudine conjunctam, quæ tunc à *Saturno* distabat 24′ 48″ versus Boream. Locus autem prioris Stellæ tunc fuit *Libr.* 28. 18′ 30″ cum Lat. Bor. 2. 41′ proxime.

Sept. 7. circa meridiem incidit conjunctio *Jovis* & *Veneris* arctissima, cujus quidem spectaculum Astronomis nostris inviderunt Nubes. Die autem sexto præcedente mane, vel 5^d 22^h 57′ 30″ T. æq. apud *Wansted*, *Venus* occidentalior distabat à *Jove* 1. 3′ 28″. Die autem 7. 17^h 21′, *Venus* jam facta orientalior à *Jove* aberat 43′ 18″; ac 17^h 34′, *Venus* australior erat *Jove* differentiâ declinationum 14′ 23″. Et 17^h 39′ capta est distantia Planetarum 44′ 4″. Hinc calculo accuratissimi Observatoris conjuncti sunt *Sept.* 7. 0^h 9′ T. æq. *Veneris* centro tum *Jovis* australiore non nisi 1′ 42″.

Denique *Sept.* 18. mane, apud *Wansted*, *Jupiter* visus est prope *Cor Leonis*, quocum die præcedente conjunctus fuerat. *Sept.* 17. 16^h 51′ T. æq. *Jovis* centrum aberat à *Corde Leon.* 24′ 22″; & 17^h 6′ 20″ erat diff Declin. 12′ 43″. Dein post Horam, nempe 17^h 54′, facta est distantia 24′ 44″; ac 18^h 7′ differentia Declinationum inventa est 12′ 35″. Hinc supputante Dom *Hounâ*, fit *Sept.* 17. 18^h 00′ T. æq. *Jovis* locus ♌ 26 11′ 7″ cum Lat Bor. 45′ 39″.

Observationes

Obſervationes Lunæ & Eclipſium.

Anno 1717. *Jan.* 12. *Weſtmonaſterii* obſervavit Dom. *Stephanus Gray* Lunæ appulſum ad quatuor Stellas contiguas ſub cornu Auſtrino *Tauri*, apud quas obſervata eſt Luna Anno 1683. *Mart.* 23. ſt. v. ab *Hevelio* & *Flamſtedio*. Itaque $9^h\,45'$ T. app. Luna gibba viſa eſt quaſi conjuncta cum Stella è quatuor præcedente, quæ eſt *Tauri* 107. *Catal. Brit.* quæque tunc Auſtralior erat Lunæ limbo Auſt. ſeſquialtero minuto. $11^h\,29'$ altera, quæ minor eſt, & ideo in Catalogo omiſſa, occultabatur paulo infra medium obſcuri limbi. Ad $12^h\,24'$ Tertia & clarior (110. *Tauri*) in ipſa fere conjunctione ſex minutis diſtabat à limbo boreo. Denique $12^h\,54'$ ſequens è quatuor (111. *Tauri*) limbo Boreo ſuperior erat $3'\,30''$. Locus autem præcedentis, ſive 107. *Tauri*, ex dicto Catalogo tunc erat *Gemini* 18. 12. Lat. Auſt. 5. 18′; *Tauri* autem 110 habuit *Gem.* 19. $26'\frac{1}{4}$ cum Lat. Auſt. 4. 44′: Sequens vero, ſive 111 *Tauri*, erat in *Gem.* 19. 45′. Lat. Auſt. 4. $48'\frac{1}{6}$. Secunda parvula, ut ex aliis obſervationibus conſtat, Locum tunc habuit *Gem.* 19. 17′. Lat. 5. 5′ ferè.

Eodem anno *Mart.* 16. mane, erat Eclipſis Lunæ partialis, apud nos ob cœlum nubilum inconſpicua. At apud *Cambridg Nov-Anglorum*, Dom. *Robie* Aſtronomiæ peritiſſimus vidit Eclipſeos initium circa horam nonam. Finem vero, juxta *Paludem Mæotida*, ad $11^h\,42'\,30''$ ſat accurate. Eſt autem *Cambridg* ſub altitudine Poli 42. 25′, *Londino* 71 grad. ſive $4^h\,44'$ occidentalior, ut ex pluribus antea obſervatis conſtat.

Dein *Sept.* 9. veſperi, in ædibus Societatis Regiæ *Londini*, obſervarunt nonnulli è Sociis finem Eclipſeos Lunaris $7^h\,26'$. Luna autem orta eſt juxta medium Eclipſeos,

Eclipſeos, nec niſi paulo ante finem è nubibus horizontem obſidentibus ſeſe extricaverat.

Sept. 14. Veſperi, hac prima vice poſt longum intervallum rediit Luna ad occultandum *Palilicium*. Favit autem admodum cœlum *Londini* præter ſolitum purum, ita ut Luna & Stella exorientes in ipſo quaſi Horizonte ſimul conſpicerentur. Incidit Immerſio Stellæ 9^h 6′ 20″, Lunâ nondum 3° altâ, in ipſo quaſi medio Limbi orientalis, è regione ſcilicet Boreæ partis maculæ illius exiguæ quam *Hevelius* Stagnum *Mœridis* vocat, quamque *Ricciolus* ſui ipſius nomine inſignivit. Emerſit autem paulo infra medium limbi obſcuri ad 9^h 58′ 20″, in ictu oculi tota ſua claritate effulgens; unde etiam in tam illuſtri Stellâ quaſi nullitas diametri demonſtratur.

Septembris 23. veſperi, incidit Eclipſis Solis vix ullibi in *Europâ*, conſpicua. Ex *America* autem noſtrâ duplicem obtinuimus ejus obſervationem; alteram ex literis illuſtris Viri D. *Keith* Provinciæ *Penſylvaniæ* Præfecti digniſſimi, qui *Philadelphiæ*, ſub altitudine Poli 40° 00′ fere, vidit Eclipſin jam cœptam (ſed quæ ante minutum temporis nondum inceperat) ad 11^h 55′. Circa medium Digiti erant quaſi decem. Finis autem viſus eſt accurate ad 2^h 46′ 35″.

Altera autem hujus obſervatio habita eſt ad *Cambridg Novæ Angliæ* Academiam, à Dom. *Robie*, de quo ſupra: Initium Eclipſeos ibi obſervatum eſt 0^h 23′ 00″ poſt meridiem. Ad 1^h 47′ defecêre IX Digiti. Ad 3^h 5′ 10″ deſiit Eclipſis, Sole integro per Tubum 24 pedum conſpecto. Hæc ex literis accurati Obſervatoris communicavit cum Reg. Societate Reverendus Vir D. *Guil. Derham*, R.S.S. Eccleſiæ apud *Windſor* Canonicus, &c.

Dec. 5. Luna paulo ſupra *Palilicium* invecta eſt: Tranſitum autem ſatis arctum obſervavit D. *Jac. Bradley*, A.M. eruditus Juvenis, qui ſimul ingenio & induſtria pollens

his

his ſtudiis promovendis aptiſſimus natus eſt, idemque Reverendi D^ni *Pound* ex ſorore nepos. Hic, cum Luna jam propemodum plena eſſet, Stellam contulit cum inſigni illa Macula quam *Ricciolus Tychonem*, *Hevelius Sinam* appellat, & ex pluribus æqualibus diſtantiis Micrometro ante & poſt captis, Stellam dictæ maculæ centro proximam apparuiſſe concluſit ad $11^h\ 15'\ 8''$ T. æq. apud *Wanſted*. Ad $11^h\ 15'\ 42''$ diſtabat Palilicium à limbo Lunæ proximo & Auſtrino $5'\ 55''$. Macula autem *Tycho* ab eodem limbo aberat $4'\ 16''$. Ad $11^h\ 18'\ 42''$ Stella erat in lineâ rectâ cum maculis *Tychonis* & *Copernici*, ſive *Sinæ* & *Ætnæ*; & $11^h\ 25'\ 27''$ T. æq. erat in rectâ cum *Tychone* & *Keplero*. Inter hæc obſervata eſt Lunæ diameter $32'\ 45''$.

Anno 1718. *Jan.* 29. veſperi, DD. *Deſaguliers* & *Gray*, *Weſtmonaſterii* alteram *Palilicii* Occultationem præſtolabant; ſed nubium interventu impediti, viderunt ſaltem quod $5^h\ 52'$ nondum immerſerat Stella; attenuatis autem poſtea nubibus concluſa eſt Emerſio ad $7^h\ 20'$, è regione Promontorii *Sarmatiæ Aſiaticæ Hevelii*.

Feb. 19. manè. Iidem obſervatores ibidem variè cum nubibus colluctati Eclipſin Solis ægre conſpexerunt: Horâ tamen 6. $59'$ viſi ſunt deficere duo Digiti, & poſt unum temporis minutum chorda inter Cuſpides viſa eſt æqualis ſemidiametro Solis.

Apud *Wanſted* autem D. *Pound* notavit ad $6^h\ 54'\ 7''$ T. app. chordam inter Cuſpides $18'\ 30''$. Ad $7^h\ 17'\ 00''$ erat $10'\ 18''$. Ad $7^h\ 19'\ 30''$ eadem inventa eſt $8'\ 05''$. Deſiit autem Eclipſis ad $7^h\ 23'\ 20''$.

Feb. 25, veſperi, $6^h\ 44'$ T. app. *Weſtmonaſterii*, Stella prima *Hyadum* in Naribus *Tauri* (γ *Bayero*) viſa eſt in rectâ per cuſpides Lunæ, adeoque propemodum conjuncta; diſtabat autem à limbo Lunæ Auſtr. $5'\ 51''$. Diameter Lunæ $31'\ 45''$ menſurata Micrometro.

Feb. 28,

Feb. 28. 8^h 36′ T. app. etiam *Weſtmonaſterii*, viſa eſt Immerſio Stellæ in Poplite *Pollucis* (λ Geminorum *Bayero*) ſub limbi *Lunæ* obſcuri ea parte, quæ paulo Borealior erat macula quam *Hevelius Cretam* vocat. Emerſio ipſa ob cœlum minus purum non conſpecta eſt: ſed ad 9^h 51′ egreſſa erat Stella è limbo lucido, à quo diſtabat 3 min. circiter, è regione Boreæ partis Inſulæ Majoris *Caſſii*.

Aug. 8. Luna orta eſt paulo infra *Palilicium*, cum quo tamen ob nubes conferri non potuit. Apud *Wanſted* autem 13^h 2′ 00″ T. app. viſa eſt Præcedens contiguarum ad σ *Tauri Bayero*, (ſive Penultima in noſtro *Hyadum* Catalogo, in Num° 354. *Tranſact.* litera *q* notata) in linea recta per cuſpides *Lunæ*, diſtans ab Auſtrino 4′ 36″. Ad 13^h 7′ 25″ Stella *p* ejuſdem Catalogi emerſit paulo infra medium obſcuri limbi. Ad 13^h 19′ 4″ emerſit Sequens contiguarum dictarum, tantum diſtans à Cornu Auſtrino quantum contiguæ illæ inter ſe, hoc eſt 7 min.

Aug. 29. Veſperi, Luna fere Apogæa paſſa eſt deliquium totalem ac fere centralem: orta autem eſt Eclipſi jam cœptâ. Hujus obſervationes maximè luculentas Regiæ Soc. exhibuit toties laudatus Rev. D. *Pound*, eo ordine quo notatæ ſunt, nempe

Obſerv.	*Tempus apparens* h ′ ″	*Eclipſis* Lunæ *obſervata apud* Wanſted, 29. *Auguſti*, 1718.	′ ″
1	6 53 38	Chorda inter Cuſpides Micrometro	22 37
2	55 8	Eadem repetita (menſurata	21 14
3	56 31	Repetita —— —— —— ——	19 51
4	57 49	Iterum —— —— —— ——	18 28
5	59 38	Denuo—— —— ——	15 00

h.	T. appar. h	'	"	Eclipſis Lunæ, *Aug.* 29. 1718.	'	"
6	7	2	41	Immerſio Totalis in Umbram ——		
7	8	36	13	Stella clara in Catalogis omiſſa occultata eſt à *Lunâ*, infra *Paludem Mareotida Hevelii* —— ——	10	2
8	8	48	18	*Luna* cœpit emergere ex Umbra —		
9		51	13	Terminus Umbræ per med. *Mareotidis*; ſimul Chorda inter Cuſpides	5	0
10		53	7	Chorda inter Cuſpides — ——	18	28
11		54	16	Eadem repetita —— —— ——	9	51
12		54	59	Iterum —— —— —— ——	21	14
13	8	56	18	Denuo —— —— —— ——	22	37
14	9	0	48	*Porphyrites* emerſit ex umbra.		
15		8	3	Mons *Sinai* incepit emergere.		
16		9	17	Umbra per medium *Sinæ*.		
17		10	6	Jam totus *Sinai* extra Umbram.		
18		11	20	Umbra per medium *Ætnæ*.		
19		17	23	Per medium *Corſicæ*.		
20		20	0	Per medium *Lacus Nigri majoris*.		
21		27	54	Per medium *Beſbici*.		
22		28	45	Emerſit Stella prædicta.		
23		32	34	*Byzantium* & *Horminius* ſimul emergunt.		
24		33	58	Stella eandem habuit Declinationem cum Cuſpide Auſt. Eclipſeos.		
25		43	28	Chorda inter Cuſpides —— 18′ 28″.		
26		47	2	Eadem repetita —— —— 15 ,00		
27	9	53	0	Deſiiſſe videbatur Defectus.		

10^{h} 30′, Capta eſt Lunæ diameter 29′ 45″. Collatis autem inter ſe Obſervationibus, ubi Chordæ partis deficientis æquales deprehenſæ ſunt, provenit Eclipſeos medium.

Ex Obſerv.

	Medium. h	'	"
Ex Obſerv. prima & decima tertia —	7	54	58
Ex ſecunda & duodecima — —	7	55	3
Ex tertia & undecima — —	7	55	24
Ex quarta & decima — —	7	55	28
Ex quinta & nona — —	7	55	25
Ex ſexta & octava — — —	7	55	29
Quorum omnium Medium fit —	7	55	18

Non minore cum curâ eandem Eclipſin, *Londini* in vico *Fleetſtreet*, inſtrumentis & Teleſcopio optimo D. *Geo. Graham* Automatopœi præſtantis, obſervavit D. *Martinus Folkes* Armig. cum aliis quibuſdam Regiæ Societatis Sodalibus, ut ſequitur,

h	'	"	
6	38	0	Luna per fumum Urbis & Vapores ægre viſa.
6	54	13	Chorda inter Cuſpides utcunque, 21′ 27″.
7	2	0	Immerſio Totalis in umbram.
7	42	15	Stella fixa ſatis clara diſtabat in limbo Lunæ orientali 19′ 21″.
8	35	18	Eadem fixa occultata eſt, 10′ circiter minutis centro Lunæ Auſtralior.
8	45	50	vel, ut quibuſdam viſum eſt, uno minuto tardius Luna cœpit emergere.
8	49	38	Palus *Mareotis* primo margine emerſit.
8	50	14	Integra Palus extra Umbram.
9	0	5	Montis *Porphyritidis* medium emerſit.
9	7	39	Primus margo *Sinæ* emerſit.
9	9	8	Mons *Sinai* totus extra umbram.
9	10	35	Umbra per medium *Ætnæ*.
9	12	0	Totus mons *Ætna* extra umbram.
9	18	51	Umbra per medium *Lacus Nigri* majoris.
9	27	35	Inſula *Beſbicus* tota emerſit.

9 42 21

9 42′ 21″ Chorda inter Cuſpides 19′ 9″.
9 51 25 Finis Eclipſeos ut quibuſdam viſum eſt.
9 52 45 Finis ex præcedente diſtantia Cuſpidum concluſa.
9 56 45 Lunæ diameter 29′ 45″, iterumque 29′ 48″.

Erat autem Umbra admodum diluta, unde orta eſt difficultas in dijudicandis Emerſionis & Finis momentis: Atque Maculæ etiam obſcuriores clarè conſpectæ ſunt, pluribus minutis antequam Umbræ marginem attingerent. Stella vero quæ durante Eclipſi occultata eſt, locum tunc habuit ♐ 17° 16′ ½ cum Lat. Auſt. 1° 6′ 30″ proximè.

Recepimus etiam Obſervationes hujus Eclipſeos à Rev. D^n *Derham*, apud *Upminſter* in agro *Eſſexienſi* habitas; à D^n *Wright* apud *Crew* in agro *Ceſtrienſi*; & à D *Hawkins* apud *Wak field* in *Eboracenſi*, cum præmiſſis ubique fere conſentientes, ſi adhibeantur meridianorum differentiæ: poſito ſcil quod *Upminſter* ſit 1 ⅓ min. *Londino* orientalius, *Crew* vero 10 min. & *Wak field* 5 min. occidentaliora.

Denique Coronidis loco obſervationem adjiciamus, eximiam quidem, ſuique generis, quod ſcimus, ab inventio Teleſcopio primam; quamque indefeſſæ D. *Jac Bradley* debemus diligentiæ. Quinto enim *Septembris* manè, ſole jam fere 30 *gr.* alto, vidit apud *Wanſted* arctiſſimum Lunæ infra *Palilicium* tranſitum, cujus diſtantiam à limbo proximo, ad 7^h 59′ 00″ T.æq. Micrometro invenit 5′ 38″. Ad 8^h 17′ 5″ diſtabat à limbo 1′ 25″. Stella autem ad 8^h 33′ 15″ erat in linea recta per Lunæ Cuſpides tum obtuſiuſculas, nec niſi 0′ 13″ diſtabat à Boreâ. 8^h 41′ 00″ jam Cuſpidem illam reliquerat 3′ 42″. Et 8^h 45′ 37″ ab eadem diſtabat 5′ 36″. Lunæ diameter ad 8^h 58′ capta eſt 31′ 7″.

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Fig. 1,

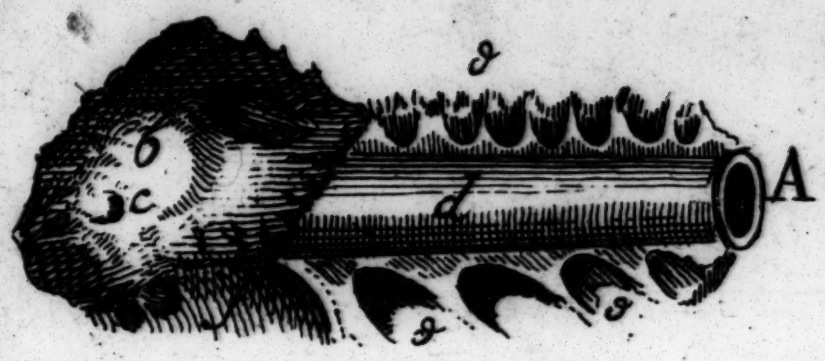


Fig. 11.

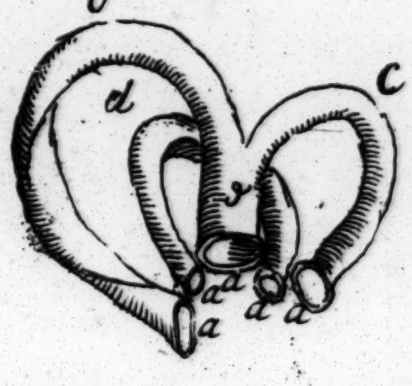
Fig. III

V

VI

VII.

IX.



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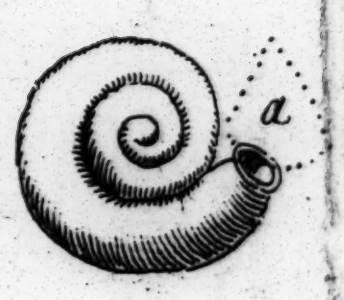


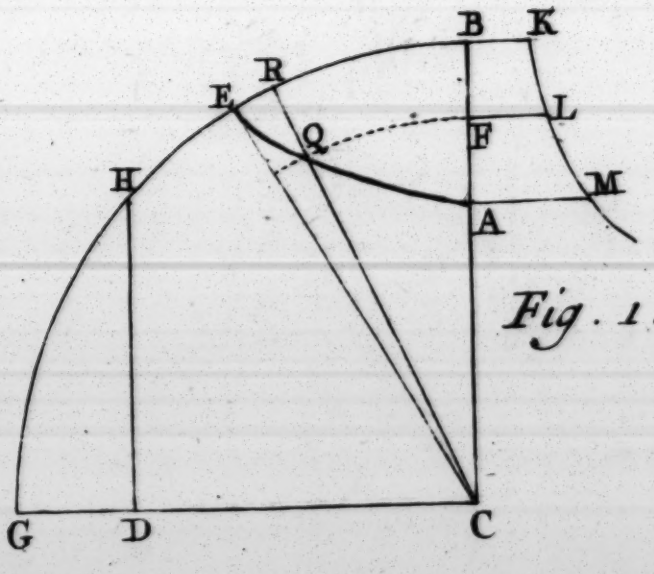


Fig. 1.

Fig. 2.

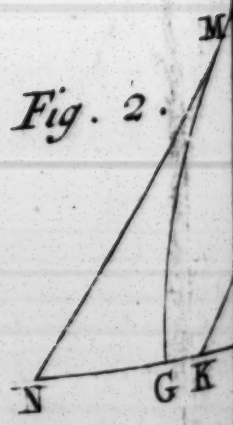
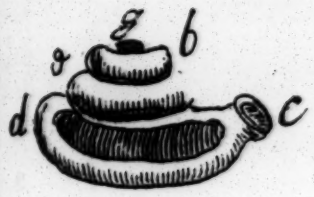
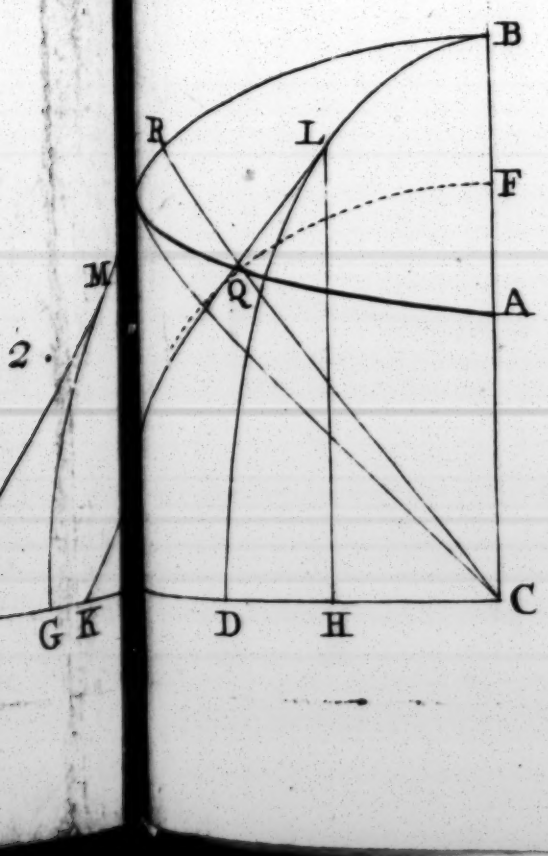


Fig. III

VIII.

Fig. XI





PHILOSOPHICAL TRANSACTIONS.

For the Months of *Octob. Nov.* and *Dec.* 1718.

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I. *Inventio Curvæ quam Corpus descendens brevissimo tempore describeret; urgente Vi Centripetâ ad datum punctum tendente, quæ crescat vel decrescat juxta quamvis Potentiam distantiæ à centro; dato nempe imo Curvæ puncto & altitudine in principio Casus.* *Per* Johan: Machin, Astron. Profess. Gresh. *&* Reg. Soc. Secret.

II. *De Potentiâ Cordis. Dissertatio Authore* Jac. Jurin, *M. D.* Reg. Soc. Soc.

III. *A Brief Account of the Contagious Disease which raged among the Milch Cows near* London, *in the Year* 1714. *And of the Methods that were taken for suppressing it. Communicated to the* Royal Society *by* Thomas Bates *Esq; Surgeon to His Majesties Houshold, and* R. S. S.

IV. *A Description of the* Organ *of* Hearing *in the* Elephant, *with the Figures and Situation of the* Ossicles, Labyrinth *and* Cochlea *in the Ear of that large Animal. Communicated to the* Royal Society, *by Dr.* Patrick Blair, *R. S. S.*

Jo. Banks

I. *Inventio Curvæ quam Corpus descendens breviss. mo tempore describeret ; urgente Vi Centripetâ ad datum punctum tendente, quæ crescat vel decrescat juxta quamvis Potentiam distantiæ à Centro ; dato nempe imo Curvæ puncto & altitudine in principio Casus.* Per Joh. Machin, Astron. Profess. Gresh. & Reg. Soc. Secret.

SIt centrum Virium C, (*Fig.* 1. 2), quo centro ad distantiam CB æqualem altitudini unde Corpus casurum est, describatur Circulus BEG, & fiat angulus BCG rectus. Ponatur A punctum Curvæ infimum, ubi axi CB occurrit ad datam distantiam CA. Oportet invenire punctum Q, ubi Curva celerrimi descensûs EQA occurrit circulo QF, ad datam aliam distantiam CF. Problema hoc duos habet Casus, quorum alter pendet ab Hyperbola & Circulo, alter ab Ellipsi & Circulo.

Cas. 1. Si fuerit Vis centripeta reciproce ut distantia à Centro. Sit KLM (*Fig.* 1.) Hyperbola quævis rectangula centro C & Asymptoto CB descripta, quæ occurrat normalibus BK, AM super ipsam BC ad puncta B, A erectis, in K & M ; ordinatæ vero cuilibet intermediæ FL ad punctum F erectæ, in L. Fiat CD ad CG ut ✓AFLM ad ✓ABKM, & sit DH normalis super CG : dein capiatur Sector RCB ad Aream HDCB ut data Area Hyperbolica ABKM ad datum Rectangulum CA×AM. Tum recta RC occurret circulo FQ in puncto Q, quod quidem est ad Curvam celerrimi descensus EQA.

Hab

Habebitur autem punctum E, à quo inciperet Corporis casus, capiendo Sectorem B C E ad Aream Quadrantis B C G, in eadem ratione Areæ Hyperbolicæ A B K M ad rectangulum sub C A & A M contentum.

Coroll. Hinc si recta R C, circa centrum C revoluta, faciat Sectores R C B proportionales Areis H D C B, in quibus quadrata Basium C D sumuntur in progressione Arithmeticâ: tum rectæ C R intersecabunt Curvam E Q A ad distantias à centro C Q, quæ decrescant in progressione Geometricâ

Cas. 2. Si vero Vis centripeta fuerit reciproce ut alia quævis Potestas distantiæ à centro; sit $n+1$ Index istius Potestatis (ubi n potest esse Numerus quilibet integer vel fractus, affirmativus vel negativus) sitque H = C B altitudo maxima Curvæ quæsitæ E Q A, h = C A altitudo minima ejusdem, & A = C F altitudo alia quævis intermedia. *Fig.* 2.

In recta C G capiatur C D ad C B ut $\surd h^n$ ad $\surd H^n$, atque etiam C H ad C D ut $\surd \overline{A^n - h^n}$ ad $\surd \overline{H^n - h^n}$. Centro C, semiaxibus C D, C B, describatur Ellipsis B L D, cui occurrat ordinatim applicata H L in puncto L; & ducatur recta L K, quæ Ellipsin tangat in L, & Axi minori C D producto conveniat in K: dein Tangenti K L parallela ducatur N M, circulum B E M G tangens in M & ipsi C D occurrens in N. Denique capiatur Sector R C B, qui sit ad Aream N M B L K N, inter Circulum & Ellipsin & utriusque Tangentes rectamque N K comprehensam, in ratione Numeri binarii ad Numerum n. Tum recta R C intersecabit Circulum F Q in puncto Q, quod erit ad Curvam celerrimi Descensus E Q A.

Quod si fiat Sector B C E ad aream B D G, inter Ellipseos & Circuli Quadrantes interceptam, in ratione dictâ Binarii ad Numerum n, coeuntibus scilicet punctis L, D & M, G; (ob $A^n = H^n$) erit punctum E unde inchoa-

choaretur Caſus Corporis breviſſimo tempore deſcendentis ad A, deſcenſuque ſuo Curvam EQA deſcribentis, quam tangit recta CE in E, quamque ad angulos rectos ſecat CB in A.

Harum Conſtructionum Demonſtrationes è Celeberrimi D. *Newtoni Quadraturis*, ejuſdemque *Philoſ. Nat. Principiis* (Prop. XXXIX. & ſequentibus aliquibus) petitæ, aliâ datâ occaſione oſtendentur. Problema autem eſt alterius generis, Deſcribere Curvas per quas Corpora, de puncto ſummo E, ſeu principio caſus, demiſſa, celerrimo deſcenſu ad inferiora data puncta Q, urgente qualibet Vi centripeta, ferrentur; cujus quidem ſolutio in poteſtate eſt. In præſentia ſufficiat generalem hujuſmodi Curvarum tradidiſſe Ideam, earumque ad Circuli & Hyperbolæ Quadraturas relationes indicaſſe, abſque quibus eaſdem Geometrice conſtruere haud adeo proclive eſt.

II. De

II. *De Potentiâ Cordis.*

Dissertatio Authore Jac. Jurin, *M.D.*

Reg. Soc. Sodale.

Viro Eruditissimo

Richardo Mead, *M.D.*

S. P. D.

Jacobus Jurin.

Disquisitionem istam, Vir Clarissime, utut rudem & imperfectam, acri tamen ac perspicaci tuo Judicio multis nominibus non illibenter permitto. Quem enim mihi potero aut Judicem æquiorem præoptare, aut Cognitorem deligere magis idoneum, quam cujus Viri candorem animi singularem, morumque humanitatem, non minus atque Mentis dotes præcellentes illas, & optimo quoque Literarum genere perpolitas, omnes suspicimus; cujusque tum acumine Ingenii, tum Judicii subtilitate, Theoriam Medicam videmus clarissimâ luce perfusam & illustratam, Usum vero medendi confirmatum pariter tenemus & expeditum? Nec sane quisquam est Mortalium, cujus calculo cogitata ista nostra comprobari magis studeamus, aut cujus auctoritate, si tibi forte fortuna minus displicuerint, ea contra Hominum quorundam perversorum iniquitatem tutiora sint futura. Ex quibus alii præjudicio ducti & famâ magnorum Nomi-

num, quorum ſententias in ſequentibus paſſim redargui. mus, noſtra forſitan ne examine quidem aut perlectu digna cenſuri ſunt. Alii vero, ut ſive labore diſcendi, ſive imperitiæ pudore ſe expediant, omnia ſcilicet, quæcunque ipſi non intelligunt, videri volunt alto ſupercilio contemnere. Quibus uti non gravate concedimus doctos Viros & olim exſtitiſſe, & hodie reperiri non paucos, qui nullâ inſtructi diſciplinâ Mathematicâ medendi Artem tamen feliciter & cum laude exerceant; ita viciſſim ipſos fateri æquum eſt eam doctrinam in Praxi expediendâ non inutilem; ad naturam vero & cauſas Morborum explorandas plane eſſe neceſſariam. Corpora enim Animalium, quod tu profecto, ſi quis alius, optime intelligis, cum partim ſolidis canalibus, partim fluido conſtent per eoſdem jugiter propulſo, Machinas eſſe patet, ac proinde opus eſſe, ad eorum Fabricam, Vires, Actiones, & agendi Impedimenta ſive Morbos rite perſpiciendos, rei Mechanicæ peritiam.

De quibus tamen multa traduntur etiam à Mathematicis Scriptoribus adeo parum accurata, ſecumque invicem & cum ratione pugnantia, ut nobiliſſimæ ſcientiæ non modo commendationem non addant & dignitatem, ſed etiam contemptui & hominum indoctorum ludibriis eandem objiciant. Quis enim, non ipſe doctrinâ Mathematicâ imbutus, cum videat, Exempli gratiâ, Cordis Humani vires jam ponderi 3,000 librarum pares, jam 180,000 pondo ſuperantes, jam vero ad uncias 5 vel 8 deductas; Aerem quoque ex Pulmone inter exſpirandum propulſum modo 100, modo 50,000 librarum vi; Quis inquam, qui iſtas concluſiones legerit diſcrimine tam immani à ſe invicem remotas, & tamen omnes demonſtrationibus ſuis munitas, ſi forte ſe à riſu temperet, non tamen inutilem plane & ineptam pronuntiaverit ad explorandas Corporis facultates ſcientiam Mechanicam? Sed meminerint oportet æqui rerum Judices neutiquam

mirandum

mirandum esse, si quandoque in difficili Problemate vel summa Ingenia allucinentur, neque errores, siqui forte inciderint, Arti ipsi, sed Artifici imputandos. Quod ut Exemplo manifestius declaretur, libet celeberrimi Problematis de Cordis viribus indagandis solutionem novam proponere. Utque facilius mihi temeritatis opinionem detraham, qui ejusmodi inceptum post *Alphonsum Borellum* aggredi ausim, utque viam simul Lectori expediam ad æquam certamque sententiam in tantâ scriptorum dissensione ferendam, primo loco ostensurus sum, quæ in *Borelli* demonstratione reprehendi debeant, deinde Virorum Doctissimorum, *Morlandi*, & *Keilii* solutiones, cum eâdem philosophandi libertate ad examen revocabo.

Primum nobis, & quidem longe præcipuum videtur *Borellianæ* solutionis vitium, quod Cordis Potentiam per pondus iners & quiescens exposuerit. Cor enim cum & ipsum inter contrahendum movetur, & corpora opposita, Sanguinem nempe & Arteriarum tunicas, in motum impellit, patet ejus Potentiam non aliâ ratione sciri posse quanta sit, quam ut motus hujus quantitatem cognitam teneamus. Motus autem quilibet cum pondere quiescente comparari non magis potest, quam Linea cum Rectangulo.

Secundum, quod in ipso Experimento à Circulatore instituto, neutiquam constet pondus illud suspensum fuisse à solâ Musculorum vi contractrice; quum etiam vis illa, quâ tum Musculi adhibiti, tum genæ quoque, & ipsa forsitan ligamenta divulsioni sui ipsorum & fibrarum ruptioni obstiterint, quâque Musculi etiam ex cadavere exsecti pondera satis magna sustinent, venire in subsidium potuerit.

3. Quod vires Musculorum pondere æqualium a *Borello* pares statuantur: quod profecto dubium admodum videtur, præsertim ubi Musculi sunt figurâ dissimiles.

4. Quod integram Cordis Potentiam, quanta maxima exeri poteſt cum ſummâ fibrarum contentione & molimine, ad ſingulas Syſtoles adhiberi poſuerit. Quum ipſe Circulator, ſi pondus ſuſpenſum vel continenter, vel alternis vicibus breviſſimâ quiete interpoſitâ, ſublevare contenderet, non ita longo tempore plane ſuccubiturus labori fuiſſet.

5. Quod Sanguinis & Arteriarum reſiſtentiam ſexagecuplam ſtatuerit totius Potentiæ Cordis, loco ejus Potentiæ, quæ ad ſyſtolem peragendam à Corde impenditur, quæque forte totius Potentiæ minima pars eſt.

6. Quod in eâ ratione ſexagecuplâ definiendâ errorem inſignem admiſerit. Nam in *Prop.* 60, loco rationis, quam obtinet Summa Potentiarum *P* & *Q* ad Summam *R* & *S*, adhibuit rationem, quæ eſt inter Rectangulum ex Potentiis *P*, *Q* confectum, & Rectangulum ex *R*, *S*. Quod errati ſi per Propoſitiones ſubſequentes corrigatur, habebitur in *Prop.* 73, reſiſtentia longe major, quam ab ipſo *Borello* definita eſt, nempe pondus librarum 1,076,000, loco librarum 180,000, idque ſecundum poſitiones ab ipſo Viro Clariſſimo uſurpatas.

7. Denique quod pondus illud librarum 180,000, quum à Cordis Potentiâ libris 3,000 æquali ſuperetur, miraculi cujuſdam aut monſtri loco Lectoribus obtrudat; & Vim Percuſſionis, quaſi quendam Θεὸν ἀπὸ μηχανῆς in auxilium advocet. Reipsâ enim nihilo plus hic ineſt prodigii, quam ubi pondus 3,000 librarum pondus aliud 180,000 librarum, ad ſubſexagecuplam diſtantiam à centro Libræ inæqualium radiorum appenſum, in æquilibrio ſuſtinet.

Minora aliquot Sphalmata, & Hypotheſes plures tum prorſus arbitrarias, tum alias aliis contrarias, non illibenter omittimus. Et quidem delicta ſupra reprehenſa, aut ſaltem majorem eorundem partem, non tam ipſi Viro

ro Doctiſſimo imputandam cenſemus, quam Operi Poſthumo condonandam.

Proximus ſequitur Vir Doctiſſimus *Joſephus Morlandus*, qui in Diſquiſitionibus de Cordis vi Sermone Anglicano editis, Methodum peringenioſam expoſuit Potentiam Cordis ad Experimentum revocandi. Hic autem, præter delictum ſupra in *Borello* reprehenſum, quod Cordis vires cum pondere quieſcente contulerit, nobis videtur eo quoque nomine notandus, quod integram Cordis actionem in tunicas Arteriarum diſtendendas impendi poſuerit. Cor enim non ſolum Arterias tendit, ſed Sanguinem quoque certâ velocitate per totum Arteriarum & Venarum tractum propellit.

Superest, ut Viri Acutiſſimi *Jacobi Keilii* ſolutionem, in Tentaminibus Medico-Phyſicis ad Oeconomiam Animalem pertinentibus, non ita pridem cum Publico communicatam, expendamus. Qui primus omnium auſus eſt Potentiam Cordis à *Borello* definitam, ac magno Scriptorum conſenſu exceptam & laudatam, non ſolum rejicere, ſed aliam eidem infinito prope diſcrimine minorem numeris diſertis expreſſam ſubſtituere.

Hunc autem cenſemus, præterquam quod primum illud *Borellianæ* ſolutionis vitium imitatus ſit, in ſequentibus etiam à vero aberraſſe.

Quod Corollarium *Newtonianum*, quo utitur ad Cordis vires definiendas, aut male intellexerit, aut certe non ſatis apte uſurpaverit. Pondus enim illud ab *Archimede Britannico* determinatum, quo Motus aquæ ex vaſe effluentis generari poteſt, nequaquam generat Motum aquæ; quippe quæ gravitatis vi cadendo ipſa Motum ſuum acquirat. Sed hoc pondus per datum tempus cadendo, Motum concipit Motui aquæ eodem dato tempore effluentis æqualem.

Præterea ponit Vir Clariſſimus velocitatem Sanguinis ex Corde effluentis perpetuo æqualem per totam Syſto-

les

les durationem, quam nos insigniter inæqualem fieri in sequentibus ostendemus.

In Methodo illa simpliciore, quam postea adhibet Vir Doctissimus, præter delicta hactenus reprehensa alia etiam bina admittit.

Adsumit enim Vires Cordis in diversis Animalibus eam inter se rationem obtinere, quæ est inter pondera eorundem; quod infra falsum esse demonstrabimus. Tum ponit velocitatem Sanguinis ex sectâ Iliacâ Arteriâ profluentis, eandem esse quâ ex Corde in Aortam emittitur. Atqui cum omnis fere sanguis ex Corde expulsus per Iliacam alteram resectam emittitur, patet ejus velocitatem tanto esse majorem in Iliacâ quam in Aortâ, quanto sectio Iliacæ circularis à sectione Aortæ superatur. Præterquam quod velocitas æquabilis, quâ Sanguis per Aortam fluit, longe distet ab eâ velocitate, quâcum exit ex ipso Corde.

Similiter fere redargui potest & illa Methodus, quâ usus est Vir Cl. ad rationem definiendam inter velocitates diversas Sanguinis, resistentiâ nunc oppositâ, nunc sublatâ, per Aortam profluentis. Sed cum isto Experimento non altera solum, sed utraque velocitas major æquo reperiatur, unde ratio, quæ est inter ipsas, non magnopere perturbetur, poterit satis tuto proportio ab ipso exposita, tanquam veræ propinqua, usurpari.

Cursu hactenus expedito, scopulisque detectis, in quos impegerunt Viri egregii supra laudati, erit modo nobis ipsis, ut in viâ difficili & erroribus plenâ, summâ adhibitâ cautione progrediendum. Et primo quidem loco ad ambiguitatem præcidendam necesse est, ut id, quod quæritur, quale sit, accuratius paulo declaretur.

Cordis Virium, sive Potentiæ, nomine significamus vel ipsum Cordis Motum, dum in contractionem agitur, vel Motum ponderis cujuslibet, quod Sanguini objectum

ex

ex Corde proruenti, & velocitate idoneâ delatum in partes contrarias, Sanguinis effluxum, adeoque ipsam Cordis contractionem, æquali vi librare valet & sistere.

Potentiam istam, cum à priori vix sperandum sit ut definire possimus, quod neque fabricam Cordis interiorem, neque causæ contrahentis naturam, aut vires satis habeamus exploratas, relinquitur, ut eandem per effecta, sive à posteriori, æstimemus.

Cordis actio in Ventriculorum suorum contractione omnis consistit. Ventriculi autem inter contrahendum in sanguinem impingunt, eique Motus sui partem communicando, eundem magna vi, qua datur porta, urgent, & expellunt. Sanguis hoc modo in Arterias, Aortam & Pulmonalem, protrusus, impetu in omnes partes facto, partim in tunicas Arteriarum ex Systole sua prægressâ collapsas & flaccidas, partim in Sanguinem priorem tardius fluentem impingit. Unde gradatim extrorsum truduntur Arteriarum tunicæ, & Sanguis antecedens cursu celeratur. Quod si animo concipiantur Arteriæ sectionibus transversis minimis distinctæ, primâ Sanguinis portiunculâ ex Corde in primam sectionem irruente, partim distenditur ista sectio, partim Sanguis eâdem antea contentus in sectionem proximam detruditur, eamque distendit, atque ista actio per succedentes Arteriarum sectiones continuatur. Deinde secunda, & tertia sanguinis portiuncula, & cæteræ deinceps, in primam Arteriæ sectionem incidunt, eamque paulo magis dilatant, & sanguinem eâdem contentum in proximas sectiones successive propellunt; idque fieri pergit, donec omnis sanguis ex Ventriculis fuerit ejectus. Cæterum id utique observandum est Arterias, quo magis contractæ & flaccidæ fuerint, eo minus dilatationi obsistere, quanto autem magis fuerint dilatatæ, tanto fortius ulteriori distractioni reniti; atque idcirco Vim Sanguinis ex Corde prorumpentis primo magis impendi in distentionem Arte-

riarum, quam in Sanguinis præcedentis protruſionem, ſub finem vero magis propelli Sanguinem antecedentem quam diſtendi Arterias, quippe quæ jam rigidæ factæ majorem dilatationem vix admittant.

Sanguis autem ex Corde proſiliens, cum, uti dictum eſt, Motus ſui partem Arteriarum tunicis, partem Sanguini præcedenti communicat, ipſe neceſſario de priſtinâ celeritate remittit; adeoque dum Ventriculorum contractionem moratur, novum ab iis impulſum excipit, ejuſque partem, eâdem ratione atque antea, tunicis Arteriarum & præcedenti Sanguini impendit, unde iterum retardatur, & alium Ventriculorum ictum ſuſcipit, & ſic deinceps, donec omnis ex Ventriculis fuerit expulſus.

Præter cauſam ſupra expoſitam, ſupereſt alia, quâ Sanguis ex Corde effluens gradatim retardatur, adeoque novos ſucceſſive impetus excipit ex Ventriculis ſeſe contrahentibus. Nam Sanguis in Arteriam Aortam influens, etiamſi nulli omnino reſiſtentiæ occurrere ponatur, adeoque nullam pati Motus ſui imminutionem, tamen, cum ex lato in anguſtum fertur, longitudine perpetim creſcit, donec totus in Aortam pervenerit; cumque ſectio Aortæ non minuatur, neceſſario minuitur Sanguinis velocitas. Motus enim Sanguinis eſt in ratione compoſitâ, ex ratione Sectionis Aortæ, velocitate in eâdem, & longitudine Columnæ Sanguineæ, per Theorema noſtrum III. De Motu Aquarum fluentium. Cum vero ea Sanguinis portio, quæ jam pervenerit in Aortam, gradatim retardetur, retardabitur inde Sanguis iſte qui adhuc Ventriculo continetur, & hinc retardabitur ipſius Ventriculi contractio. Unde Ventriculi perpetuo aliam atque aliam Motus ſui partem Sanguini contiguo, his de cauſis perpetim retardato, communicabunt. Patet vero iſthinc, ut id obiter notemus, alium eſſe Motum Sanguinis ex Corde erumpentis, alium ejuſdem jam ex Corde expulſi, & intra Arterias fluentis.

fluentis. Item ictum. ſive impulſum Ventriculorum in Sanguinem impreſſum, qui alioqui unicus eſſet futurus, & puncto temporis tranſigeretur, tamen cauſarum ſupra dictarum vi, quibus Sanguis perpetim retardatur, per totam Cordis Syſtolen continuari.

Ventriculum itaque alterutrum Cordis Sanguinem impellentem licebit ſpectare, ut datum corpus cum datâ celeritate impingens in aliud corpus quieſcens, cui Motus ſui parte communicatâ ambo corpora communi velocitate deferuntur. Æquatur autem Potentia ejuſdem, vel Facto ex pondere Ventriculi & velocitate ejus initiali, priuſquam in Sanguinem impingat; vel Summæ Motuum ipſius Ventriculi ac Sanguinis ex eodem profluentis, & Motûs qui tunicis Arteriarum & Sanguini præcedenti communicatus eſt; vel etiam, ſi abeſſe ponatur omnis Arteriarum & Sanguinis præcedentis reſiſtentia, Summæ Motuum ipſius Ventriculi & Sanguinis effluentis.

Theorema I.

Motus, quo Machina cava inæqualiter contractilis in contractionem agitur, æqualis eſt Summæ Factorum ex ſingulis Machinæ particulis ductis in velocitates reſpectivas.

Patet ex Mechanicâ.

Corol. 1. Machinæ Motus minor eſt Facto ex pondere Machinæ ducto in velocitatem earum Machinæ partium, quæ omnium celerrime moventur inter contrahendum.

2. Motus Machinæ æquatur Facto ex pondere ejuſdem, ducto in velocitatem aliquam mediam inter velocitates earum Machinæ partium quæ omnium celerrime, & earum quæ omnium tardiſſime, moventur.

3. Si Machinæ plures ſimiles ſimiliter ſeſe contrahant, velocitate mediâ, vel æquabili vel inæquabili, ſimiliter tamen auctâ vel imminutâ in omnibus Machi-

nis; Motus, quo Machina quæque in contractionem agitur, rationem obtinet compoſitam ex ratione quadruplicatâ Diametri homologæ ipſius Machinæ, & ratione inversâ temporis, quo Machinæ contractio perficitur; vel rationem compoſitam ex ratione ponderis Machinæ, ratione ejuſdem ponderis ſubtriplicatâ, & ratione temporis inversâ. *Theoremata reliqua huc ſpectantia in* Tranſactione *proximè edenda exhibebuntur.*

III. *A Brief Account of the Contagious Diſeaſe which raged among the Milch Cowes near* London, *in the Year* 1714. *And of the Methods that were taken for ſuppreſſing it. Communicated to the* Royal Society *by* Thomas Bates *Eſq; Surgeon to His Majeſties Houſhold, and* R. S. S.

ABout the middle of *July* the Diſtemper appeared at *Iſlington*, and thereupon their Excellencies the Lords Juſtices having notice of it, were pleaſed to Command that I ſhould examine into the truth of the Report of its being Contagious; and order'd the Lord *Harcourt*, then Lord *High Chancellor*, to grant ſuch Authority as wou'd be proper to make the Diſcovery. Accordingly Mr. *Milner*, Mr. *Offley*, Mr. *Richardſon*, and Mr. *Ward*, four Juſtices of the Peace for the County of *Middleſex*, were appointed to make the neceſſary Examinations.

Purſuant to thoſe Orders we went to *Iſlington*, where Mr. *Ratcliff* had loſt 120 out of 200; Mr. *Rufford* 62 out of 72; and Mr. *Pullen* 38 out of 87. They were very unwilling to own it, becauſe ſo ſoon as it ſhould be known, none wou'd buy their Milk; but Mr. *Ratcliff*, a Man of good Judgment in Cattle, after much perſwaſion, gave us the following account, *viz.* That they

first refuſed their Food; the next Day had Huskiſh Coughs, and voided Excrements like Clay; their Heads ſwelled, and ſometimes their Bodies. In a Day or two more there was a great diſcharge of a Mucous Matter by the Noſe, and their Breaths ſmelled offenſively. Laſtly, a ſevere Purging (ſometimes Bloody) which terminated in Death. That ſome died in three Days, and others in five or ſix, but the Bulls lived eight or ten. That during their whole illneſs, they refuſed all manner of Food, and were very hot.

We then adviſed with ſeveral of the *Cow-leeches*, or Doctors, who all agreed that it was a Murrain, or rather a Plague; and that the Methods they had tryed for a Cure, had proved unſucceſsful. This Diſeaſe was ſo ſurpriſing, that ſome of thoſe Men who uſed to look after them, were afraid to go near them.

We then ordered ſome of the ſick Cows to be Houſed, and ſeveral ſorts of Cattle to be kept with them, to ſee whether the Contagion would affect any other Species.

The next Day I made a Verbal Report to their Excellencies, of all the ſeveral Opinions and Diſcourſes which I have had about it, and left them debating what Method to take; at laſt I was called in, and Ordered to conſider of it again the next Day, and to deliver to them in Writing what would be proper to be done. Accordingly I drew up, and gave them the following Propoſals.

I. That all ſuch Cows as are now in the poſſeſſion of Mr. *Ratcliff*, *Rufford*, and *Pullen*, be Bought, Kill'd, and Burnt: or, at leaſt, that the Sick be Burnt; and the Well kept and ſecured on the Grounds where they now are, that ſuch of them as Sicken or Dye of this Diſtemper may be Burnt.

 II. That

II. That the Houſes in which thoſe Sick Cows have ſtood be Waſhed very clean, and then ſmoaked by the burning of Pitch, Tarr and Wormwood, and be kept three Months at leaſt before any other Cows are put therein.

III. That the Fields where thoſe Sick Cows have Grazed, be kept Two Months before any other Cows are ſuffered to ſtand or Graze thereon.

IV. That the Perſons looking after ſuch as are Ill, ſhou'd have no Communication with thoſe that are well.

V. That the ſame Methods be Obſerved if any other of the Cow-keepers ſhou'd get this Diſtemper among them; and that they be all Summoned and told, that as ſoon as they perceive any of their Cows to refuſe their Meat, or have any other Symptoms of this Diſtemper, that they immediately ſeparate them from their others, and give notice to ſuch Perſons as your Excellencies ſhall appoint, that they may be Burnt; and the places where they have ſtood or Grazed to be ordered as before.

VI. That the Cow-keepers be required to divide their Cows into ſmall Parcels, not more than ten or twelve in a Field together; and that they be allowed ſuch ſatisfaction for complying with theſe Propoſals, as your Excellencies ſhall think fit; all which is moſt humbly ſubmitted, *&c.*

The next day their Excellencies conſulted the four Gentlemen before-named, and gave them Orders to comply with the preceeding Propoſals, and to allow *Forty Shillings* for every Sick Cow which they Burnt, that belonged to Mr. *Ratcliff*, *Rufford*, and *Pullen*; but the free intercourſe which both Maſters and Servants had had with each others Cows (before we were appointed) had

had ſpread the Contagion; and the Diſeaſe began ſoon to appear in ſeveral other Neighbouring places.

The Gentlemen then ſummoned all the Cow-keepers in the County, and acquainted them with the above-named Propoſals (to moſt of which they readily Complyed, as being viſibly their intereſt) and offered them *Forty Shillings* for every Cow which they Burnt, that had not been Sick above twenty-four Hours; but for ſuch as had been longer Ill, or were Dead, they wou'd allow them only the value of their Skins and Horns.

Some of the Cow-keepers appeared not content with this Regulation, and believing that the Diſeaſe wou'd become general, deſign'd to have ſold their Cows at ſome diſtant Market; which the Gentlemen having notice off, appointed ſeveral Butchers to Watch near their Grounds, and count their Numbers every Morning, with Orders to follow ſuch as they ſent to any Market, and prevent their being ſold, by telling the people what they were.

Another great Obſtacle at the firſt was the Cow-keepers not owning the Diſeaſe, till they had loſt ſeveral of their Cows; for ſo ſoon as it was known that any Man had but one Sick, none wou'd buy his Milk; and to thoſe who kept many Cows, that loſs was conſiderable.

Nor was there ever wanting one or other who gave them hopes of a Cure.

To obviate theſe three difficulties, the Gentlemen encouraged them to hope for a Brief, but aſſured them that ſuch only as complyed with theſe Directions, ſhou'd have any benefit by it. Accordingly they ordered a daily account to be taken of the Conduct of each Cow-keeper, and allowed or diſallowed their pretenſions to this Brief, as well as to the *Forty Shillings per* Cow, as they complyed or diſregarded theſe Directions.

This

This had a pretty good effect; but here in *England*, where every Man is at liberty to diſpoſe of his Cattle as he pleaſes, nothing but making them ſenſible that it was each Mans particular intereſt to comply with theſe Methods cou'd do; this, tho' true in fact, yet the Reader will readily judge to be very difficult among ſuch a Number; but the Gentlemen ſpared no labour to accompliſh it; for that purpoſe they ſummoned them once or twice every Week, urged all that cou'd be ſaid to induce their Complyance, and omitted no warrantable means to fruſtrate their Folly.

I had Orders from the beginning to aſſiſt thoſe Gentlemen with my Advice, which I did at moſt of their Meetings; as alſo to make a ſtricter enquiry into the Diſeaſe by Diſſections, *&c.*

Accordingly I diſcourſed the *Cow-leeches* about the Cuſtoms and Diſeaſes that Cows were ſubject to, and conſulted ſuch Books as treated of them; but concerning this Diſeaſe, I cou'd gain but ſmall aſſiſtance from either.

I then made Diſſections of ſixteen Cows, in different degrees of Infection; and found the Putrefaction of their *Viſcera* to encreaſe, in proportion to the time of their Illneſs.

The firſt five that I opened, had hearded with thoſe that were Ill, and the Symptoms of this Diſtemper were juſt become viſible; in theſe, the Gall-bladders were larger than uſual, and filled with Bile of a natural Taſte and Smell, but of a greener Colour. Their *Pancreas*'s were ſhrivelled, ſome of the Glands obſtructed and tumified. Many of the Glands in their *Meſenterys* were twice or thrice their natural bigneſs. Their Lungs were a little inflamed, and their Fleſh felt hot. All other parts of their *Viſcera* appeared as in a healthful State.

The

The next ſix that I opened, had been ill about two Days; in them the Livers were blacker than uſual, and in two of them, there was ſeveral Cyſts filled with a Petrified Subſtance like Chalk, about the bigneſs of a Pea. Their Gall-bladders were twice their uſual bigneſs, and filled with Bile of a natural Taſte and Smell, but of a greener Colour than the firſt. Their *Pancreas*'s were ſhrivelled, ſome of their Glands very large and hard, and of a blackiſh Colour. The Glands in their *Meſenterys* were many of them five times their natural Bigneſs, and of a blackiſh Colour. Their Lungs were inflamed, with ſeveral ſmall Cyſts forming. Their Inteſtins were full of red and black Spots. Their Fleſh was very hot, tho' not altered in Colour.

The five laſt that I opened, were very near dying; in them I found the Liver to be Blackiſh, much Shrivelled and Contracted, and in three of them, there was ſeveral Cyſts as big as Nuts or Nutmegs, filled with a Petrified Subſtance like Chalk. Their Gall-bladders were about three times their uſual bigneſs, and filled with Bile of a natural Taſte and Smell, but of a deep Green Colour. Their *Pancreas*'s were Shrivelled and Contracted, many of their Glands very large and hard, and of a black Colour. The Glands in their *Meſenterys* were many of them diſtended to eight or ten times their natural bigneſs, were very Black, and in the *Pelvis* of moſt of thoſe Glands in two Cows, there was a yellow Petrefaction, of the conſiſtence of a ſandy Stone. Their Inteſtines were the Colour of a Snake, their inner Coat excoriated by Purging. Their Lungs were much Inflamed, with ſeveral Cyſts containing a yellow Purulent Matter, many of them as big as a Nutmeg. Their Fleſh was extream hot, tho' very little altered in Colour.

I have

"I have here only given you a general account of my Diſſections, in the three different Stages of the Diſeaſe; for as the difference was but ſmall, and the Diſeaſe incurable, it could neither be uſeful nor pleaſant to the Reader, to have each particular Diſſection at large, tho' I have now the Minutes by me. But the following Caſes being very extraordinary, I cou'd not omit the mention of them, *viz.* In one of them the Bile was Petrified in its Veſſels, and reſembled a Tree of Corral, but of a dark yellow Colour, and brittle Subſtance.

In another there were ſeveral Inflammations on the Liver, ſome as large as a half Crown, cracked round the Edges, and appeared ſeparating from the ſound part, like a Peſtilential Carbuncle.

In a Third, the Liquor contained in the *Pericardium* (for Lubricating the Heart in its Motion) appeared like the ſubſidings of *Aqua Calcis*; and had excoriated, and given as yellow a Colour to the whole Surface of the Heart and *Pericardium*, as *Aqua Calcis* cou'd poſſibly have done.

In giving my Opinion of this Diſtemper, I muſt beg leave to premiſe, that all Cows have naturally a Purgation by the *Anus* for five or ſix Weeks in the Spring, from (as the Cow-keepers term it) the frimneſs of the Graſs; during which time they are brisk and lively, their Milk becomes thinner, and of a blewiſh Colour, ſweeter to the Taſte; and in greater Plenty: but the Spring preceeding this Diſtemper, was all over *Europe* ſo dry, that the like has not been known in the Memory of any one living; the conſequence of which was little Graſs, and that ſo dry and void of that frimneſs which it has in other Years, that I could not hear of one Cow keeper, who had obſerved his Cows to have that Purgation in the ſame degree as uſual; and very few who had obſerved any at all. They all agreed

that

that their Cows had not given above half ſo much Milk that Summer as they did in others; that ſome of them were almoſt dry; that the Milk they did give was much thicker, and yellower than in other Years. It was obſerved by the whole Town, that very little of the Milk then ſold wou'd Boyl without turning; and it is a known Truth, that the weakeſt of the common Purges you can give a Cow entirely takes away her Milk; from all which Circumſtances, I think it evident, that the want of that natural Purgation was the ſole cauſe of this Diſeaſe; by producing thoſe Obſtructions, which terminated in a Putrifaction and made this Diſtemper Contagious.

During my daily Converſation at that time with Cowkeepers, *&c.* there occurred many other Circumſtances of leſs Moment, to confirm me in this Opinion: but as there was no one reaſon to give me the leaſt notion of any other Cauſe, I ſhall not trouble the Reader with a uſeleſs detail of them.

Cows are likewiſe ſubject to a Purgation (tho' in a leſs degree) from the ſame quality in the Graſs, about the latter end of *September*; which is called the latter Spring; and which I believe contributed not a little, to the preventing the encreaſe of this Diſtemper; for this Purgation coming ſo ſoon after the Diſeaſe appeared, it is not unreaſonable to ſuppoſe, that it freed ſuch Cows as were not much injured, from the ill effects of thoſe Obſtructions, occaſioned by the want of their Vernal Evacuations.

Several Phyſitians attempted the Cure, and made many Eſſays for that purpoſe; but the Diſſections convinced me of the improbability of their ſucceeding, with which I acquainted their Excellencies. However they having received the following *Recipe* and Directions from ſome in *Holland*, ſaid to have been uſed

there with good ſucceſs, gave me Orders to make tryal of it: But the effect was anſwerable to my expectation, for in very many Inſtances, I was not ſenſible of the leaſt Benefit.

Herb. *Ariſtoloch. Rotundæ,*
Veronicæ, ãa M.viij
Pulmonariæ,
Hyſſopi,
Scordij, ãa M. 4
Rad. *Gentianæ,*
Angelicæ,
Petaſitidis,
Tormentillæ,
Carlinæ, ãa ℔ ſs.
Bacc. *Lauri,*
Juniperi, ãa ℥xij *Miſce fiat Pulv.*

See *Phil. Tranſact.* N°. 338. *in fine.*

This Powder is to be given in Water, one Ounce at a time, three or four Mornings ſucceſſively; then reſt four Days, and if the Diſeaſe continues, repeat the Powders in warm Water, as before.

I think there is no one Method in Practice, but what was tryed on this Occaſion, tho' I cannot ſay that any of them was attended with an appearance of Succeſs; except that of Bleeding plentifully, and giving great quantities of Cooling and Diluting Liquids. But by this Method, the inſtances of Succeſs were ſo few, that they do not deſerve any further mention.

Their Excellencies being informed that the feeding Cows with Diſtillers Grains was a new Cuſtom, and was the cauſe of this Diſeaſe, gave me Orders to examine into the Truth of it; but upon enquiry, I found it to have been the Practice of ſeveral of the Cow-keepers above twenty Years, without the leaſt appearance of any

any inconvenience; and that ſome of thoſe Perſons who had ſuffered moſt, had never given any. Nor is there any difference between thoſe of Brewers and Diſtillers, only that the latter are the dryer.

It was likewiſe ſaid, that the want of Water was the cauſe of this Diſeaſe, for that the Springs and places where People uſed to Water their Cowes, were almoſt every where dry; and that many were obliged to ſend them ſeveral Miles for Water. This might produce ſome Diſeaſes, but ſuch only as they got by the fatigue of being driven ſo far; for Mr. *Ratcliff*, Mr. *Rufford* and Mr. *Pullen*, the three Perſons where this Diſeaſe firſt appeared, had the New River Water running thro' the very Grounds where their Cows conſtantly Grazed, and cou'd drink at their Pleaſure, and ſo had moſt of the Cow-keepers at *Iſlington*.

There were at that time ſeveral other reports of the cauſe of this Diſeaſe, but none that had a ſhew of Reaſon.

About the latter end of *September*, the Diſeaſe increaſed, and the Numbers brought to be burnt were ſo great, that it cou'd not be well executed; therefore it was judged proper only to bury them fifteen or twenty Foot deep: but firſt to make large Inciſions in their moſt fleſhy parts, and to cover them with quicklime.

At the ſame time, having notice that it was a Cuſtom with the Cow-keepers, to ſend their Calves when a Week old to *Rumford*, *&c.* to be Sold; and apprehending by this means that the Contagion might be carried into the Country, I required all ſuch as had Sick Cows, to bring their Calves to be buried; to which they readily conſented, and were allowed from *Five* to *Ten Shillings per* Calf.

In the beginning of *October*, being informed that ſome of the Cows in *Norfolk*, *Suffolk*, and *Hertfordſhire*, had

got this Diſeaſe, and apprehending that it wou'd become general: I gave in the following Report to a Committee of Council.

The Diſtemper among the Cattle encreaſing, and beginning to appear in ſeveral other Counties, I thought it my Duty to acquaint your Lordſhips, with the hazard that may attend their not being duely buried. It is the Opinion of all Authors in Phyſick that treat of Contagious Diſeaſes, as well as of ſeveral of the Phyſitians in Town, that a Putrifaction of ſo many Cows as there is reaſon to fear will dye of this Diſtemper, may produce ſome Contagious Diſeaſe among Men; unleſs they are buried ſo deep that the Infectious *Effluvia* cannot injure the Air, which I am certain has very ſeldom been complyed with, except in the Counties of *Middleſex*, *Eſſex*, and *Surry*, the Gentlemen employed being capable of acting in thoſe Counties only. It is affirmed by ſeveral now living, that there was a Mortality among the Cattle, a little before the laſt great Plague in the Year 1665, which was imputed to the want of a due Care in burying them. And your Lordſhips may know of what importance it was judged by the King of *Pruſſia*, the States of *Holland*, and ſeveral other Princes and States, by the Care they took to publiſh Decrees and Placarts, commanding them to be buried upon pain of Death, or other ſevere penalties; and I humbly conceive it wou'd be neceſſary, not only to bury thoſe which ſhall Dye, but that ſuch as are already Dead may have the ſame Care; as alſo that they be buried nine or ten Foot deep at leaſt. All which is moſt humbly ſubmitted, *&c.*

Their Lordſhips thought fit to defer all proceeding upon this Report, till the Diſtemper becoming more general ſhou'd make it Neceſſary; but I thank God that Neceſſity never happened, for within three Weeks

or

or a Month after the giving in of that Report, the following particulars concurred to put an end to the Diſeaſe.

The Cows began their latter Purging, which contributed much to prevent the Diſeaſe from appearing in freſh Places; and the Cow-keepers were convinced that the Diſeaſe was incurable.

The knowledge of the Diſeaſe was ſpread all over *England*, ſo that none wou'd buy a Cow in the Country; and the Gentlemen prevented their being kill'd in Town, by having the Markets examined daily; and ſuch Meat condemned as appeared Suſpicious.

They now divided their Cows into ſmall Parcels, by which they loſt only that in which the Diſeaſe happened; whereas before that Method, when one Cow got this Diſeaſe, if ſhe had herded with One, Two, or Three Hundred (the Contagion was ſuch) ſcarce one did eſcape.

Thoſe who had no Sick Cows avoided all Communication with ſuch as had.

They likewiſe found that the keeping their Cows ſo long when Ill, had been the chief Cauſe of their Loſs; they therefore now brought them to be Buried on the firſt appearance of the Diſeaſe, before the Contagion cou'd poſſibly have got to any great height.

Theſe were the effects of the Cow-keepers dear bought Experience; but it was the indefatigable Care and Diligence of thoſe four Gentlemen, who gave a daily Attendance, both early and late, that ſecured *Great Britain* from that terrible Ravage, which was made by this Diſtemper in ſeveral parts of *Europe*.

The ſeverity of this Diſeaſe in *England* did not laſt above three Months; tho it was not entirely ſuppreſſed till about *Chriſtmas*: But in ſeveral other Countries it continued two or three Years; and I am credibly aſ-

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ſured,

ſured, that in *Holland* it now rages with as much violence as ever; and that they have loſt in Cows, Oxen and Bulls, above Three Hundred Thouſand.

The Providence of God has ſo diſpoſed the matter of Animal Bodies, as to render Contagious Diſeaſes very ſeldom infectious to different Species; but Experience demonſtrates, that Contagions may be communicated to the ſame Species, by touching the Woolen, Linnen, *&c.* to which the Infectious *Effluvia* of the Diſeaſed had adhered, tho' the two Bodies ſhould be at a very great diſtance; and I verily believe that more Hundreds died from the Infection, which was carried by the Intercourſe that the Cow-keepers had with each other, than ſingle ones by the original Putrifaction.

The Nature of Contagious Diſeaſes are but little underſtood, and it would neither be agreeable to my Deſign, nor uſeful to the Publick, to ſay more of this than what was evident: But I have been particularly careful, not to omit any thing Material, either for deſcribing the Diſeaſe, or manifeſting the Methods that were taken for ſuppreſſing it; becauſe it is more than probable that the ſame Care wou'd be equally ſucceſsful in any other Species of Cattle.

The number of Bulls and Cows loſt by this Diſeaſe, in the Counties of *Middleſex*, *Eſſex* and *Surry*, were Five Thouſand Four Hundred and Eighteen; and of Calves, Four Hundred and Thirty Nine; and the Money iſſued for them, at *Forty* or *Ten Shillings per* Cow, *&c.* was the Royal Bounty of his Majeſty, from his own Civil Liſt: and tho' neither the four Gentlemen, nor I, made any demand for a Reward, or for Expences, yet it amounted to 6774 *l.* 1 *s.* 1 *d.* But the entire loſs to the Cow-keepers, as delivered in upon Oath, was 24500 *l.* (excluſive of the 6774 *l.* 1 *s.* 1 *d.*) tho' computed but at *Six Pounds per* Cow; which at a Medium, was

was not more than their Prime Coſt; the dearneſs of keeping them near *London* neceſſitating the Cow keepers to buy the very beſt.

His Majeſty was further pleaſed, on the Sollicitation of the four Gentlemen, to grant a Brief fot the 24500 *l.* but the many falſe Reports that were then induſtriouſly propagated, to leſſen the value of thoſe poor Mens loſſes, ſo fruſtrated that Charity, that the entire ſum Collected (the charges of Collecting being firſt paid) was but 6278 *l.* 2 *s.* 6 *d.* which on a Dividend, amounted to *Five Shillings* and *Three Half Pence* in the Pound, computing their Loſs as above, at *Six Pounds per* Cow; tho' if we conſider their Contracts with Brewers for Grains, their Rent of Grounds which lay uſeleſs, Servants wages, *&c.* their real Loſs may (by a modeſt Computation) be allowed to be *Ten Pounds* for every Cow that died.

IV. *A Deſcription of the* Organ *of* Hearing *in the* Elephant, *with the Figures and Situation of the* Oſſicles, Labyrinth *and* Cochlea *in the Ear of that large Animal. Communicated to the* Royal Society, *by Dr.* Patrick Blair, *R. S. S.*

IN the Deſcription I formerly wrote to the Honour'd Sir *Hans Sloane*, Bart. of the *Elephant* I Diſſected in *Dundee*, *Anno* 1706. which he was pleaſed to Communicate to the *Royal Society*, as you have it in *Philoſ. Tranſ.* N°. 226. 227. I treated of the Bony part of the Ear of that prodigious Animal a little too ſuperficially; becauſe I was unwilling at that time to break up the *Os Petroſum* of the right Ear, which had accidentally been

been ſeparated on dividing of the Scull, by which the account I then gave of the *Lineæ Semilunares*, or *Labyrinth* and *Cochlea* was but Lame. But I have choſen ſince rather to deſtroy that Bone (however ſeldom ſuch Bones are to be met with) than that the Publick ſhould be depriv'd of an exact Deſcription of that curious Organe, and that I may give a clear Idea of all its Bony parts, I ſhall repeat what I formerly advanc'd upon that Subject, and add what Improvements I have made upon it ſince.

Before I proceed, 'tis fit I obſerve that the *Auris Externus* of this big Creature lyes flat, and not Protuberent as in other Quadrupeds, whoſe Cartilaginous Subſtance is capable of divers Motions perform'd by ſeveral Muſcles, whereby the inner Ear is preſerv'd from the great violence of the External Air, which upon ſome occaſions might perhaps injure or break the thin and delicate Membrane of the *Tympanum*. It is alſo for this reaſon that the *Meatus* is further guarded, by the Contorſions and oblique Poſition of the Cartilage at the Orifice of the *Meatus*, which only admits of a determinate quantity of Air, ſufficient for the vibration of the *Membrana Tympani*, by which a diſtinct ſound is convey'd to the *Senſorium commune*; whereas did the Air admitted exceed its due proportion, nothing but the confus'd Idea of a Sound would follow, ſuch as reſemble the ruſhing of Waters, *&c.* or that noiſe often obſerv'd when, by a ſupervenient Cold or the like, obſtructions are generated within the Ear it ſelf. And in Man, becauſe the *Auris externus* is alſo flat, not only are theſe turnings and windings obſervable in the Cartilage at the entry, but the *Meatus* it ſelf is likewiſe obliquely Situated, to prevent the aforeſaid Inconveniencies. But there is no need for ſuch a contrivance in the *Elephant*, whoſe external Orifice of

the *Meatus* is patulent, open (ſcarſe being guarded by the Cartilage) and ſtreight, whoſe length (it reaching from the external to the internal Table of the Scull) is ſufficient to prevent the acceſſion of too great a quantity of Air to the *Tympanum*; for in its progreſs moſt of the *Columnæ Aeris* beat againſt one or other of the ſides of the *Meatus*, in ſo much that their force is inhibited, and only ſo many as ſuffice to convey the Sound, can reach the *Tympanum* it ſelf.

Meatus Auditorius.

The *Meatus Auditorius* then is a long ſtreight Tube or *Canule* ſituated Horizontally, and reaching from the outer to the inner Table of the Scull in Figure not unlike the Barrel of a Piſtol, but ſomewhat Oval, the ſides of whoſe Cavity are hard and ſolid, about the thickneſs of a Halfpenny, from whoſe outer Part ſeveral of the *Laminæ* betwixt the two Tables of the Scull do ariſe, (*Fig.* I.) Its Cavity is an Inch or $\frac{3}{4}$ of an Inch Diameter, and length $9\frac{1}{2}$ Inches; being ſomewhat enlarg'd as it arrives at the *Crena* for the *Membrana Tympani.* (*Fig.* II.)

Crena and *Cavitas Tympani.*

This *Crena* is two Inches in Circumference, within which is the *Cavitas Tympani*, conſiſting of two different Surfaces; the one much deeper and Cellulous, the other more ſuperficial and Smooth. The firſt runs perpendicularly down $\frac{1}{2}$ Inch from the *Crena Tympani.* Its bottom is variouſly divided into ſeveral *Celluls*, not unlike a Hony Comb, but irregularly diſpos'd. Its Bony *Laminæ*, by which theſe Cellules are diſtinguiſh'd from each other, are thicker at the Top than at the Bottom, they being one Line, two Lines, or $1\frac{1}{2}$ Line diſtant from each other, and about $\frac{1}{4}$ Inch deep. Could I have got it ſo well cleans'd as I wiſh'd for, doubtleſs I might have obſerv'd their Communication with each other, by means of certain Orifices which ſerve to convey what ſuper-

fluous Moiſture is contain'd in them, for we may reaſonably ſuppoſe, as in all other Cavities of the Body, there are certain Glands for ſeparating proper Liquors convenient for the uſes deſign'd; ſo here there ſeems to be a neceſſity for ſeparating a certain quantity of Moiſture, fit to lubrifie the Muſcles of the Oſſicles, and facilitate their Motion; as alſo to preſerve the *Membrana Tympani* from becoming too dry. This dryneſs of the *Membrana Tympani*, and the thickneſs of the Liquor ſeparated by theſe Glands, is often the cauſe of a Deafneſs in Human Subjects; eſpecially thoſe that are advanc'd in Age. This cellulous Structure of the *Cavitas Tympani*, ſeems to be very proper for receiving of the ſuperfluous Humidity; and theſe Communications are requiſite for conveying it from one Cellule to another, till it is emptyed into the *Receptaculum Commune* the *Aqueduct*, whereof hereafter.

This firſt or cellulous Cavity is two Inches broad, and reaches from the *Crena Tympani* to the *foramen Ovale*, or entry into the *Veſtibulum*, which is ſhut by the *Stapes*. The ſecond Part of this Cavity is more ſuperficial (*Fig.* II. (*e*), in form not unlike a Pear, from a narrow beginning becoming broader and more ſuperficial, terminating Semicircularly, ſmooth in the Bottom, and having ſeveral incurvated Lines running acroſs it; it reaches much farther than the *Veſtibulum*, being one Inch five Lines from before to behind, and one Inch tranſverſly where broadeſt. What ſuperfluous Moiſture it contains is diſcharg'd into the fore-named Aqueduct.

Beſide the above-mentioned uſes for theſe two Cavities, *viz.* to receive and diſcharge the ſuperfluous Moiſture; they are alſo moſt beneficial and aſſiſting to the Hearing: for, no ſooner is the external Air modulated, and the *Membrana Tympani* mov'd thereby, than

than the Sound is conveyed by the *Officles* to the *Nervus Auditorius*, and the Undulation continued, firſt by the *Anfractuoſities* of the firſt Cavity, and then by the *Gyres* and incurvated Lines of the ſecond, ſo that we may eaſily account for the acute Senſation of Hearing, wherewith *Elephants* are ſaid to be endow'd: For as the Tame ones are moſt exact in obeying their Maſters commands; ſo the wild Ones are ſoon aware of what Traps or Snares are laid to catch them, by the tremulous Motion convey'd to their Ear from the Cavous parts of the Earth, where the Pit into which it is expected they ſhould fall, is digg'd. It is eaſy therefore to explain whence the acuteneſs of the Senſation of this Animal may proceed; for as the *Nervus Olfactorius* has a large Space and Bounds wherein to be diſpers'd, *viz.* the two Cavities of the *Proboſcis*, which are both long and large, ſo that ſcarce any *Columna aeris* can enter them, but ſome one or another of the Filaments of the *Nervus Olfactorius* diſpers'd in theſe Cavities muſt be toucht, whereby the Idea of ſmelling muſt be conveyed to the *Senſorium commune* in a more intenſe Degree, and the Animal ſoon become ſenſible of whatever approaches that is noxious or nauſeous to it, and thereby is taught how to avoid it; ſo this Structure, for a quick conveyance and long continuance of the Sound, is a great means both to make the *Elephant* ſoon receive the Sound and have a deep impreſſion of it.

The Aqueduct.

The *Aqueduct* is a flat Tube or Pipe, whoſe Orifice is ſo ſituated betwixt the two forementioned Cavities, that if there be any ſuperfluous Humidity contain'd in them, it muſt needs be diſcharg'd (at leaſt in this Animal) into the Mouth; for as it is ſituated where the firſt Cavity terminates, ſo the ſecond, from a broader and more ſuperficial be-

ginning muſt needs diſcharge its Moiſture, by its more narrow and deeper termination, into this receptacle; alſo it deſcends directly towards the Mouth, paſſing through the Scull below the hole for the Jugular Vein (*mm*) betwixt the hole for the *Carotid* Artery, (*p p*) and that for the *Arteria duræ matris* (*q q*) whence deſcending (*n n*) it is joyn'd with its Fleſhy part, which diſcharges it ſelf into the Mouth on each ſide, behind the back part of the inner Teeth of the upper Jaw. This ſituation of the Aqueduct makes it plainly appear, that its Uſe is to receive the ſuperfluous Moiſture from the *Cavitas Tympani*; for beſide the Glands above-mentioned, fit for ſeparating ſuch a quantity of Humidity as may lubrifie the Muſcles, and facilitate both their Motion and that of the *Offcicles*; the very Vapours that ariſe in ſuch a Cavity as that of the *Tympanum* in this Animal, muſt at laſt be converted into a Liquor, and that muſt either again be receiv'd into the Blood Veſſels, or otherwiſe diſcharg'd by ſuch a Receptacle as this. Further if there be a neceſſity for Glands in the *Meatus Auditorius* without the *Tympanum*, to ſeparate a certain Liquor, by which the acrimonious Particles of the Air are obtunded, and hindred from being offenſive to the Nervous Membrane of the *Tympanum*, (which muſt be of a moſt acute Senſation) and for moiſtning it, by which it the more eaſily receives the Vibration of the Air; ſo ſuch Glands as theſe ſeem to be moſt requiſite in the *Cavitas Tympani* for the Uſes above nam'd. And ſince what ſuperabounds of this Moiſture, cannot be diſcharg'd outwardly as that of the *Meatus*, this Aqueduct ſeems to be moſt convenient for that purpoſe. Some are of opinion that this Aqueduct is alſo aſſiſting to the Hearing, eſpecially in Men; becauſe it is generally obſerv'd

Oſteographia Elephantina, or *Philoſ. Tranſact.* No. 327. Tab. 3. Fig. 3.

that

that they who are Deaf, open their Mouths wide, when they are desirous to hear more distinctly: But I see not how that can be, for tho' the Cavity of the Bony part of the Aqueduct, in most of Animals, is proportionally large enough; yet its carnous or fleshy Part lyes for the most part so flat, and its two sides are so collaps'd together, that scarce any Air can be admitted, at least so far as to be subservient to the Hearing.

The four Bones of the Ear.

The *Ossicles* in this as in other Animals are three or rather four in number; for though I did not procure the *Os quadrangulare* of *Du Verney*, yet I have good reason to believe it was there; because there is a conspicuous *Sinus* in the extremity both of the *Incus* and *Stapes*, where they are articulated, so big as to contain the Head of an ordinary Pin; and when I consider the Angle which must have been form'd by the articulation of these two Bones, I look upon this small Bone to serve for the same purposes as the *Patella* in the Knee, and *Sesamoide* Bones in the Fingers and Toes.

The Malleolus.

The *Malleolus* is an irregular Bone, and doubtless has been endow'd with pretty large Muscles, because of the rugosities, protuberances and *Sinus*'s observable in it. It has a protuberant Head (*Fig.* IV. (1) four Lines broad, next to that a *Crena* or semicircular *Sinus*, (2) after which the Bone is rais'd, affording a protuberant Margin to an oblong *Sinus* (3) for receiving the head of the *Incus*, four Lines broad. The opposite part of this *Sinus*, or back part of the Bone, is convex of an unequal rugous Surface, with a great many protuberances and depressions, for the Origins and insertions of the Muscles, for the space of five Lines; where it forms an Angle, from whence it becomes Flat and Smooth, being three Lines broad and reaching four

Lines

Lines to another Angle, (5.) where the *Manubrium Malleoli* begins, and where it becomes more round; from whence it gradually Tapers to the Point, being ſix Lines in length.

The head of the *Incus* is four Lines broad, (*Fig.* VI. (1.) below which is the Neck or an oblique *Sinus*; (2) next to that are two *Apophyſes*, one on each ſide. Theſe deſcending obliquely outwards, and becoming flat, meet in a Point, (*Fig.* VII. (5.) whence aſcending obliquely inward, this Production is join'd to another ſmall round one, like the *Manubrium Malleoli* $4\frac{1}{2}$ Lines long (6.) This has the fore-mentioned ſmall excavation or half round *Sinus*, (7.) which with the extremity of the *Stapes*, I ſuppoſe to have contain'd the *Os quadrangulare*, or rather *Orbiculare*, according to the Figure of the *Sinus*.

The *Stapes* differs much in Figure from the Human one. From its Concave extremity 'tis enlarg'd on each ſide by two ſmall ſlender Productions, not unlike the Proceſſes of the *Vertebræ* of ſome Fiſhes (*Fig.* VI. (2 2) to which is join'd the *Baſis*, (3.) ſo thin almoſt as the Scale of a Fiſh. This was accidentally ſeparated from its two Sides, and remain'd in the *Foramen Ovale*, from whence I pull'd it with a Pin; 'Tis Concave towards the *Stapes*, and Convex toward the *Veſtibulum*.

The *Foramen Ovale* lyes ſo hid and obliquely in the ſide of the *Cavitas Tympani*, that it could not be delineated in its true Dimenſions. Near to it is another hole Oblong and Sharp at both ends, both which give an entry into the *Veſtibulum*.

The *Veſtibulum* is of an irregular Figure, (*Fig.* X. (*a*) 'tis for the moſt part three Lines from the one ſide to the other, and perforated by eight Orifices, *viz.* five for the Canals of the *Labyrinth*, (*Fig.* IX. X. (*a*)

one

one for the *Cochlea*, (*Fig.* X. (*h*) and two for the *Feneſtræ* (*b*, *c*.)

The *Cochlea* is a long Cavity conſiſting of three *Gyres* or *Meanders*; (*Fig.* XI. (*d e f*) Its Orifice where it proceeds from the *Veſtibulum* is but ſmall; but it afterwards widens, ſo that the firſt courſe of this Cavity is a third part larger than the ſecond (*e*), and proportionally the third is leſs than the other two (*f*), till it terminates in an Orifice (*g*) ſituated in the Top, for receiving a branch of the ſoft portion of the *Nervus Auditorius*, which accompanies and paſſes along all its *Gyres*.

The hardneſs and ſolidity of the Bone (for which it may be juſtly called *Os Petroſum* in this Subject) was ſuch that I could not ſo exactly trace the three Canals or Ducts of the *Labyrinth*, ſo as to give a true Idea of the manner of their ſeveral Turnings. But *Valſalva*'s Figures of the Humane Ear directed me ſo exactly, that I eaſily found out the ſeveral Orifices, and opened them ſo far as to find out their ſituation and true Dimenſions, by introducing a Hogs briſtle, then cutting it off and ſtretching it out to the Scale. Thus after laying open the two *Foramina* which gave an inlet to the *Veſtibulum*, I ſoon perceiv'd the ſeveral Orifices which in ſo large a Subject were pretty conſpicuous. I firſt turn'd to the one hand and diſcovered the *Duct* of the *Cochlea*; this I purſued all along the Protuberance, (*Fig.* III. (*d*) in doing of which I laid wholly open the *Leſſer Duct* of the *Labyrinth*. (*Fig.* IX. X. (*d*) Then turning up the other ſide of the Bone, I trac'd the ſoft Portion of the *Nervus Auditorius* divided into two Branches, one whereof was diſtributed into the *Cochlea*, and the other to the *Labyrinth*. In filing the Bone a little further, I opened a ſmall part of the *Middle Duct*, and in a ſhort time I diſcovered the *Ductus Major*;

Major; after which I meaſured their ſeveral lengths as is ſaid.

Labyrinth. The *Labyrinth* then conſiſts of three *Lineæ Semilunares* or incurvated *Ducts*, whereof the *Major* lyes in that part of the *Proceſſus Petroſus* which regards the ſeat of the Brain (*b*) This is twenty Lines or one Inch eight Lines long. The *Medius Ductus*, one part whereof regards the Orifice of the *Cochlea*, and the other is common with the *Major* for the ſpace of three Lines; (*e*) this is fifteen Lines or one Inch three Lines long: And the *Minor* which regards the *Cavitas Tympani*, has one Orifice which is near to the *Medius*, were it approaches the *Cochlea*; and the other near to the Orifice of the *Major*. This is one Inch long.

The ſeventh pair of Nerves called in general the *Nervus Auditorius*, enters the *Proceſſus Petroſus*, and is divided into the hard and ſoft Portions, as in other Animals. In this Subject I find one Canule entring the Bone from the ſides of the Orifice for the *Carotide* Artery, about three Lines diameter, (*e*) (*h*) from thence running forward for the ſpace of one Inch four Lines, then bending downwards one Inch, till it meets with the Orifice at the Sides of the *Meatus Auditorius*, by which it pierces the Scull, and paſſes outward. This Canule, after it is entred the *Proceſſus Petroſus* for the ſpace of eight Lines, communicates with the Orifice which uſually enters the foreſaid Proceſs from the Baſe of the Scull; and both theſe Orifices, after they have accompany'd one another about five Lines, are ſeparated, and the ſoft Portion penetrates the Bone at two places, as is ſaid.

Oſteog. Eleph. Tab. 3 Fig. III. Fig. IV.

I have now endeavoured to give ſuch a Deſcription of the *Offeous* or Bony part of the Ear of this ſtupendious Animal, as I am in hopes may be uſeful for the

clearing

clearing up of ſome *Phænomena* in leſſer Subjects. At leaſt we may hereby obſerve, what a variety of Mechaniſm the great Author of Nature has thought fit to employ, in the ſeveral parts of different *Species* of Animals. Thus both the external Ear of Man, and of the *Elephant* lye flat, as being moſt convenient: for if they had been Protuberant as in moſt Quadrupeds, how unſuitable would it have been in Man, who is the moſt perfect of all Creatures, not upon the account of his Reaſon alone, but alſo as he is a Pattern for Beauty and the Symmetry of his Parts; and how unſeemly would it have been in the *Elephant*, if his external Ear had ſtuck out, and been proportional to his other Parts; conſidering what an extraordinary aſpect he makes already by his Trunk and Tusks? But the Ears in theſe two Subjects differ by the tortuoſity of the Cartilage, and oblique *Meatus*, to prevent the injury of the Air, by its immediate acceſs into the inner Ear in Man: whereas in the *Elephant* the external Orifice is fully expos'd to the Air; but then the length of the *Meatus* hinders any more Air than is convenient from arriving at the *Tympanum*. We likewiſe ſee in the *Seal* and *Otter*, that thoſe two Amphibious Quadrupeds have no external Ear further protuberant than the other Parts of their Head; for had it been otherwiſe, their ſwimming and diving would have been much hindred: But its two ſides are ſo collaps'd, that no Water can enter in when in the Deep, though it can receive ſufficient Air when aſhoar. The cellulous Cavity of the *Tympanum* in the *Elephant*, may well be compar'd to the *Apophyſis Maſtoides* in Man; and the ſecond Cavity of a plain Surface ſeems to be Analogous to the Cavous *Maſtoides* in *Sheep*, *Cats*, *Dogs*, *&c.* So that we ſee that whereas other Animals have but one Cavity for aſſiſting the Vibration of the Air, and continuation of the

Sound in the *Tympanum*; this Animal has two, or a large one with two different Surfaces. The Aqueduct both by its Figure and Poſition in this Animal doth plainly ſhew us the Uſe of it in other Animals, which is to receive the ſuperfluous Humours in the *Tympanum*, and convey them to be diſcharg'd in the Mouth.

Explication of the FIGURES.

Figure I.

Repreſents the Bony part of the Meatus Auditorius *of the Right Ear.*

a *The external Orifice of the* Meatus Auditorius.
b *The* Proceſſus Petroſus.
c *The Orifice where the* Nervus Auditorius *enters.*
d *The* Meatus Auditorius.
e *A part of the* Laminæ *which proceed from it on each ſide, by which the* Cellules *betwixt the two Tables of the Scull are form'd; thoſe ſituated above the* Meatus *being remov'd.*
f *Part of the inner Table of the Scull.*

Fig. II.

Repreſents part of the Meatus Auditorius *opened, with other parts of the inner Ear.*

a *The ragged part of the Bone from whence the* Os Petroſum *was ſeparated.*
b *The* Proceſſus Petroſus *opened.*
c *The* Crena *for the* Membrana Tympani.
d *The Hony-comb Cavity of the* Tympanum.
e *Its inner Cavity of a ſmooth Surface.*
f *Its Semicircular or undulated Lines.*
g *The Orifice of the Aqueduct.*
h *The Orifice of the hard Portion of the* Nerve.

Fig. III.

Repreſents the lower Surface of

of the Os Petroſum, *as it was ſeparated from above the* Tympanum *and other parts of the inner* Ear.

a a *The ragged Margine of the Bone.*

b b *The upper part of the* Cavitas Tympani.

c *The* Foramen Ovale.

d *The Protuberance in which the* Labyrinth *and* Cochlea *are lodg'd.*

e *The Orifice of the hard portion of the* Nervus Auditorius.

Fig. IV.

Repreſents the Malleolus *alone in its true dimenſions.*

1 *The Protuberant Head.*

2 *The Semicircular* Sinus *betwixt it and the Margin.*

3 *The* Sinus *which receives the head of the* Incus.

4 *The angle below the* Sinus *for the head of the* Incus.

5 *The angle where the* Manubium Malleoli *begins.*

6 *The* Manubrium Malleoli.

Fig. V.

Repreſents the Incus.

1 *The head of the* Incus.

2 *The* Sinus *or neck of the* Incus.

3 *Two* Apophyſes.

4 *A long protuberance with the* Sinus *for the* Os quadrangulare *at its extremity.*

Fig. VI.

Repreſents the Stapes.

1 *The ſmall part of the* Stapes, *where it is articulated with the* Incus, *with a* Sinus *at its extremity, being the other half of the Cavity for the* Os quadrangulare.

2 2 *Two ſmall portions of the* Stapes, *where it is articulated with the* Baſis.

3 *The* Baſis *of the* Stapes *ſeparated.*

4 *The whole* Stapes.

Fig. VII.

The Malleolus *and* Incus *join'd together, with their lower ſide turn'd up.*

1 *The* Malleolus.

2 *Its articulation with the* Incus.

3 *The* Incus.

4 *The* Manubrium Malleoli.

5 *A point of the* Incus, *fram'd by the other two Productions.*

6 *The*

6 *The long protuberance of the* Incus.
7 *The* Sinus *in the extremity of its long Production.*

Fig. VIII.

The Malleolus, Incus *and* Stapes *articulated together.*

1 *The* Incus.
2 *The* Malleolus.
3 *The* Stapes *where it shuts the* Foramen Ovale.

Fig. IX.

Represents the upper part of the Lineæ Semilunares, *or that side which is towards the passage of the* Nervus Auditorius.

a *The five extremities cut off.*
b *The* Linea Semilunaris Major.
The Semilunaris Media
d *The* Minor.
e *The common Canule between the* Major *and* Media.

Fig. X.

Represents the Cochlea *and* Labyrinth *together.*

a *The* Vestibulum.
b *The* Foramen Ovale.
c *The* Foramen Oblongum.
d *The* Linea Semilunaris Minor, *which is towards the* Cavitas Tympani.
e *The common Canule to the* Major *and* Media.
f *The* Major.
g *The* Media
h *The* Cochlea.

Fig. XI.

Represents the Cochlea.

a *The* Vestibulum.
b *The third* Gyre *or turning.*
c *The Orifice.*
d *The first* Gyre *or turning opened*
e *The second turning.*
g *The Orifice at the top of the* Cochlea.

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PHILOSOPHICAL TRANSACTIONS.

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IV. *Pars reliqua Diſſertationis* De Potentiâ Cordis. *Authore* Jacobo Jurin, *M. D. &* R. Societatis Soc.

V. *Nova Methodus univerſalis Curvas Omnes cujuſcunque Ordinis Mechanice deſcribendi, ſola datorum Angulorum & Rectarum ope. Per D.* Colin Maclaurin *in Collegio Novo* Abredonenſi *Matheſeos Profeſſore.*

VI. *Extract of a Letter of the Reverend Mr.* William Rice, *Rector of* Caerleon *upon* Usk, *to* Charles Williams *Eſq; giving an account of an ancient* Roman *Inſcription lately found there. With ſome Conjectures thereon, by the Reverend Dr.* John Harris, *R. S. S.*

Jos: Banks

I. *Curious Obſervations of the Tranſit of the Body and Shade of* Jupiters *Fourth Satellite over the Diſque of the Planet. Communicated by the Reverend Mr.* James Pound, R.S.S.

Finding by the Tables of *Jupiter*'s Satellites that the Fourth Satellite was to paſs over the Diſque of *Jupiter* the 16*th* of this preſent *February*, at Night; we were very deſirous to obſerve the ſame with the *Hugenian* Teleſcope, having never before, ſince I have had the Uſe of it, been able, by reaſon of the foulneſs and inconſtancy of the Weather, to make any tolerable Obſervation of this kind.

At $6^h \frac{1}{4}$ thro' a ſhort Tube, we ſaw all the 4 Satellites, the 3 outermoſt on the *Eaſt* ſide of *Jupiter*, and the innermoſt near the *Weſtern* Limb approaching to an Eclipſe. The Fourth at that time was about half a Semidiameter of *Jupiter* from the *Eaſtern* Limb. Then it proved Cloudy till about 8^h, at which time (thro' the long Glaſs) we could ſee only the ſecond and third Satellites, the firſt being behind *Jupiter* in the Shadow, and the fourth entred upon the Diſque. We ſaw at this time a dark Spot, a little *Northward* of the great *Northern* Zone, and near the *Eaſtern* Limb, where the Satellite was to enter on the Diſque; which Spot we took for the Shade of the Satellite. The Clouds then again intercepted our View, till $8^h.\ 53'.$ *Æq. T.* at which time the firſt Satellite was lately emerged out of the Shadow, and the Spot advanced ſo far, that we perceived it would arrive at the middle of *Jupiter*, near two Hours ſooner than the

the Shade ought to have done by our Computation; but not imagining that this dark Spot could be any thing elſe but the Shade, we concluded there had been ſome Error in the Calculation, which we thought to re-examine afterwards. On this preſumption we left off obſerving till $9^h.\ 35'$. at which time we were ſurprized to ſee a Notch in the Limb of *Jupiter*, near the place where the former Spot entred. This laſt appearance agreeing well with the time that the Shade of the Satellite ought to have entred the Diſque, ſoon made us alter our former Opinion, and conjecture that this and not the other Spot was the ſaid Shade. At $9^h.\ 39'\frac{1}{2}$ *Æq. T.* the Notch vaniſhing, a round black Spot appeared within the Limb, but in contact with it. At $9^h.\ 45'.$ we judged the firſt Spot, and at $11^h.\ 45.$ the ſecond, to be in the middle of *Jupiter*.

At $11^h.\ 50'.$ the firſt Spot touched the Limb, being within the Diſque; ſoon after which the Limb in that place ſeem'd a little protuberant. At $12^h.\ 5'.$ appeared the fourth Satellite juſt come out of the Diſque, and touching the Limb in the place where the Protuberancy was. At $12^h.\ 7'$, we could perceive the Satellite ſeparated from the Limb At $13^h.\ 56'.$ the ſecond black Spot, ſtill within the Diſque, juſt touched the *Weſtern* Limb; ſoon after which there appeared a Notch in this part of the Limb, as it did on the other at the coming on of this Spot. At $14^h.\ 6'.$ the Spot was all gone off, and the Limb appeared clear and entire The firſt Spot, when in the middle of *Jupiter*, was almoſt as black as the ſecond when near the Limb, but ſomewhat leſs and a little more *Northerly*.

At the time that the firſt Spot was in the middle of the Diſque, the three innermoſt Satellites appeared

to

to the *East* of *Jupiter*; the firſt (as aforeſaid) having lately emerged out of the Shadow; the ſecond being almoſt at its greateſt diſtance; and the third having paſſed the Axis of the Shade about twelve Hours before, and appearing at this time about three Diameters of *Jupiter* from his Limb. The times that theſe Spots arrived at the middle of the Diſque are agreeable to the times found by Calculation, in which the fourth Satellite and its Shade ought to have appeared there. From all which 'tis very plain, that the firſt of theſe Spots was the fourth Satellite itſelf, and the ſecond its Shadow.

We have ſeen the firſt and ſecond Satellites appearing not as dark Spots, but as bright ones (ſomewhat different from the light of *Jupiter*) for ſome little time after they entred his Diſque, but as they approached nearer the Middle we loſt ſight of them. And we have frequently obſerved that the ſame Satellites appear brighter at ſome times than at others; and that when one of them hath ſhined with its utmoſt Splendour, the Light of another hath been conſiderably diminiſhed. From whence 'tis very probable at leaſt, not only that the Satellites revolve upon their proper Axes, but alſo that ſome parts of their Surfaces do very faintly (if at all) reflect the Solar Rays to us.

All which hath for ſome time ſince been obſerved and taken notice of by Meſſ. *Caſſini* and *Miraldi*, as may be ſeen in the *Memoirs* of the *Academie Royale*, for the Years 1707 and 1714.

Philosoph.: Transact N.° 359

SPAIN

Cattaia
Lepe
Taron
Guelva
Palos
Zavega
EL CONDADO
R. Saltese
R. de Palos
R. Guadalquivir
S.t Lucar
Rota
Xeres
P. S.t Maria
R. Guadalera
Puerto Real
Cadiz
Pont Suaco
I. S.t Pedro
Conil
Bejer de la Miel
C. Trafalgar

Ximena
R. Jolgarganta

COAST OF SPAIN

COAST OF SPAIN

Beier de la Miel

R. Barbate

la Barca

C. Trafalgar

R. Alguariate

Balonia

Arroyo de S. Juan Francisco

Playa de Orimel

Tariffa

Val de Vaca

Algezeira

R. Guadarangue

Rocadillo

Gibraltar

R. Guadiaro

C. Europe

C. Cabrito

Lelius's Fleet

STREIGHTS OF GIBRALTAR

Asdrubals Fleet

Ceuta Pt

Ceuta

Apes Hill

Alcafer

C. Spartel

Tangir

COAST OF BARBARIE

II. *A Diſcourſe tending to ſhew the Situation of the ancient* Carteia, *and ſome other* Roman *Towns near it. By* John Conduitt *Eſq*; Fellow *of the* Royal Society.

ABout four *Engliſh* Miles *N.W.* from *Gibraltar*, at the end of the Bay, there are conſiderable Ruins. The place is called at preſent *Rocadillo*, and conſiſts of a few Huts, and a Modern Square Tower, which appears to have been raiſed on the Foundation of a much greater Pile. The Walls of the old City are very eaſy to be traced. They ſeem to have been about two *Engliſh* Miles in Circumference, and were built upon the Brow of a riſing Ground. The ſpace within is covered with Ruins, among which are a great many pieces of very fine Marble well wrought; and innumerable fragments of Veſſels of that kind of red Earthen Ware, which *Ambroſio Morales* in the firſt Chapter of his *Diſcurſo de las antiguedades de las ciudades de Eſpanna*, lays down for a certain mark of a *Roman* City, and takes to have been a Compoſition of the Clay of *Saguntum*, often mentioned among the *Romans*.

Ficta Saguntino pocula malo luto. Mar. Lib. VIII. Ep. 6.

Sume Saguntino pocula ficta luto. Lib. XIV. Ep. 108.

There are remains of a rude Semicircular Building, raiſed on Arches, which deſcends gradually into an Area, and ſeems to have been a kind of Theatre. I brought away with me a Marble Pedeſtal of a Statue, dug up near to the Square Tower. The Marks where

the Feet and the extremities of the Drapery were fastened to it, are ſtill to be ſeen, and the following Letters finely cut VARIA MARCE. It was given me by the owner of the Ground, who ſaid he had read upon it formerly three other Letters L L A ſince broken off There are other Inſcriptions, but ſo Defaced and ill Cut, that they do not deſerve a particular mention. I have a conſiderable number of Medals, that were found among theſe Ruins; moſt of them have a *Caput turritum* with CARTEIA in very legible Characters. The Reverſe is generally a *Fiſh*, a *Neptune*, or a *Rudder*. Towards the *Weſt* there is an eaſy Deſcent to the River *Guadarranque*, which takes its Source at *Caſtellar*, about four Leagues in the Country, and is very deep at *Rocadillo*. There is a Bar where the River falls into the Bay; but it does not hinder the entrance of Veſſels of 15 Tun, to load Charcoal and other neceſſaries, that are Shipt off from thence for *Ceuta*. Along the ſide of the River there is ſtill a great deal of Stone Work and viſible remains of an Ancient Key. At a ſmall diſtance to the Eaſt, upon an Eminence, there are conſiderable ruins of a Square Caſtle, which appears to have been an ancient Building of very great Strength. The Country People now call it *Caſtillon*, but the *Corrigidor* of that Diſtrict told me he remember'd it called *Torre Cartagena*. The Situation agrees exactly with the Tower of that Name, mentioned in the 274*th* and 316*th* Chaprers of the Chronicle of *Alphonſo* XI of *Caſtile*. A Book of great Authority among the *Spaniards*, who are generally of Opinion that it was formed upon the Memoirs of *Fernando Nunnez de Valladolid*, a Favourite and Miniſter of that King, tho' it goes under the name of another Perſon.

All

All the *Spaniards* who live about the Ruins I have been deſcribing, ſay they are the remains of a City of the *Gentiles* called *Cartago*. The corruption of *Carteia* into a name ſo much more talked of, might eaſily happen, in an Oral Tradition of ſo many Years; and I cannot help thinking that, where other Circumſtances concur, an account deliver'd down from Father to Son is an evidence not to be ſlighted, in matters of ſo much obſcurity.

Frequent mention is made of *Carteia* by the Ancient Geographers and Hiſtorians. I build ſo much on two Paſſages of *Livy*, that I am obliged to inſert them at length. The firſt is Lib. XXVIII. C. 30. (*Livy* does not mention from what Port *Lælius* ſailed for *Carteia*, but by what goes before, it ſeems to have been from *Cartagena*, at that time *Scipio*'s Head Quarters) Lælius *interim, freto in Oceanum evectus, ad* Carteiam *claſſe acceſſit. (Urbs ea in orâ Oceani ſita eſt, ubi primum è faucibus anguſtis panditur mare)* Gades *ſine certamine proditione recipiendi (ultrò, qui eam rem pellicerentur, in caſtra* Romana *pervenientibus) ſpes, ſicut antea dictum eſt fuerat. Sed patefacta immatura proditio eſt, comprehenſoſque omnes* Mago Adherbali *Prætori* Carthaginem *devehendos tradit.* Adherbal, *Conjuratis in quinqueremem impoſitis, præmiſſaque eâ, quia tardior quam triremis erat, ipſe cum octo triremibus modico intervallo ſequitur. Jam fretum intrabat quinqueremis, cum* Lælius *& ipſe in quinqueremi, è portu* Carteiæ, *ſequentibus ſeptem triremibus, evectus, in* Adherbalem *ac triremes invehitur; quinqueremem ſatis credens deprehenſam rapido in freto, in adverſum æſtum reciprocari non poſſe.* Pœnus, *in re ſubitâ parumper incertus, trepidavit utrum quinqueremem ſequeretur, an in hoſtes roſtra converteret. Ipſa cunctatio facultatem detrectandæ pugnæ ademit: jam enim ſub ictu teli erant, & undique inſtabant hoſtes. Æſtus quoque arbitrium moderandi*

di naves ademerat; neque erat navali pugnæ ſimilis, quippe ubi nihil voluntarium, nihil artis aut conſilii eſſet. Una natura freti æſtuſque totius certaminis potens, ſuis, alienis navibus, nequicquam remigio in contrarium tendentes invehebat; ut fugientem videres retro vortice intortam victricibus illatam, & ſequentem, ſi in contrarium tractum incidiſſet maris, fugientis modo ſeſe avertentem. Jam in ipsâ pugnâ hæc, cum infeſto roſtro peteret hoſtium navem, obliqua ipſa ictum alterius roſtri accipiebat: illa, cum tranſverſa objiceretur hoſti, repente intorta in proram circumagebatur.

The other Paſſage is Lib. XLIII. C. 3. *Et alia novi generis hominum legatio ex* Hiſpaniâ *venit: Ex militibus* Romanis *& ex* Hiſpanis *mulieribus, cum quibus connubium non eſſet, natos ſe memorantes, ſupra quatuor millia hominum, orabant ut ſibi oppidum in quo habitarent daretur. Senatus decrevit, uti nomina ſua apud L.* Canuleium *profiterentur, eorumque ſiquos manumiſiſſet, eos* Carteiam *ad Oceanum deduci placere Qui* Carteienſium *domi manere vellent, poteſtatem fore uti numero colonorum eſſent, agro aſſignato.* Latinam *eam coloniam fuiſſe, Libertinorumque appellari.*

The beſt *Spaniſh* Authors, and *Ortelius* and *Cellarius* truſting to them, take this *Carteia* of *Livy* to be different from that which was the next to *Calpe*, and place it generally about *Conil. Rodrigo Caro* in his *Convento Juridico de Sevilla* C. 24. applies the *Carteia* in the XLIII Book of *Livy* to *Rocadillo*, and in Cap. 74. to *Cartaia* near *Lepe*. It is ſurprizing he takes no notice of the Paſſage in the XXVIII Book, For the particular mention of *ad Oceanum*, and *Urbs ea in orâ Oceani ſita eſt*, implies they both relate to the ſame place; perhaps it was becauſe he could not reconcile it with his *Cartaia* near *Lepe*. *Cellarius* makes *Bæſippo* this *Carteia* of *Livy*. l. 2. c. 1. *Bæſippo, quæ videtur* Carteia Livii *eſſe, extra*

fretum

fretum & columnas posita. Aliam pro Livio Carteiam *non invenio*; tho' in all the ancient Geographers *Bæsippo* is mentioned by it self as a distant Town. I am so far from seeing any necessity of erecting a new *Carteia* in the Ocean for these Passages in *Livy*, that I take that in Lib. XXVIII. to be rather a proof that the City there mentioned, stood at *Rocadillo*. It certainly agrees much better with that Situation than with *Conil* or *Cartaia* near *Lepe*. It is not to be reconciled with the latter, because that lies *North West* of *Cadiz*, from whence *Adherbal* set out for *Carthage*, and is a good way up the Country, on the side of a River, and not *in orâ Oceani*. Neither can *Conil* be said properly to be Situated *Ubi primum è faucibus angustis panditur mare*; for the Sea widens considerably before it reaches the Capes *Spartel* and *Trafalgar*, and becomes an Ocean where that Town stands. It is observable that *Mela* applies words of the same import with those of *Livy* to the Sea between *Calpe* and *Abila*. *Barbesul*; *Aperit deinde angustissimum pelagus*. There is no Harbour at *Conil*, or any other place between Cape *Trafalgar* and *Cadiz*. If the *Carthaginian Quinqueremis* had only *been going into* (*intrabat*) the Mouth of the Streights between Capes *Spartel* and *Trafalgar*, *Lælius* could not have believed it *satis deprehensam rapido in freto, in adversum æstum reciprocari non posse*, for there is no such strong Current there; and the action between him and *Adherbal's Triremes*, which were at some distance behind the *Quinqueremis*, must have happened *Westward* of those Capes; which is inconsistent with the description *Livy* gives of it; because in that part of the Ocean there are none of those Eddies, that appear to have had so particular an effect on both the Fleets, during the Engagement, and are peculiar to the Middle of the Gut.

This

This general miſtake ſeems to have been occaſioned by giving too eaſily into the opinion, that *Livy* underſtood by the *Fretum* all the Sea between the Capes *Spartel* and *Trafalgar*, and the Rock of *Gibraltar* and *Apes-Hill*; when it is more probable that he termed ſtrictly ſo only the narroweſt Part, which was generally reckoned to be between the two latter: *Mela. Proxima Africæ & Europæ littora montes efficiunt Calpe & Abila.* *Pliny* takes *Mellaria* to be neareſt to *Africk*, and therefore places there the *Fretum ex Atlantico mari Lib.* 3. which is an argument his *Fretum* was not the ſame with our Streights, and that he carried the Atlantick Ocean much farther *Eaſt* than the Capes *Spartel* and *Trefalgar*.

Other Authors ſeem to make the Pillars of *Hercules* the Boundary of the Mediterranean and the Ocean. *Marcianus Heracleotes.* Ἐνταῦθα πέρας ἔχει τῆς Βαικκῆς Ἱσπανίας τὸ μέρος. τὸ παρῆκον πὰρ ἑκατέρας τὰς θαλάσσας τὰς περὶ τὸν Ἡράκλειον πορθμὸν τυγχανούσας, τήν τε καθ' ἡμᾶς & τὴν ἔξω, τοῦτ ἔστι τὸν Ὠκεανόν. *Hic finem habet Hiſpaniæ Bæticæ pars contingens utraque maria quæ circa fretum Herculeum, tam mare noſtrum quam mare exterius, h. e. Oceanum.* Τῆς μὲν Βαικκῆς τὸ πλεῖσον πρὸ τῆς καθ' ἡμᾶς κεῖται θαλάσσης, τῆς Ἡρακλείων ἐντὸς στηλῶν, μέρος δέ τι πρὸς τὸν ἐκτὸς Ὠκεανόν. *Bæticæ quidem pars maxima prætenditur noſtro mari, Herculeas intra columnas, pars vero quædam occidentali Oceano.*

Polybius L. III. Καλεῖται δὲ τὸ μὲν πρὸς τὴν καθ' ἡμᾶς παρῆκον ἕως Ἡρακλείων στηλῶν, Ἰβηρία· τὸ δὲ πρὸς τὴν ἔξω & μεγάλην πελαγορευομένην κοινὴν μὲν ὀνομασίαν οὐκ ἔχει, διὰ τὸ πελσφάτως καταπτεύεσθαι. *Quæ porrigitur ſecundum mare noſtrum portio ad columnas uſque Herculis Iberia nominatur; quæ ſecundum mare externum quod & magnum appellatur, communem appellationem nondum invenit, quia non diu eſt cum fuit explorata.*

Appian

Appian L. II. Ἐμφυλ. πρός τε τὰς στήλας τὰς Ἡρακλείους τὸν ὠκεανὸν ἐπέρων. *Trajecto ad Columnas Herculis Oceano.*

Florus Lib. IV. C. 2. *In ipso ostio Oceani* Varus Didiusque *legati conflixere; sed acrius fuit cum ipso mari, quam inter se navibus bellum: Siquidem velut furorem civium castigaret Oceanus, utramque classem naufragio cecîdit. Quinam ille horror, cum eodem tempore fluctus, procellæ, viri, naves, armamenta confligerent? Adde suîs ipsius formidinem, vergentia in unum hinc* Hispaniæ *inde* Mauritaniæ *littora; Mare & intestinum & externum, imminentesque* Herculis *speculas; cum omnia undique simul prælio & tempestate sævirent.* Here the Pillars of *Hercules* are made the very Mouth of the Ocean. If you understand the *Fretum* of *Livy* in this Sense, and reckon it to signify only the Sea between *Calpe* and *Abila*, and the Ocean to begin from thence *Westward*, the Passage in the 28*th* Book is an accurate description of *Rocadillo*. *Lælius interim freto in Oceanum evectus ad Carteiam classe accessit. Urbs ea in orâ Oceani sita est, ubi primum è faucibus angustis panditur mare.* And allowing *Lælius* to set out against *Adherbal* from thence, every circumstance mentioned by *Livy* is so easy to be accounted for, that it is needless to make Application. A Passage in *Dio Cassius* Lib. XLIII. induces me to believe the Vessels anchored in the *Guadarranque*, and that that River and not the Bay, was properly *Portus Carteiæ* Οὐαρ. δὲ ὑπὸ τ Διδίυ περὶ Κραντίαν ἐναυμαχήθη, κ. ἔως μὲν περιπατοῦν ἐς τὴν γῆν, ἀγωνας ἐς τὸ σῶμα τ λιμένος ἄλλας πρὸς ἄλλας προσενεβεβλήκει. κ. περὶ αὐτὰς οἱ πρῶτοι τῶν διωκόντων σφᾶς, ὥσπερ περὶ ἕρμα ἐστηκεισαν, πᾶν ἄν τὸ ναυικὸν ἀπολωλέκει. Varus *vero à* Didio *apud* Crantiam *navali prælio superatus in terram evasit, conjectisque in introitum portûs anchoris, ita ut una ab aliâ teneretur, cum ad eas, tanquam ad septum quoddam, primæ insequentium*

naves

naves offendiſſent, periculum totius claſſis amittendæ declinavit. This cannot be underſtood of the Bay, becauſe that is three Leagues over at the narroweſt part, and much too deep for a work of ſuch a Nature, which might eaſily have been effected upon the Bar of the River *Guadarranque.*

There is no room to doubt of the emendation *Luis Nunnes*, in his *Hiſpanica*, has made here of Καρτηία for Κεαντία; for no ancient Author mentions any other Town or Harbour thereabouts of a name like that; and *Carteia* was the place which held out the longeſt for the younger *Pompey*, and where he kept his Fleets.

Florus in the Paſſage I have already quoted, relating the ſame Action between *Didius* and *Varus*, repreſents in very lively Colours, the very Scene near *Rocadillo. Adde ſitus ipſius formidinem, vergentia in unum hinc Hiſpaniæ inde Mauritaniæ littora; mare inteſtinum & externum, imminenteſque Herculis ſpeculas* &c.

Hirtius, in the latter part of his Book *de bello Hiſpanico*, ſays *Cn. Pompeius ad navale præſidium parte alterâ contendit* Carteiam, *quod Oppidum abeſt à* Corduba *millia paſſuum* CLXX. which diſtance, as well as the circumſtance of *navale præſidium*, agrees with the Situation of *Rocadillo* The ancient Geographers place *Carteia* next to *Calpe* Weſtward. *Pomponius Mela*, after having given us a perfect Picture of *Calpe*, and deſcribed thoſe laſting Marks, in which ſo many Centuries have made no alteration, ſays—*Sinus ultra eſt, in eoque Carteia.* *Strabo* L. III. Ἐνταῦθά δὴ ὄρος ἐστὶ τῆς Ἰβήρων ἡ Καλπὴ, &c. καὶ πρὸς αὐτὸ Καλπὴ πόλις ἐν τετταράκοντα σταδίοις, ἀξιόλογος καὶ παλαιὰ, ναυσταθμός ποτε γενομένη τῆς Ἰβήρων. Ἔνιοι δὲ καὶ Ἡρακλέους κλίσμα λέγουσιν αὐτὴν, ὧν ἐστι καὶ Τιμοσθένης, ὅς φησι καὶ Ἡρακλείαν ὀνομάζεσθαι τὸ παλαιὸν, δείκνυσθαί τε μέγαν περίβολον καὶ νεωσοίκους. *Ibi mons* Hiſpanorum *eſt* Calpe, &c. *& ad* XL *inde Stadia Urbs* Calpe

veluſta

vetuſta & memorabilis, olim Statio navibus Hiſpanorum. *Hanc ab* Hercule *quidam conditam aiunt, inter quos eſt* Timoſthenes, *qui eam antiquitùs* Heracleam *fuiſſe appellatam refert, oſtendique adhuc magnum murorum circuitum & Navalia.* *Caſaubon*, in his Notes on this Paſſage is of Opinion it ſhould be Καρτηΐα πόλις. *Legendnm cenſeo* Καρτηΐα πόλις, *nam eam urbem hic intelligi res ipſa docet; & ex eo colligi poteſt, quod toties eam infra nominans nihil tamen de ejus ſitu alibi dixiſſe reperitur.* *At* Calpen *Urbem nemini Veterum ne nominatam quidem reperio.*

Marcianus Heracleotes makes *Carteia* 50 Stadia from *Calpe.* Either of theſe Diſtances agrees with *Rocadillo*, according to the part of the Rock from which they reckon; for it is above ſix Miles from *Europa* Point to *Rocadillo.*

Bochart in his *Geographia Sacra* Lib. I. Cap. 34. ſtrengthens *Caſaubons* Opinion. *Nec fruſtrà* Heraclea Carteiæ *fuiſſe vetus nomen, tanquam ab* Hercule *conditore.* Herculem *enim ſuum* Phœnices Μέλκαρθον *appellabant.* Philo Biblius *ex* Sanchoniathone *apud* Euſebium L. I. *Præparat.* τῶδε Δημαροῦντα γίνεται Μέλκαρθος ὁ καὶ Ἡρακλῆς. *Ex* Demarunte *autem natus eſt* Melcarthus *qui &* Hercules. Μέλκαρθος *autem eſt* מלך קרתא Melech Kartha. *Rex Urbis, i. e.* Tyri. *Idem* Græcis Melicertes *ſive* Palæmon *Maris Deus, quem* Cadmi *nepotem eſſe fingunt. Hinc* Heſychius *rurſus* Μάλικα τὸν Ἡρακλέα Ἀμαθούσιοι. *Omnino igitur ex* Melcartho, *vel* מלך קרתא Melech Kartha. *Urbs quam ad* Calpen *condidit* Hercules Phœnicius, primo Melcartheia *vocata eſt*, Melech Karthiia, *quaſi* Ἡρακλέαν *dixeris; deinde per Apharefin* Cartheia *vel* Carteia. *Apud* Hebræos *frequens eſt hæc Apharefis in nominibus locorum compoſitis. Tale* Sittim *pro* Abel-Sittim, Salem *pro* Jeruſalem, &c.

I have ſome Medals that were dug up at *Rocadillo*, with the Head and Club of *Hercules* upon them;

which ſeem in ſome meaſure, to ſupport that great Man's Aſſertion. Upon the Reverſe are Tunny Fiſhes, which according to *Strabo* and *Pliny* abounded formerly near *Carteia*, and are ſtill taken in great quantities near the Shoar of the Eaſt Sea, at a ſmall diſtance from *Rocadillo*.

Bernardo Aldrete an Author of ſuch Weight, that *Bochart* does not diſdain to copy him on ſeveral occaſions, in the ſecond Book and ſecond Chapter of his *Antiguedades de Eſpanna*, accounts for the Addition of *eia* to *Cartha*; which in the *Syriack* and *Chaldean* ſignifies *Pulcher*, *Formoſus*, and was affixed to the Name of this City to diſtinguiſh it from the *Cartha* in *Syria*, mentioned in the 21*ſt* Chapter and 34*th* Verſe of *Joſhua*.

By all accounts, the *Phœnicians* founded moſt of the Cities on this Coaſt, and probably *Carteia* was one of their earlieſt Settlements; for it lies very near *Africk*, in a moſt inviting Situation, having on one ſide a Bay, and on the other a River, which waters a rich Country. Its height gave it ſtrength and a very beautiful Proſpect; circumſtances which ſeem to juſtifie *Aldretes* interpretation of the latter part of it's name.

In the Itinerary of *Antoninus*, it is *Calpe - Carteiam*, not *tanquam duæ urbes diverſæ*, as *Caſaubon* intimates in his Notes on the third Book of *Strabo*, for then it would be *Calpen Carteiam*; nor, according to *Surita's* Comment on that part of the Itinerary, *ut ſignificet non rectâ iter ex* Suel Carteiam *deduci, ſed paululum ad* Calpen *deflecti*; becauſe *Calpe* ſtands at the end of a narrow neck of Land, which projects to the *Southward* a great way from the reſt of the Continent; and conſequently is quite out of the Road from *Suel* to any other place *Weſtward* of it; probably *Calpe - Carteia* is for *Carteia ad Calpen*, to diſtinguiſh it from the other *Carteia* in *Celtiberia*, mentioned in the 21*ſt* Book and

5*th*

5th Chapter of *Livy*: for, as *Caro* obſerves, there is no neceſſity for the alteration *Sigonius* has made in that paſſage of *Althæa* for *Carteia*, from the Text of *Polybius*; becauſe *Livy* never mentions the other *Carteia* without adding *ad* Oceanum, *Urbs ea in ora Oceani ſita eſt*; which diſtinction were needleſs, had there been only one City of that Name. *Strabo* in his third Book mentions a City called Καρταλίας, and places it near *Saguntum*, which is agreable to the Situation given this *Carteia* by *Livy*.

I am very much ſurprized that *Mariana*, and ſeveral others, ſhould take the preſent *Gibraltar* to have been the ancient *Heraclea*; when neither *Pliny*, who reſided ſo long in thoſe Parts, *Mela* who was born there, nor any ancient Geographer or Hiſtorian that I have met with, makes the leaſt mention of ſuch a City thereabouts, except *Strabo*; and he places it 40 Stadia from *Calpe*, at the Foot of which *Gibraltar* is ſituated. The *Spaniſh* Hiſtorians give good ground to believe there was no Town upon that Mountain till the *Moors* invaded *Spain* under *Tariff*, who gave it the name it has retained ever ſince. I ſhall not enter into the detail of the reaſons of thoſe Authors who place *Carteia* at *Tarifa* or *Algezeira*: the true one ſeems to have been their not knowing any other place which agreed better with the old accounts of *Carteia*, or where the ruins of a City, which made ſo great a Figure, could be buried; the common practice of Authors who deſcribe places they have not ſeen. This appears to have been the caſe of moſt of thoſe, eſpecially *Mariana*; who, had he been in theſe Parts, would not have been guilty of the overſight he has committed Lib. XVI. C. 9. where he places two Bays in the *Streights*, one at *Gibraltar*, and the other at *Tarifa*; which error he was probably led into (as it often happens) by another.

For, giving into the Opinion that *Tarifa* was the antient *Carteia*, and finding that City placed in a Bay by *Mela*, he concluded there muſt be one at *Tarifa*, which is an open Road, and ſo much expoſed, that in the leaſt bad Weather, the ſmalleſt Veſſels muſt be haul'd aſhore. Which Circumſtance alone is a ſufficient proof of its not being *Carteia*, by all accounts, a famous Harbour.

Tho' there are very great Ruins at *Algezeira*, they are not ſuch as give any room to believe they are the remains of a *Roman* City. For neither pieces of Marble, nor Inſcriptions are found there, nor any *Roman Coins*. The Circumſtance of *Varus* his ſhutting up the Mouth of the Harbour of *Carteia*, and the diſtance of 40 or 50 *Stadia* from *Calpe*, are not applicable, either to *Tarifa* or *Algezeira*; and if one of thoſe Towns was *Carteia*, to what City belong thoſe Ruins I have been deſcribing? ſince all the ancient Geographers make *Carteia* not only the neareſt Town to *Calpe*, but the only one in that Bay. There is better ground to believe *Tarifa* ſtands on the Ruins of an other Town, as I ſhall endeavour to ſhew preſently.

But before I proceed to a Deſcription of the Coaſt, it may not be improper to mention ſome Ruins I ſaw at *Ximena*; an inland Town, about five Leagues *North* from *Gibraltar*, ſituated on a Rocky Hill, at the bottom of which to the *Eaſtward* is a very plentiful Country, waſhed by the *Joſgarganta*, a ſmall Branch of the River *Guadiaro*. On the top of the Hill is the old Town, which by the Arches and Vaults, appears to have been built by the *Moors*. On the right-hand Corner of the ſecond Gate of it, there is a courſe Stone with Mouldings on the Edges, which has the following Inſcription.

L. HERENNIO HE
RENNIANO
L. CORNELIVS HEREN.
NIVS RVSTICVS
NEPOS EX TESTA
MENTO POSVIT
NONIS MARTIIS
SEX. QVINTILIO CON.
DIANO SEX. QVIN
TILIO MAXIMO COSS.

Rodrigo Caro in his *Convento Juridico de Sevilla* C. 13. ſays he ſaw the beginning of this Inſcription in *Bejer de la miel*; but when I was in that Town, I was informed by a very intelligent Perſon, that there is no *Roman* Inſcription in any part of it. The Author of *Cadiz el Emporio del Orbe*, when he inſerts this inſcription, makes it SEXOVINTILIO CONDIMIO; But the Daſh of the Q is very plain, and the other word ſeems rather CONDIANO. The *Latin Faſti* in A. U. C. 903. place Conſuls

SEX. QUINTILIUS GORDIANVS
SEX. QUINTILIUS MAXIMUS.

But the very learned Dr. *Bently* has obſerved to me that the *Greek Faſti* and *Dio* call him Κονδιανὸς, which reading is confirmed by this Inſcription.

I have brought with me from this Town a piece of Marble with the following Words upon it.

AVCTINVS CLEMEN
TIS SIBI
ET SVIS BRITTÆ
MATER AN LX
H.S.E. SIT T.T. LEVIS.

I ſaw

I ſaw another on the Wall of the great Church which ſeems to have been the Baſe of a Statue; the Inſcription is as follows.

RESPVBLICA OBEN
SIS E..LO DATO
DEDI...VIT CVRAT
LIBE..OR H..REN
NIO RVSTICO H.M.
SINILO RESTITVTO
II VIR.

The manner in which the *Moors* have placed theſe Inſcriptions plainly ſhews the little value they ſet upon them, and there is ſo great a plenty of Stone on the Rock where *Ximena* ſtands, that it is not to be thought they would fetch them for ſuch an uſe, from any diſtant place; which induces me to believe a *Roman* Town formerly ſtood there called O B A.

I do not find any Town of that name in the ancient Authors. *Strabo* L. III. mentions Σόγοβα Μαίνοβα καὶ ἄλλας πλείους, which may poſſibly comprehend *Oba*. The *Geographia Nubienſis*, in the fourth *Clima*, makes a Town called *Rotban*, the firſt Station from *Algezeira* to *Seville*, which perhaps may have been this *Oba*; for it is about a Days journey from *Algezeira*, and in the direct Road from thence to *Seville*.

Mariana places Lib. III. C. 2. the Cave where *Craſſus* hid himſelf, near *Ximena*; the Marks of it, given by *Plutarch*, are common to moſt others. I went three Leagues in ſearch of it; but the Country People having a notion that there is a Treaſure in it, and not being to be perſuaded that I would take ſo much Pains out of pure Curioſity, would not ſhew me the Way, tho' they acknowledged there were ſeveral ſuch Caves thereabouts. I cannot help taking notice of one very

odd

odd tho' trifling circumſtance. The name of the Perſon who owns the Land where thoſe Caves are, is *Pachieco*, which is very near the ſame with that of the *Spaniard*, who is ſaid by *Plutarch* to have entertained *Craſſus* ſo courteouſly, Παυανής. *Hirtius* in the beginning of his Book *de bello Hiſpanico* mentions a *Spaniard* of Note, *in provinciâ Bæticâ*, called *Patiecus*. *Quibus præfecit hominem ejus provinciæ notum & non parum ſcientem, L. Julium Patiecum*, which was probably the *Roman* Name; and therefore I am ſurprized the *Latin* Tranſlator of *Plutarch* makes it *Pacianus*.

Moſt of the antient Geographers deſcribe the Coaſt Weſtward of *Carteia* in the following manner. *Julia Traducta, Mellaria, Bælo fluvius & oppidum, Portus Bæſippo, Promontorium Junonis*, &c. The Itinerary of *Antoninus*, makes no mention of *Julia Traducta*, and *Pliny* places it on the *African* Coaſt, which *Hardouin* endeavours to account for Pag. 227. in his *Nummi Illuſtrati*. *Strabo* calls it *Juliam Jozam*, which as *Bochart* obſerves Lib. I. C. 24. ſignifies the ſame in the *Phænician* Language as *Traductam* in the Latin. *Ptolomy* calls it *Tranſducta*. He places *Barbeſula* between that and *Carteia*. But all the other old Geographers put both the Town and River of that Name Eaſtward of *Calpe*. I ſaw ſome Ruins on the *Eaſt* ſide of the River *Guadiaro*, four Leagues *Eaſt* of *Gibraltar*; which I take to be the remains of the ancient *Barbeſula*. For I find in the *Cadiz Emporio del Orbe*, mention made of two pieces of Marble, brought from thence to *Gibraltar*; on one of which was M M BARBESVLANI. I was credibly informed they were uſed for the Fountain on the Parade. The Letters probably were either ſawed off, or turned inwards; for they do not appear. This *Barbeſula* is probably the *Barbariana* placed in the Itinerary X. M. P. *Eaſt* from *Carteia*.

Pompo-

Pomponius Mela, who was born in thoſe Parts, and therefore is moſt to be depended on, gives the following account of the Coaſt, according to the Edition of *Gronovius*. *Sinus ultra eſt, in eoque* Carteia, *ut quidam putant, aliquando* Tarteſſus, *& quam tranſvecti ex* Africâ Phænices *habitant*; *atque unde nos ſumus*, Tingentera. *Tum* Mellaria *&* Bælo, *&* Bæſippo *uſque ad* Junonis promontorium *oram freti occupat.* The Text of *Mela* in this place has occaſioned great diſputes among the Learned. *Caſaubon* in his Notes upon *Strabo*, ſays, *lego autem—atque unde nos ſumus* Tingi *contraria Mellaria, aut Tingi è regione ſita Mellaria. Nam* Tingis *factam hic à* Melâ *mentionem mihi eſt perſuaſiſſimum; primùm quidem veterem lectionem ſpectanti, quæ eſt, ut dixi,* Cingenteratum; *aut etiam ut in ſuis libris doctiſſimus* Elias Vinetus *reperit* Tingentera; *ut jam de eo dubitari non poſſit. Deinde autem video morem* Melæ *hunc eſſe, ut locorum in alterâ orâ oppoſitorum mentionem faciat. Sic alibi:* Majorem Sabæi tenent partem, oſtio proximam, & Carmanis contrariam Macæ. *Nec moveri quiſquam debet quod alii* Tingin Bæloni *non* Mellariæ *faciunt contrariam. Nam* Bælo *&* Mellaria *ita ſunt vicinæ, ut mirari hoc nemo debeat. Salmaſius*, whoſe opinion is approved by *Bochart*, makes it *Tingis altera, tum Mellaria*, &c. and takes the preceding *tranſvecti* to denote *Julia Traducta. Caſaubon* ſeems to have been once of the ſame Opinion. *Sed a* Strabone *ſtare* Ptolemæus *videtur, qui in hâc* Hiſpaniæ *orâ oppidum quoddam memorat cui nomen* Tranſducta, *in quod ſcilicet collocati fuerint iſti, de quibus nunc loquitur* Strabo; *& de quibus dubitavi aliquando, an hæc* Melæ *verba eſſent accipienda*, In eoque Carteia, ut quidam putant, aliquando Tarteſſus, & quam tranſvecti ex Africâ Phænices habitant. *Nam videbatur ſatis operiè* Tranſductam Ptolomæi *παραφράζειν. Nunc iis aſſentior qui ad* Carteiam *ea referunt.* The opinion of

* *Salma-*

Salmaſius ſeems to be the moſt probable; for *Bælo* and not *Julia Traducta* is ſaid to be over againſt *Tingis*. *Marcianus Heracleotes* makes the two former about 250 Stadia diſtant from one another, and *Mellaria* is generally placed between them; therefore they could not be ſo near one another as *Caſaubon* inſinuates. Tho' *Carteia* was originally founded by the *Phœnicians*, it had been erected into a *Roman* Colony long before *Mela's* time, and therefore he could not very properly ſay *Carteia, quam Phœnices habitant*; and had he intended to take notice of the Founders of that City, it is probable that one whoſe Stile is ſo pure and accurate, would have made uſe of another word, or at leaſt another Tenſe. Beſides, if *Julia Traducta*, according to *Caſaubon*, is not meant by that paſſage, it muſt have been entirely omitted by *Mela*; which is very unlikely, conſidering he was Born in or near it; and that it is mentioned by *Strabo*, who lived before him, and *Ptolomy* and ſeveral others who were after him; and appears to have been remaining at the time the *Vandals* were in poſſeſſion of *Spain*; for *Greg. Turon.* Lib. II. ſays *Proſequentibus* Alamannis *uſque ad* Traductam, *tranſito mari*, Vandali *per totam* Africam *ac* Mauritaniam *ſunt diſperſi*. The Letters of *Tingi altera* come nearer the *Tingentera* of *Elias Vinetus*, and the *Tinge Hiera* of *Gronovius*, than *Caſaubons Tingi contraria* or *Tingi è regione ſita*. The & and the *atque*, by making the ſtop at *Tarteſſus* inſtead of *Habitant*, may very well relate to the ſame place; and it is not improbable that *Mela* was deſirous to illuſtrate the obſcure place of his Birth by a Periphraſis, and a name of ſome *Eclat*; tho' it has happened, the method he took to do Honour to it, has been the occaſion, that we are in doubt even of its Name.

I met with two Medals of *Julia Traducta* among the Braſs *Spaniſh* Coins; but as I cannot aſcertain where they were found, I will not pretend to form from thence any judgment of the ſituation of the Town to which they belong. But I preſume in matters ſo dark, a conjecture may be offered. It does not ſeem very improbable, that *Julia Traducta* ſtood where *Tarifa* is at preſent. The *Spaniſh* Authors reckon that Town to have been built by *Tarif* at his ſecond coming to *Spain*. I cannot ſee what could invite him to ſettle on a Spot which has neither the convenience of a River, nor a Harbour, and is commanded by a riſing Ground; unleſs he found ſome Tenements ſtanding, or Ruins to ſerve for Materials to Build. I have ſeveral *Roman* Coins that were found there after great Rains, in the Common Sewer; which is ſome ſlight inducement to believe it was formerly a *Roman* Town.

About a League and half to the *Weſt* of *Tarifa*, is a place which goes now by the name of *Val de Vaca*. The Country People have a Tradition among them, that it was once a conſiderable Town, ſince ſwallowed up by the Sea. There is a ſmall Brook called *el Arroyo de Juan Franciſco*, which ſerves to turn ſome Mills, that a Prieſt of that Name was encouraged to build there, by finding an antient Stone Channel for the Water. I ſaw ſome other ſmall Ruins, and was credibly aſſured there are viſible remains of an old Town a good way under Water. There is a Shoal almoſt off this place, that runs pretty far in the Sea, on which a *Hamburgher* was loſt ſome Years ago. Perhaps *Mellaria* ſtood hereabouts.

Wherever it was, the Ruins of it muſt be a conſiderable way in the Sea, if credit is to be given to *Pliny*, who upon the Teſtimony of one Born there, reckons only five Miles from thence to *Afric*. Lib. III.

whereas

whereas it is at preſent five Leagues over at the narrow Part. *Caſaubon* is miſtaken in that Note on *Strabo* L. II. where he ſays, *At Maris Mediterranei oſtium vix LXX Stadia latum eſt* ἡ στενότατον.

I cannot help obſerving that the beſt Hony in all *Spain* is made in theſe Parts, and that the ſame cauſe to which the ancient *Mellaria* ow'd its Name, ſtill ſubſiſts, and has given a modern Appellation to ſeveral places hereabouts, as *Playa de Orimel*, *Rio de la Miel*, *Bejer de la Miel*. The latter of theſe is generally reckoned by the *Spaniards* to be the old *Mellaria*, for no other reaſon, that I can ſee, but the Name. For it is at leaſt two Leagues from the Coaſt of the Streights, and, by what I could judge when I was on the Spot, as near the Ocean, and therefore may as well be aſcribed to the one as the other. Whereas *Mellaria*, according to all the old Geographers, was ſituated on the Sea ſide in the Streights, and is reckoned by *Pliny* the neareſt Town to *Afric*; a plain proof that it was not what is now *Bejer de la Miel*.

About a League and half further *Weſt*, in a ſmall Bay, there are very great Ruins, which appear evidently to be the remains of a *Roman* Town. A League *Eaſtward* from that place, upon an Eminence, are to be ſeen the Quarries from which the Stone was fetched for building it; and all the way from thence are large remains of an Aqueduct, of which in ſome places there are entire Arches ſtill ſtanding. Among the Ruins of the old Town, I ſaw the Body of a *Roman* Statue of fine Alabaſter, ſomething bigger than the Life. Our Guide ſaid his Father had ſeen it entire; but as it was an Idol of the *Gentiles*, they, like good Catholicks, had broken it to pieces. He likewiſe told us that Urns of old Coins had been found there;

but not being Current in *Spain*, they had thrown them away. The place is called *Balonia*. It is over againſt *Tangier*, and frequently infeſted by the *Moors* from thence; on which account it is uninhabited. A ſmall River, called *Alpariate*, runs by it: all which circumſtances correſpond with the ancient accounts of *Bælo*. I have a Medal that was given me at *Tarifa*, with the following Letters upon it BAILO, which probably belonged to this City, called by *Ptolomy* Βαίλων. *Martianus Capella* Lib. VI. mentions it under the name of *Velonenſis Bæticæ Civitas*. The Itenerary of *Antoninus* places *Bælo* VI. M. P. *Weſt* of *Mellaria*, which is about the diſtance of theſe Ruins from *Val de Vaca*.

About five Leagues farther is the Cape of *Trafalgar*; the ſight of which immediately brought to my mind *Mela*'s deſcription of it. *Illud jam in Occidentem & Oceanum obliquo jugo excurrens, atque ei, quod Ampeluſium eſſe dixeramus, adverſum*, &c. Near the Capes Point are the Ruins often mentioned by the *Spaniſh* Authors, under the name of *Aguas de Mecca*. I was not there, but was aſſured at *Bejer de la Miel*, that there were ſtill ſome Ruins on the Shore, and more in the Sea, that run all along under the Cape; particularly remains of a Mole, which muſt have made it a tolerable Harbour. Theſe Ruins ſeem to be the remains of old *Bæſippo*. *Plin.* L. III. *Portus Bæſippo*. Mela *Bæſippo uſque ad Junonis promontorium, oram freti occupat*.

The placing of Watch-Towers along the Coaſt of *Spain* to Alarm the Country, upon any Deſcent, ſeems to have been a practice of a long ſtanding. *Livy* Lib. XXII. C. 19. *Multas & locis altis poſitas turres* Hiſpania *habet, quibus & ſpeculis & propugnaculis contra latrones utuntur: inde primo, conſpectis hoſtium navibus, datum ſignum* Aſdrubali *eſt*, &c.

III. *A Letter of M.* l' Abbé Conti, R. S. S. *to the late M.* Leibnitz, *concerning the diſpute about the Invention of the* Method *of* Fluxions, *or* Differential Method ; *with M.* Leibnitz *his Anſwer.*

J'Ai differé juſqu' à cette heure de repondre à votre Lettre, parce que j'ai voulu accompagner ma Réponſe de celle que M. *Newton* (*a*) vient de fair à l'Apoſtille que vous y avez ajoutée. Je n' entrerai dans aucun detail à l'égard de la diſpute que vous avez avec M. *Keill,* ou plutôt avec M. *Newton.* Je ne puis dire qu' hiſtoriquement ce que j'ai vû, & ce que j'ai lû, & ce qu'il me manque encore de voire & de lire, pour en juger comme il faut.

J'ai lû avec beaucoup d' attention, & ſans la moindre prevention, le *Commercium Epiſtolicum,* & le petit Livre (*b*) qui en contient *l' Extrait.* J'ai vû à la *Societé Royale* les Papiers Originaux des Letters du *Commercium* ; une petite (*c*) Lettre écrite de votre main à M. *Newton* ; & l' ancien Manuſcript (*d*) que M. *Newton* envoya au Docteur *Barrow,* & que M. *Jones* à publié depuis peu.

(*a*) In his Letter dated *Feb.* 26. 1715-16. *ſt. vet.* and Printed at the end of *Raphſon's* Hiſtory of Fluxions.

(*b*) Printed in the *Philoſ. Tranſ.* N. 342. and in Tome VII. *du Journal Literaire.*

(*c*) Dated 17 *March,* 1693. and Printed at the end of *Raphſon's* Hiſtory of Fluxions.

(*d*) Entituled *Analyſis per Series numero terminorum infinitas.*

De

De tout cela j'en infere, que ſi on ôte à la diſpute toutes les digreſſions étrangeres, il ne s'agit que de chercher ſi M. *Newton* avoit le Calcul des Fluxions ou infinitéſimal, avant vous, ou ſi vous l'avez eu avant lui. Vous l'avez publié le premier, il eſt vrai; mais vous avez avoué auſſi que M *Newton* en avoit laiſſé entrevoir beaucoup dans les Letters qu'il a écrites à Mr. *Oldenbourg* & aux autres. On prouve cela fort à long dans le *Commercium*, & dans ſon *Extrait*. Quelles ſon vos Réponſes? Voila ce qui manque encore au Public, pour juger exactement de l'affaire.

Vos amis attendent votre réponſe avec beaucoup d'impatience, & il leur ſemble que vous ne ſauriez vous diſpenſer de répondre, ſi non à M. *Keill*, du moins à M. *Newton* lui même, qui vous fait un deffi en termes exprès, comme vous verrez dans ſa Lettre.

Je voudrois vous voir en bonne intelligence. Le public ne profite guere des diſputes, & il perd ſans reſſource, pour bien de ſiecles, toutes les lumieres que ces mêmes diſputes lui dérobent.

Sa Majeſté a voulu que je l'informaſſe de tout ce qui s'eſt paſſé entre M. *Newton* & vous. Je l'ai fait de mon mieux, & je voudrois que ce fut avec ſuccès pour l'un & pour l'autre.

Votre Probleme a été reſolu fort aiſément en peu de tems. Pluſieurs Geometres à *Londres* & à *Oxford* en ont donné la Solution. Elle eſt générale, car elle s'étend à toutes ſortes de Courbes, ſoit Geometriques, ſoit Méchaniques. Le Probleme eſt un peu équivoquement propoſé: mais je croi que M. *de Moivre* ne ſe trompe pas, en diſant, qu'il faudroit fixer l'ideé d'une ſuite de Courbes. Par exemple ſuppoſer qu'elles ayent la même Soûtangeante pour la même Abſciſſe; ce qui conviendra non ſeulement aux Sections Coniques, mais à une infinité d'autres tant Geometriques que

 Me-

Mechaniques. On pourroit encore faire d'autres ſuppoſitions pour fixer cette idée.

Je vous parlerai une autre fois de la Philoſopie de M. *Newton*. Il faut convenir auparavant de la Methode de Philoſopher, & diſtinguer avec beaucoup de ſoin la Philoſophie de M. *Newton*, des conſequences que pluſieurs en tirent fort mal à propos. On attribuë à ce grand homme bien de choſes qu'il n'admet pas; comme il l'a fait voir à ces Meſſieurs Francois qui vinrent à Londres, à l'occaſion de la grand Eclipſe.

Je ſuis avec tout le reſpect poſſible

A *Londres* le Monſieur, vôtre &c.
de *Mars* 1716.

N. B. *Mr.* l'Abbé Conti *ſpent ſome Hours alſo in looking over the old Letters and Letter Books kept in the Archives of the* Royal Society, *to ſee if he could find any thing which made either for Mr.* Leibnitz, *or againſt Mr.* Newton, *and had been omitted in the* Commercium Epiſtolicum Collinii & aliorum : *but could find nothing of that kind.*

A Letter of M. Leibnitz *to M.* L' Abbe Conti, *in Anſwer to the former.*

Monſieur, Hanover *ce* 14. *d'*Avril, 1716.

POur ne vous faire attendre, je vous dirai par advance que j'ai répondu d'abord à l'honneur de votre Lettre, & en même tems à celle que Mr. *Newton* vous a écrite; & j'ai envoyé le tout à Mr. *Remond* à *Paris*, qui ne manquera pas de vous le faire tenir. Je me ſuis ſervi de cette voie, pour avoir des temoins neutres & intelligens de notre Diſpute: & M. *Remond* en fera encore part à d'autres. Je lui ai envoyé en même

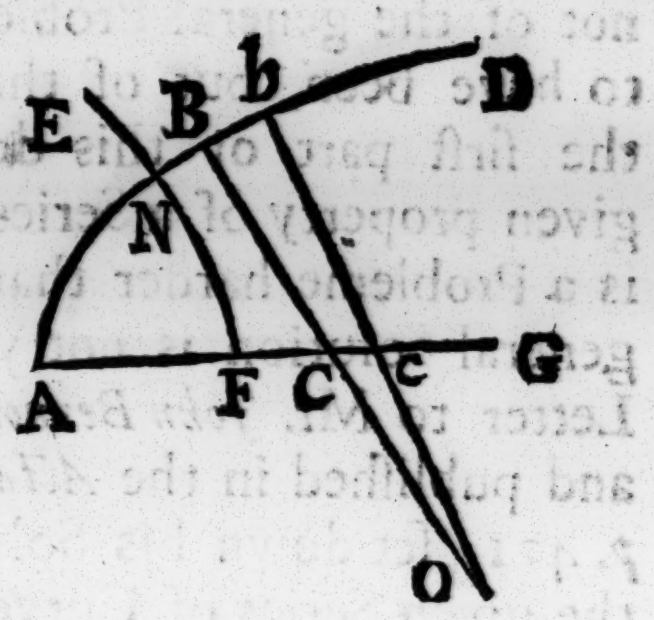
tems

tems une copie de votre Lettre & celle de Mr. *Newton*. Après cela vous pourrez juger, ſi la mauvaiſe chicane de quelques uns de vos nouveaux Amis m'embarraſſe beaucoup.

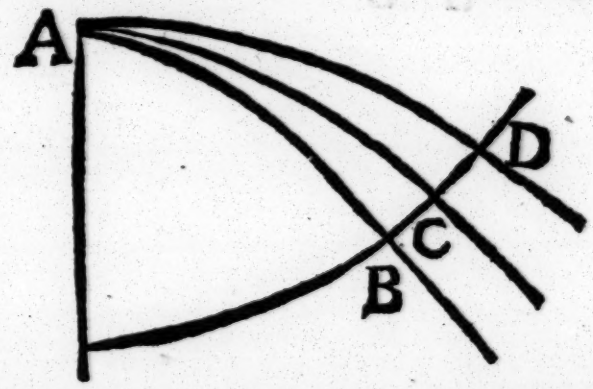
Quant au Probleme dont quelques-uns parmi eux ont voulu reſoudre des cas particuliers, pour en fixer, diſent-ils, les idées; il y a de l'apparence qu'ils ſe ſeront jettez ſur des cas faciles: car il y en a dans les Courbes Tranſcendantes, auſſi-bien que dans les ordinaires; mais il s'agit d'une ſolution generale. Ce Probleme n'eſt point nouveau. M. *Jean Bernouilli* l'a deja propoſé dans le mois de *May* des Actes de *Leipſic* 1697, *p.*211. Et comme M. *Fatio* mépriſoit ce que nous avions fait; on en repeta la propoſition pour lui & pour ſes ſemblables, dans les Actes de *May* 1700. *p.* 204. Il peut ſervir encore aujourd'hui à fair connoitre à quelques-uns, s'ils ſont allez auſſi avant que nous en Methodes: & en attendant qu'ils trouvent le moyen de parvenir à la ſolution génerale, ils pourront eſſayer ce qu'ils péuvent, en fixant les idées ſur un cas particulier, qu'on leur propoſe dans le papier cy-joint. Sa ſolution vient encore du même M. *Bernoulli*. Ainſi vous aurez la bonté de ne pas vous rendre trop tôt aux inſinuations de ceux qui nous ſont contraires; comme lorſqu'ils vous font a croire que notre Probleme leur étoit aiſé. Je ſuis avec zele, Monſieur Votre *&c.*

Problema continens caſum ſpecialem Problematis generalis de inveniendа Serie Curvarum, quarum quælibet ſit ad aliam Seriem Curvarum perpendicularis.

Super recta A G *tanquam axe, ex puncto* A *conſtructis Curvis quotcunque qualis eſt* A B D, *ejus naturæ ut radius oſculi ex ſingulis ſingularum Curvarum punctis* B *eductus* B O *ſecetur ab axe* A G *in* C *in Data ſemper conſtanti ratione: ut nempe ſit* B O *ad* B C *ut* M *ad* N. *Conſtruendæ jam ſunt Trajectoriæ qualis eſt* E N F, *priores Curvas* A B D *ſecantes ad angulos rectos.*



Thus far this Letter. Mr. *Leibnitz* firſt propoſed the general Probleme to M. *l'Abbé Conti* in theſe words; *Trouver une ligne* B C D, *qui coupe à angles droits toutes les courbes d' une ſuite determinée d' une même gendre; par exemple, toutes les Hyperboles* A B, A C, A D, *qui ont le même ſommet & le même centre; & cela par une voye generale.* And in the *Acta Eruditorum* for *October*, 1698. *p.* 470, 471. he calls the Curves in this determinate Series, *Curvas ordinatim datas, & poſitione datas, & poſitione ordinatim datas.* And by all this, the Series of Curves to be cut is given, and nothing more is to be found, than the other Series which is to cut it at right Angles. But Mr. *Leibnitz* being told that his Probleme was ſolved, he changed it into a new one, of finding both the Series to be cut and the other Series which is to cut it. And the particu-

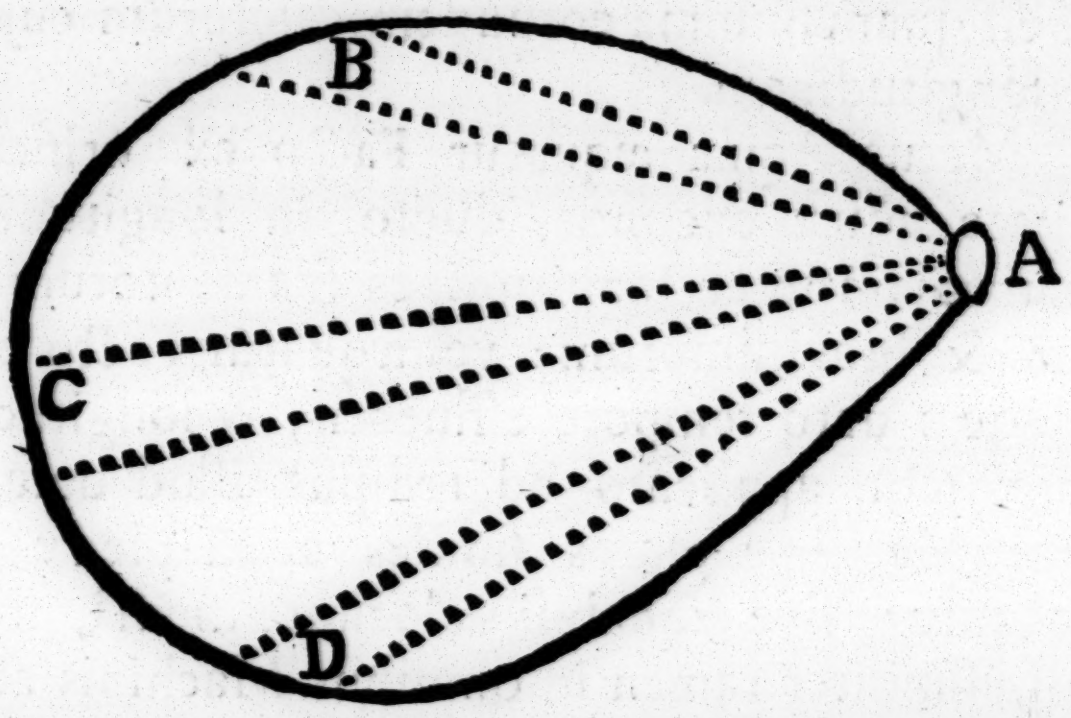


lar Probleme propoſed in this Letter is a ſpecial Caſe, not of the general Probleme firſt propoſed, as it ought to have been, but of this new double Probleme. And the firſt part of this double Probleme (*viz.* by any given property of a Series of Curves to find the Curves) is a Probleme harder than the former, and of which a general Solution is not yet given. Mr. *Leibnitz* in a Letter to Mr. *John Bernoulli*, dated 16 *December*, 1694. and publiſhed in the *Acta Eruditorum* for *October* 1698. *p.* 471, ſet down his Solution of the Probleme, when the given Series of Curves is defined by a finite Equation, expreſſing the relation between the Abſciſs and Ordinate. The ſame Solution holds when the Equation is a converging Series, or when the property of the Curve to be cut can be reduced to ſuch an Equation, by the *Analyſis per Series numero terminorum infinitas*. But Mr. *Leibnitz* was for ſolving the Probleme without converging Series.

IV. *Pars reliqua Dissertationis* De Potentiâ Cordis. *Authore* Jacobo Jurin, *M. D.* & R. Societatis Soc.

Theorema II.

SI ex Machinâ cavâ inæqualiter contractili, A B C D, *aqua per Machinæ contractionem exprimatur, Motus aquæ ex orificio* A *prosilientis æquatur Summæ Factorum ex*



Sectionibus quibusvis transversis omnium aquæ filamentorum A B, A C, A D; *singulis ductis in longitudines & velocitates respectivas.*

Demonstratio. Loco filamentorum aquæ, concipiatur Machina tubis minimis, inæqualiter amplis, A B, A C, A D, in orificium A desinentibus, tota constare.

Est aquæ Motus in quovis tubo æqualis sectioni cuivis ipsius tubi, ductæ in velocitatem aquæ per sectionem istam fluentis, & longitudinem tubi, per *Theor.* 3.

De Motu Aquar. fluent. Proinde Summa Motuum aquæ in omnibus tubis ſimul ſumptis, ſive Motus aquæ ex Machinæ orificio prorumpentis, æqualis eſt Summæ Factorum ex omnium tuborum ſive filamentorum aquæ ſectionibus, ductis in longitudines, & velocitates, reſpectivas. *Q. E. D.*

Corol. 1. Motus aquæ effluentis minor eſt Facto ex orificio A, velocitate aquæ exeuntis, & longitudine filamenti aquæ omnium longiſſimi. Eſt enim Factum ex orificio & velocitate aquæ effluentis, æquale Summæ Factorum ex ſectionibus filamentorum ſingulis ductis in velocitates reſpectivas; & Summa horum Factorum, ducta in longitudinem filamenti omnium longiſſimi, major eſt quam Summa eorundem ductorum cujuſque in ſuam longitudinem.

2. Motus Aquæ æquatur Facto ex orificio A & velocitate aquæ exeuntis, ducto in longitudinem aliquam mediam inter longitudines filamentorum longiſſimorum & breviſſimorum: vel æquatur Facto ex quantitate aquæ dato tempore effluentis, & longitudine mediâ prædictâ, applicato ad tempus illud datum.

3. Si Machinæ plures ſimiles aquâ plenæ ſimiliter contrahantur, ſive æquabili velocitate mediâ, ſive inæquabili, ſimiliter tamen in omnibus Machinis auctâ, vel imminutâ; Motus, quo aqua ex Machinæ cujuſque orificio prorumpit, rationem habet compoſitam ex ratione quadruplicatâ Diametri cujuſvis homologæ ipſius Machinæ, & reciprocâ temporis ratione, quo peragitur Machinæ contractio: vel rationem compoſitam, ex ratione ponderis Machinæ, vel molis aquæ, ſive Machinâ contentæ, ſive ex eâdem expulſæ, ratione ejuſdem ponderis, vel molis, ſubtriplicatâ, & ratione temporis reciprocâ.

PROBLEMA.

Invenire Potentiam Cordis.

Sit p = Pondus Ventriculi ſiniſtri, ſive quantitas Sanguinis eidem ponderi æqualis.

S = Superficies interna ejuſdem.

l = Longitudo media filamentorum Sanguinis ex eodem prodeuntium.

s = Sectio Aortæ.

q = Quantitas Sanguinis Ventriculo ſiniſtro contenti.

t = Tempus, quo Sanguis ex Corde expelleretur, ſublatâ Arteriarum & Sanguinis præcedentis reſiſtentiâ.

v = Velocitas variabilis, quâ Sanguis ex Corde proſiliens per Aortam flueret, ſublatâ reſiſtentiâ.

x = Longitudo variabilis Aortæ à Sanguine ex Corde effluente percurſa.

z = Tempus, quo longitudo x percurritur.

Inde velocitas media variabilis Sanguinis Ventriculo contigui, ſive media velocitas ipſius Ventriculi $= \frac{sv}{S}$.

Motus Ventriculi (per *Theor.* 1. *Cor* 2.) $= p \times \frac{sv}{S}$.

Motus Sanguinis effluentis (per *Theor*, 2. *Cor.* 2) $= sv \times \overline{l + x}$.

Horum Summa, ſive Potentia Ventriculi $= sv \times \frac{p}{S} + l + x$. Eſt autem $v = \frac{\dot{x}}{\dot{z}}$. Unde per Methodum *Newtonianam* inverſam, elicitur Potentia Ventriculi $= \frac{sx}{z} \times \frac{p}{S} + \frac{x}{2} + l$. Sed cum $z = t$, erit $sx = q$.

Hinc Potentia Ventriculi $= \frac{q}{t} \times \overline{\frac{p}{S} + \frac{q}{2s} + l}$.

Simili

Simili ratione invenitur Potentia dextri Ventriculi

$$= \frac{q}{t} \times \frac{\pi}{\Sigma} + \frac{q}{2\sigma} + \lambda.$$

Literis autem Græcis eadem ſignificantur in dextro Ventriculo, quæ Latinis in ſiniſtro.

Hinc tota Cordis Potentia

$$= \frac{q}{t} \times \frac{p}{S} + \frac{\pi}{\Sigma} + \frac{q}{2S} + \frac{q}{2\sigma} + l + \lambda. \quad Q.\ E.\ I.$$

Si ponatur

$p = 8$ unc. Avoird $= 13.128$ unc. cub.
$\pi = 4$ $= 6.564$
$S = 10$ unc. quadrat.
$\Sigma = 10$
$l = 2$ unc.
$\lambda = 1\frac{1}{2}$.
$q = 2$ unc. Avoird. $= 3.282$ unc. cub.
$s = 0.4185$ unc. quadrat. } Ex *Keillianis* Experimentis.
$\sigma = 0.583$
$t = 0.1''$

Erit Potentia Ventriculorum æqualis motui ponderum ſubſcriptorum, nempe,

	lib. unc.
Ventriculi ſiniſtri	9 . 1
Ventriculi dextri	6 . 3
Cordis totius	15 . 4

Quorum ponderum ea eſt velocitas, quâ percurratur longitudo uncialis ſingulis minutis ſecundis.

Cor. 1. Quoties Pulſus fit celerior; aut minuitur reſiſtentia, aut Potentia Cordis augetur, aut minor ſolito Sanguinis copia ſingulis contractionibus ex Corde expellitur.

2. Si Pulſus ſolito tardior fiat; neceſſe eſt, vel augeatur reſiſtentia, vel Cordis Potentia minuatur, vel major Sanguinis moles ex Corde ejiciatur.

3. Auctâ reſiſtentiâ, neceſſario vel Pulſus retardabitur, vel augebitur Cordis Potentia, vel Sanguinis quantitas ſolito minor ex Corde exprimetur.

4. Imminutâ reſiſtentiâ, vel Pulſus acceleratur, vel major Sanguinis copia quâque Syſtole ejicitur, vel Cordis vires minuuntur.

5. Auctis Cordis viribus, neceſſario vel augebitur reſiſtentia, vel Pulſus accelerabitur, vel plus Sanguinis ex Corde ejicietur.

6. Viribus Cordis imminutis, vel minuatur neceſſe eſt reſiſtentia, vel Pulſus tardior fiat, vel minus Sanguinis ex Corde exprimatur.

7. Cum minor Sanguinis moles ex Corde projicitur; vel acceleratur Pulſus, vel Cordis vires minuuntur, vel augetur reſiſtentia.

8. Cum plus Sanguinis ex Corde exprimitur; vel Pulſus tardior fiet, vel augebitur Cordis Potentia, vel reſiſtentia minuetur.

Schol. 1. Ventriculorum ſuperficies internas, cum factu difficillimum videatur, ut accurate determinentur, aut etiam ratio habeatur imminutionis, quam inter contrahendum patiuntur, contenti fuimus præterpropter æſtimare: cum ſive eaſdem 12, ſive 8 unciis quadratis ſingulas æquales ſtatueris, perparva reperiatur Potentiarum facta mutatio. Quod etiam obſervari poterit de longitudine mediâ filamentorum Sanguinis. Præterea differentias, quâ Arteriæ ambæ, earumque rami proximi à Corde progredientes, ſectione augentur, ut æſtimatu perdifficiles & pene inſenſiles, negligimus. Alioqui eſſet Cordis Potentia tantillo minor ſtatuenda, quam quæ ſupra definita eſt.

2. Determinavit Vir Celeberrimus, *Jacobus Keillius*, velocitatem Sanguinis, reſiſtentiâ ſubmotâ, ex Corde effluentis, eam circiter, quâ percurrantur pedes $6\frac{1}{2}$ ſingulis minutis ſecundis. Ponit vero ille celeritatem San-

guinis

guinis per totam Syſtolem æquabilem, quam nos inſigniter inæqualem fieri, & perpetim à Syſtoles initio retardari ſupra oſtendimus. Hanc ſi cui definire libuerit, ſubſtituenda eſt, in quartâ Æquatione ſupra poſitâ, Potentia Ventriculi proxime inventa, & ipſi x valor quivis tribuendus, ut eliciatur v, ſive velocitas eidem reſpondens. Ita, cum initio Syſtoles ſit $x = 0$, ſub finem vero $x = \frac{1}{5}$, determinatur inde ea Sanguinis velocitas initio Syſtoles, quâ pedes $14\frac{1}{4}$; in fine autem quâ $4\frac{1}{4}$, minuti ſecundi ſpatio percurrantur. Pariter in dextro Ventriculo: velocitas Sanguinis initialis pedes circiter $10\frac{5}{6}$, ultima vero 3 pedes eodem temporis ſpatio conficiet.

Adhibuimus hactenus eam Hypotheſin, quâ Muſculi Cordis Ventriculos conſtituentes Motum omnem, quo adiguntur in contractionem, Momento temporis concipiunt. Quod ſi ponamus Motum iis communicari non unico quidem Momento, ſed tantillo tamen temporis ſpatio, quod cum totâ Syſtoles duratione comparatum rationem obtineat admodum exiguam; erit Cordis Potentia paululo major ſtatuenda, quam quæ ſupra determinata eſt. Si vero ſtatuatur iſte Motus, procedente Syſtole, in ratione temporis augeri; erit totus Motus in fine Syſtoles acquiſitus duplo major quam ſupra poſuimus, ubi nulla reſiſtentia Sanguini ex Corde profluenti objicitur: Ubi autem ſolita adeſt reſiſtentia, erit idem quintuplo major; quod inſtituto calculo facile patebit. Pari ratione poterit calculus noſter ad aliam quamlibet Hypotheſin, quâ Ventriculorum Motus in duplicatâ vel ſuperiori quâvis ratione temporis augeatur, accommodari. Potentia vero in fine acquiſita ſuprapoſitâ elicietur longe major, nempe ex ratione duplicatâ Potentia tripla, ex triplicatâ quadrupla, ex quadruplicatâ quintupla, & ſic in infinitum.

Nobis

Nobis autem videtur ſecunda Hypotheſis, quâ Ventriculi parvo admodum temporis ſpatio Motum omnem concipiunt, cæteris longe veriſimilior. Quum neceſſe ſit, ut aliquid temporis impendatur ad Motum quemlibet generandum; neque videatur adeo tarde increſcere Ventriculorum Motus, ut non celerius augeatur, quam ſecundum temporis rationem. Motus enim Muſculorum impetu ſolo Fluidorum quorumcumque, quæ ex Sanguine proveniunt, perfici nequit; quum Brachio alterutro Motum exerere poſſimus Motu Sanguinis per vaſa Corporis univerſa profluentis longe majorem. Relinquitur ergo, ut Muſculorum fibræ Ventriculos Cordis conſtituentium, rareſcentiâ quâdam liquorum in eaſdem influentium, in Motum impellantur. Hæc autem, quoties vim magnam concipit, plerumque ſubita eſt, & fere inſtantanea. Adde quod Ventriculorum Motus ſecundum hanc Hypotheſin longe minor efficitur, quam in tertiâ. Non ſolet autem ſapientiſſimus Artifex, Rerum Conditor, in operibus ſuis plus Virium adhibere, quam quantum ſufficit ad finem propoſitum conſequendum.

Cæterum ſive admittatur iſta Hypotheſis, ſive alia quæcunque ex ſupra dictis verior cenſeatur, poterunt omnia Corollaria noſtra eodem jure ex Problemate deduci. Quæ utrum aliquid adjumenti afferant ad Morborum Hiſtoriam explicandam Medico ſagaci conſiderandum permittimus. Facile autem ex Morbi cujuſque Naturâ ſciri poterit, utrum aucta ſit vel imminuta reſiſtentia. Augeri vero credibile eſt vel imminui Cordis vires auctis vel imminutis Muſculorum reliquorum viribus; quamvis aliter ſtatuiſſe video Virum Celeberrimum, *Laurentium Bellinum.*

Theorema III.

Totus Motus reſiſtentiæ, quæ Sanguini ex Corde erumpenti durante Syſtole objicitur, ſive totus Motus, qui Sanguini præcedenti & Arteriarum tunicis communicatur, toti Cordis Potentiæ quamproxime æqualis eſt.

Dem. Peractâ Cordis Syſtole, quæ pars Aortæ & Arteriæ Pulmonalis Cordi proxima eſt, perſtat plena Sanguine per totam Syſtolem Arteriarum. Nec enim patitur earum fabrica & nexus, quo Cordi conjunctæ ſunt, ut tunicis in ſeſe penitus collabentibus totæ occludantur, neque poteſt earum cavum Sanguine vacare. Alioqui enim, contrahentibus ſeſe reliquis Arteriarum partibus, Sanguis iiſdem contentus retro in vacuum impelleretur motu, & inutili & motui Sanguinis naturali contrario. Tum etiam Valvulæ Semilunares non tenderentur verſus Ventriculos, adeoque Sanguis ex Auriculis in Ventriculos expreſſus, etiam in Diaſtole Cordis, in Arterias protruderetur.

Hinc patet Sanguinem proxime ex Corde expulſum Syſtole peractâ immotum in Arteriis perſiſtere, adeoque tum omnem Ventriculorum Motum excepiſſe, tum eundem totum partim Sanguini antecedenti, partim tunicis Arteriarum communicaſſe. *Q. E. D.*

Theorema IV.

Motus, qui in Syſtole Cordis communicatur Sanguini præcedenti, eſt ad Motum tunicis Arteriarum communicatum, ut tempus Syſtoles Cordis ad tempus Diaſtoles quam proxime.

Dem. Quum Sanguis per vaſa Corporis univerſa, ſi partes Arteriarum Cordi propiores exceperis, æquabili curſu deferatur; neceſſe eſt, ut tum Motus affrictu Sanguinis ad vaſorum latera deperditus, tum Motus Sanguini redditus à Syſtole ſive Cordis ſive Arteriarum

rum, æqualibus temporibus æqualis ſit. Qui autem Motus à Syſtole Arteriarum Sanguini communicatur, idem eſt præciſe, qui prius à Cordis Syſtole Arteriarum tunicis fuerat impreſſus, cum Arteriæ eodem impetu quo diſtractæ fuerint etiam reſtituantur. Et Syſtole Arteriarum cum Cordis Diaſtole duratione convenit. Unde patet Propoſitum. *Q. E. D.*

Cor. Si ponamus cum Viro Doctiſſimo *Jacobo Keillio*, Syſtolen Cordis peragi tertiâ parte temporis inter Pulſus binos intercepti ; erit Motus Sanguini præcedenti communicatus totius Potentiæ Cordis pars tertia : Motus vero Arteriis communicatus prioris duplus, ſive duæ partes tertiæ totius Cordis Potentiæ.

Theorema V.

In diverſis Animalibus Potentia Cordis rationem obtinet compoſitam, ex ratione quadruplicatâ Diametri cujuſvis homologæ Animalis, & ratione inversâ temporis, quo Cor contrahitur : vel rationem compoſitam, ex ratione ponderis vel ipſius Cordis vel integri Animalis, ratione ponderis ejuſdem ſubtriplicatâ, & ratione temporis reciprocâ.

Facile demonſtratur vel ex *Corol.* 3. *Theor.* 1 & 2. vel ex Potentiâ Cordis Problemate præcedente definitâ,

Cor. 1. Si ponatur Cordis Potentiam rationem obtinere ponderis vel ipſius Cordis, vel integri Animalis, vel Sanguinis copiæ in toto Animali ; erit Animalis longitudo in ratione temporis, quo Cordis Syſtole perficitur, ſive in ratione inversâ frequentiæ Pulſuum.

2. Si ratio longitudinis integri Animalis major fuerit ratione inversâ frequentiæ Pulſuum, neceſſe eſt major ſit ratio Potentiæ Cordis ratione ponderis ejuſdem.

Schol. Quum conſtet Experimentis Puerorum Pulſus non eſſe tanto frequentiores Pulſibus Virorum, quanto Pueri Virorum longitudine ſuperantur, concludendum

eſt, vi ſecundi Corollarii, Potentiam Cordis Virilis majorem obtinere rationem ad Potentiam Cordis Pueri, quam eſt ratio ponderum. Et par eſt ratio in cæteris Muſculis. Nam ſi Corporis robur rationem ponderis ſequeretur, poſſent Pueri æqualia itinerum ſpatia eodem tempore cum Viris conficere.

Simili ratione ac Motum Sanguinis ex Ventriculis Cordis erumpentis ope ſecundi *Theorematis* determinavimus, poterit quoque Urinæ Motus ex Urethrâ profluentis determinari. Nempe ſi ponatur Urethræ & Veſicæ longitudo 12 unciis æqualis, & binæ unciæ Urinæ minuti ſecundi ſpatio emittantur. erit Motus Urinæ effluentis æqualis Motui ponderis libræ $1\frac{1}{2}$, quod uncialem longitudinem ſingulis minutis ſecundis percurrat Quoniam vero Urina non ſolis Veſicæ Urinariæ viribus contractivis, ſed etiam Diaphragmatis & Muſculorum abdominalium ope in ſubſidium vocatâ, expellitur, nequit Veſicæ Potentia ex Motu Urinæ profluentis æſtimari.

Hæc tu, Vir Doctiſſime, æqui bonique conſulas rogo: ipſe autem ut diutiſſime valeas, utque exiſtimationem tuam, & ipſam Artis Æſculapiæ dignitatem uſque ut hactenus feciſti, inſigniter tueri pergas, ac magis indies magiſque extendere, idcirco ex animo voveo, quia publicam ad ſalutem pertinere arbitror.

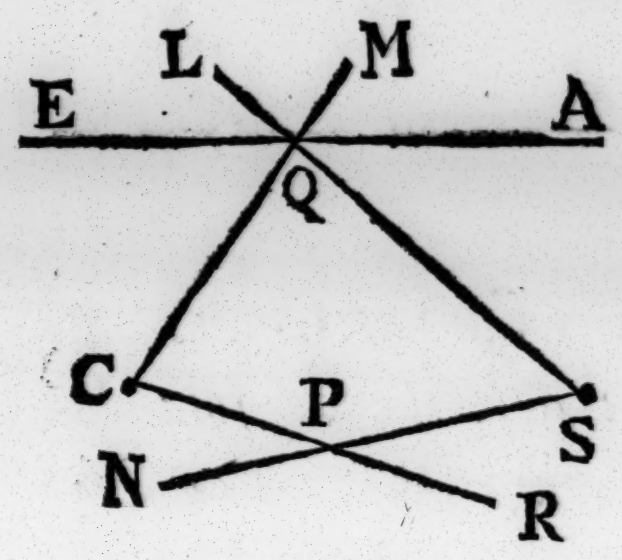
Calendis *Januarii*,
$171\frac{7}{8}$.

V. *Nova Methodus Universalis Curvas Omnes cujuscunque Ordinis Mechanicæ describendi sola datorum Angulorum & Rectarum Ope.* Per Colin Maclaurin *in Collegio Novo* Abredonensi *Matheseos Professore.*

INter innumera sublimiaque Magni *Newtoni* inventa, quibus Geometria ampliſſime ditata in immensam excrevit luculentiſſimæ Cognitionis molem, Conſtructionem exhibuit Curvarum Mechanicam, poſt Enumerationem Linearum Tertii Ordinis, ad finem *Optic.* editam, arduo ſummi Viri ingenio dignam; quas ſimpliciorem & ſimul adeo Univerſalem aliam exhibuit Nemo. Methodum vero ſuam ad Curvas Tertii Ordinis puncto duplice carentes, aut eas altiores Ordinis puncto multiplice deſtitutas, non extendit; earumque deſcriptionem Problematibus Geometriæ difficilioribus annumerandam pronuntiat. Atque hinc in ſpem venio Methodum ſequentem, qua Curvæ Geometricæ cujuſcunque Ordinis, licet puncto duplice aut multiplice quovis deſtitutæ, conſtruuntur, non fore Geometris ingratam.

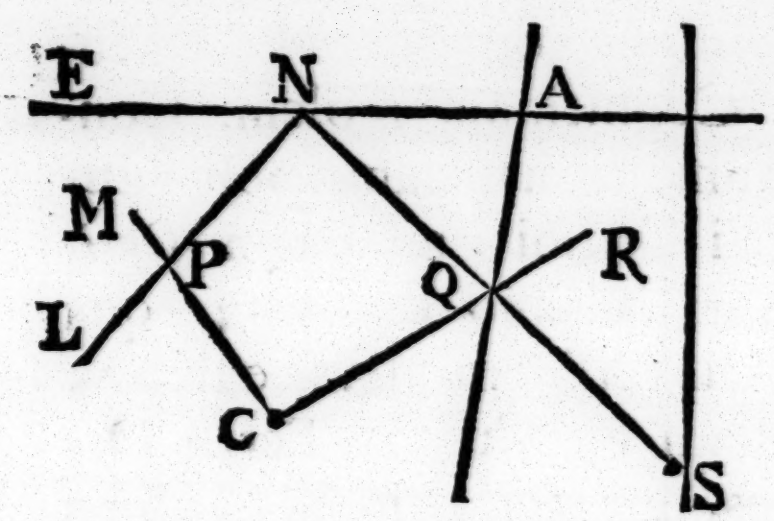
I. Lineæ primi Ordinis ipſæ ſunt Rectæ; quæ in uno ſolo puncto ſibi mutuo occurrere poſſunt. Lineæ ſecundi Ordinis ſunt Sectiones Conicæ, quæ in pluribus punctis quam duobus à rectâ quavis ſecari non poſſunt. Quæ vero omnes ſecundum Lemma 21. Lib. I. *Princip.* D. *Newtoni* ſic conſtrui poſſunt, Circa data duo puncta

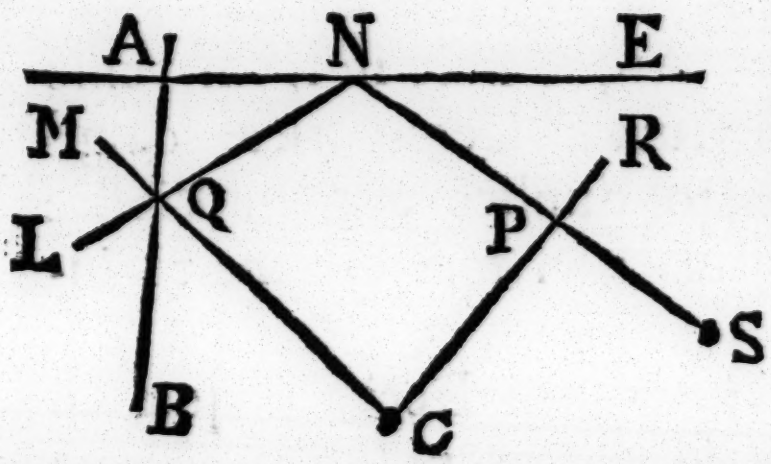
C & S



C & S moveantur Anguli dati MCR, LSN; ita ut Crurum CM & SL concurſus ſemper ducatur per rectam indefinitam poſitione datam AE; tunc crurum aliorum CR & SN concurſus in P deſcribet Lineam ſecundi Ordinis ſeu Sectionem Conicam.

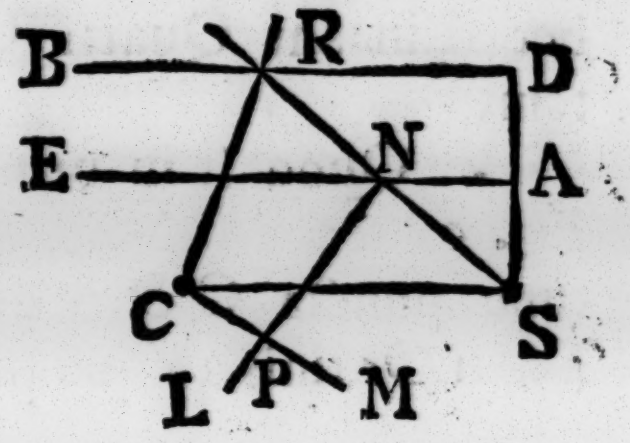
II. Moveatur ut prius Angulus MCR (*v. Fig.* 2.) circa datum punctum C; Angulus vero datus LNQ ſemper percurrat Angulari ſuo puncto N rectam datam AE, ita ut crus NQ ſemper tranſeat per datum punctum S. 1. Si concurſus crurum CR & SN, tum punctum Q ducatur per rectam infinitam AB, concurſus crurum CM & NL deſcribet Curvam lineam Tertii Ordinis punctum duplex habentem in C. 2. Reliquis manentibus, ſi crurum CM & NL concurſus (*vide Fig.* 3.) ducatur per rectam indefinitam AB: concurſus crurum CR & SN in P deſcribet Curvam Tertii Ordinis punctum duplex habentem in S.



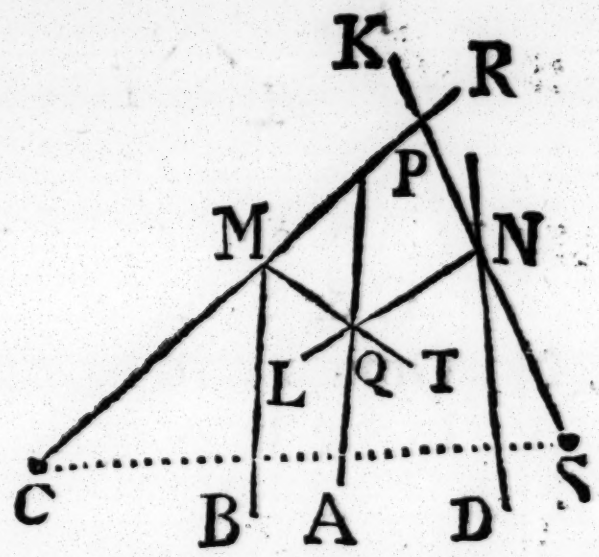


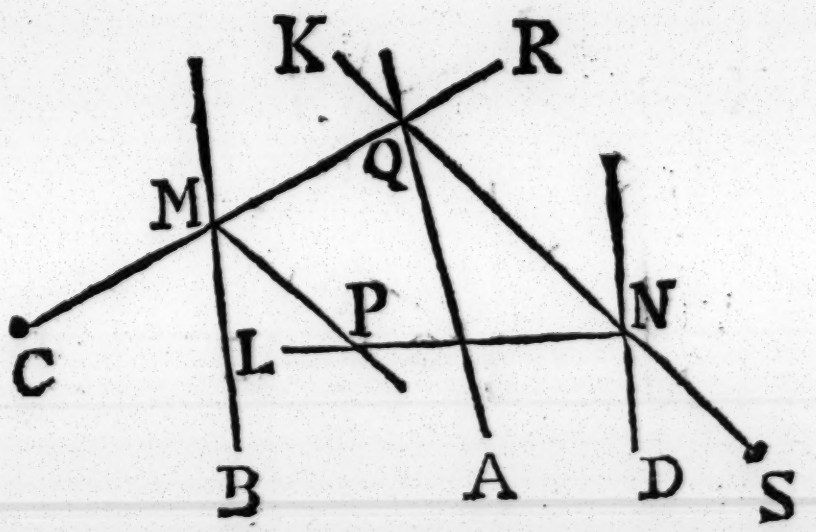
Exem-

Exemplum Caſus 1. Sint anguli MCR, LNS recti, (*vide Fig.* 4) & AE, DB, CS parallelæ; ſint quoque SA & SD normales reſpective in rectas AE & DB; ſitque SD = 2SA. Hiſce poſitis, ſi SD ſit minor recta CS, Curva ſecundum regulam Caſus primi deſcripta, erit Parabola Nodata cum Ovali, Speciei 68*væ* Curvarum D. *Neutoni*; Quod ſi SD = CS, Ovalis evaneſcit & nodus evadit Cuſpis, atque Curva deſcripta erit Parabola *Neiliana* ſeu ſemicubica; Si vero ſit SD major quam CS, erit Curva Parabola punctata Campaniformis Speciei 69*na*.



III. Moveantur Anguli dati RMT, KNL, ita ut puncta M & N percurrant rectas indefinitas BM, DN reſpective; & crura RM, KN ſemper tranſeant per data puncta C & S. Si primo Crurum MT & NL concurſus Q ducatur per rectam indefinitam AQ; tunc concurſus crurum MR & NR in P deſcribet lineam Quarti Ordinis puncta duo duplicia habentem, alterum in C alterum vero in S. Sed ſecundo ſi crurum MR & NK (*vide Fig.* 6.) concurſus ducatur per rectam indefini-

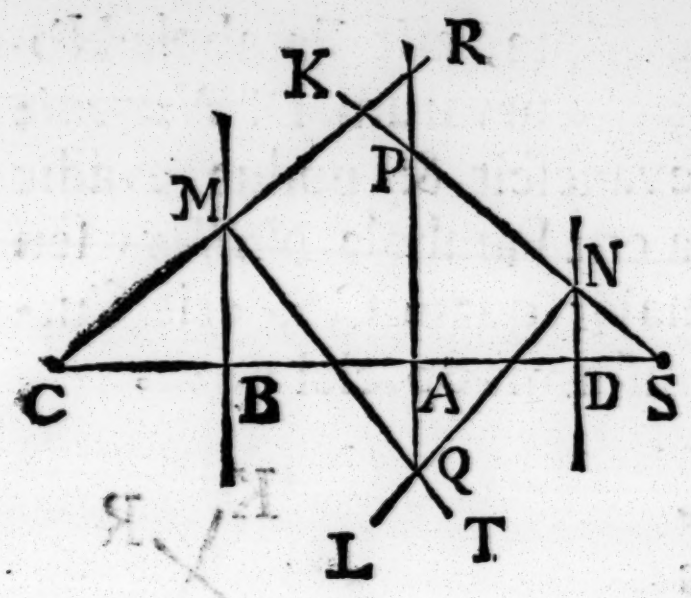




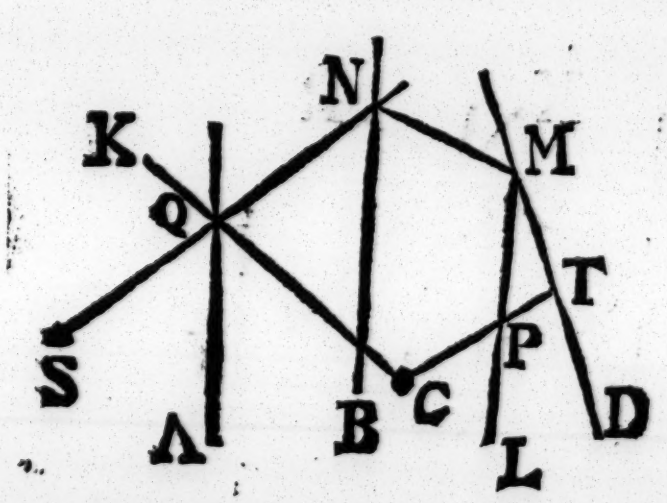
tam

tam A Q; tunc concurſus crurum M T & N L deſcribet Lineam Quarti Ordinis puncto duplice carentem.

IV. Quod ſi in primo caſu hujus Conſtructionis (*v. Fig.* 5.) rectæ C M R, S N K, una coincidant cum C S; tunc puncta C & S evadunt ſimplicia & Curva erit Tertii Ordinis abſque puncto duplice. Exemplum. Sint rectæ B M, A Q, D N, ſibi mutuo parallelæ atque omnes perpendiculares in C S. Sint quoque Anguli R M T, K N L recti, & ſi ſecundum regulam primi Caſus deſcribatur Curva, Crura C M R, S N K una coincident cum C S; & hac conſtructione deſcribi poſſunt Curvæ D. *Newtoni* 10, 11, 20, 21, 40, ſecundum varias poſitiones punctorum C & S reſpectu trium rectarum B M, A Q, D N; Omnes vero hæ Species puncto duplice carent.

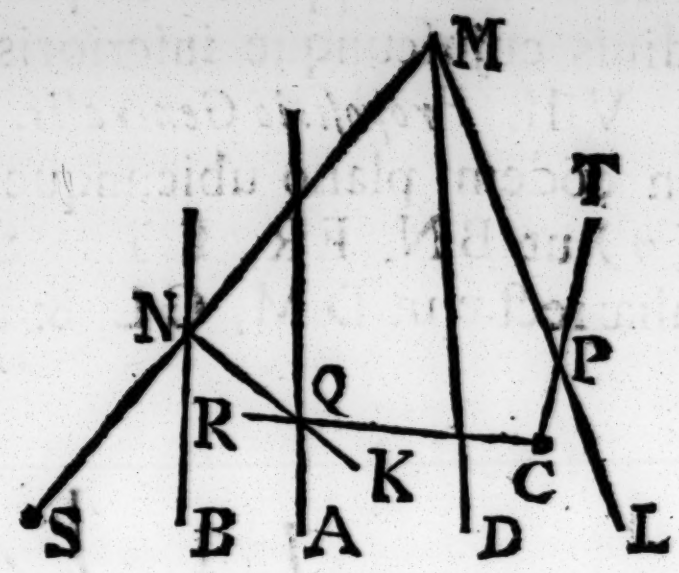


V. Lineæ vero Quarti Ordinis quæ punctum triplex habent ſic conſtrui poſſunt. Sint tres rectæ A Q, B N, D M poſitione datæ; ſint etiam Anguli Q C T, S N M & N M L dati & invariabiles; percurrant puncta N & M rectas B N & D M, ita ut crus N Q ſemper tranſeat per datum punctum S: Revolvatur Q C T circa C ita ut concurſus crurum C K, S N percurrat tertiam rectam A Q; tunc concurſus



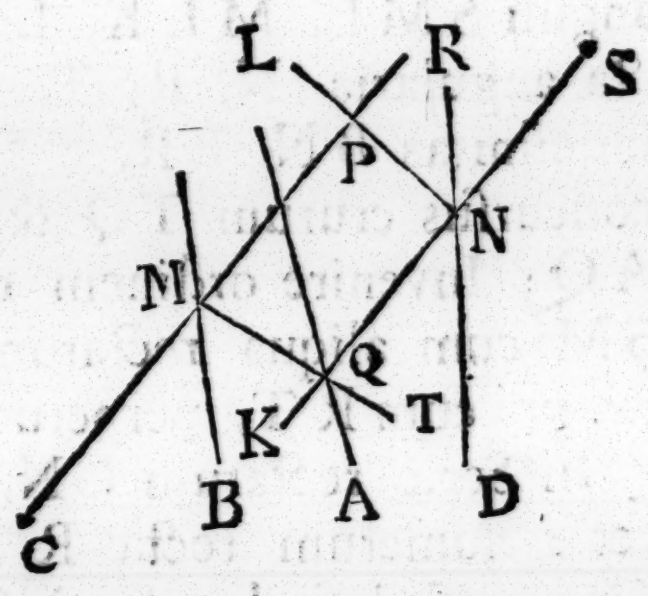
ſus crurum C T, M L deſcribet Lineam Quarti ordinis punctum triplex habentem in C.

VI. Oſtendi quo pacto Lineæ Quarti Ordinis deſcribi poſſunt, quæ punctum triplex habent aut duo duplicia; Aliæ quæ unicum habent punctum duplex ſic commode deſcribuntur.



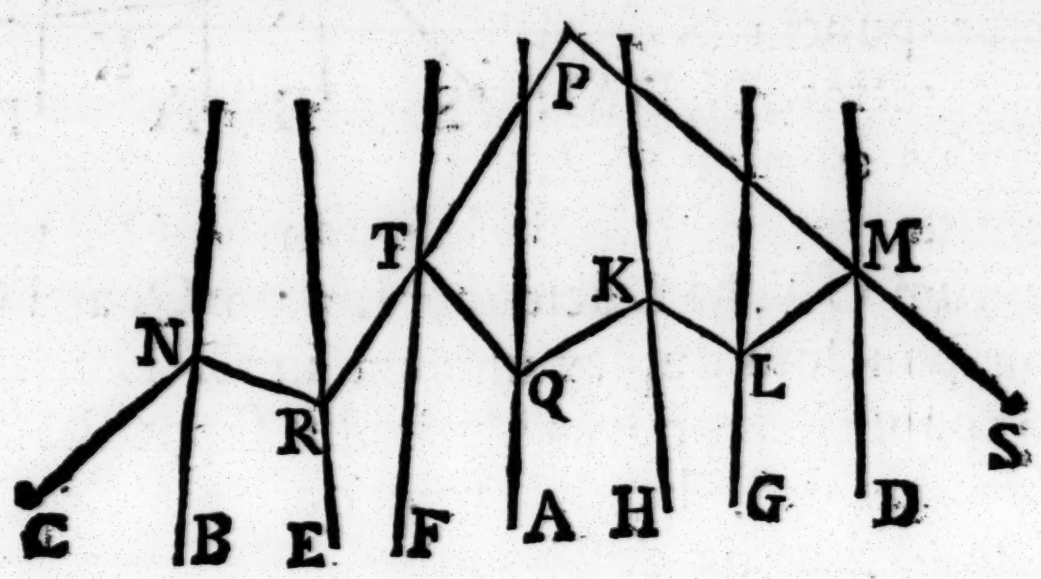
Sint tres rectæ ut prius poſitione datæ, A Q, B N, D M, dentur etiam Anguli S N K, S M L, R C T; ſint puncta N, M & S ſemper in eadem recta linea; Moveantur puncta N & M ut prius per rectas B N, D M; Si concurſus crurum C R, N K ducatur per rectam indefinitam A Q, tunc concurſus crurum C T, M L deſcribet Lineam Quarti Ordinis habentem punctum duplex unicum in C. Hæ vero duæ ultimæ Propoſitiones novas Methodus ſuppeditant lineas Tertii Ordinis deſcribendi, tum quæ puncta duplicia habent, tum quæ iis deſtituuntur; Eæ vero in brevi hoc Methodus Noſtræ ſpecimine ſunt omittendæ.

VII. Maneant Anguli atque rectæ ut in *Prop.* III. Concurſus vero nunc rectarum M T, N K ducatur per indefinitam rectam A Q; & Concurſus crurum M R & N L deſcribet Lineam Quinti Ordinis punctum quadruplex habentem in S. Habeo etiam alias Methodus curvas deſcribendi Quinti Ordinis, quæ punctum habent triplex, duplex, aut duo duplicia, vel nulla niſi puncta ſimplicia; ſed



hæc ſufficiant ad ſimplicitatem & univerſalitatem Methodus demonſtrandam. Notandum vero in ſpecialibus ſimplicioribus Angulorum & rectarum circumſtantiis, Lineam aliquando migrare in curvam ordinis inferioris quam in *Prop.* explicatur; Imo ſingulæ Propoſitiones Methodus ſuppeditant particulares, curvas aliquas ordinis cujuſcunque inferioris deſcribendi.

VIII. *Propoſitio Generalis.* Sumantur ad libitum Rectæ in eodem plano ubicunque poſitæ, quarum ſit numerus (*n*) ut B N, E R, F T. Sumantur etiam ad libitum aliæ rectæ ut D M, G L, & H K *&c.* quarum ſit numerus



(*m*). Sint Anguli C N R, N R T, R T Q *&c.* atque anguli S M L, M L K, L K Q *&c.* invariati, dum puncta angularia N, R, T, M, L, K, percurrant rectas indefinitas B N, E R, F T, D M, G L, H K; Ducatur concurſus crurum T Q & K Q per rectam indefinitam A Q; Invenire ordinem curvæ quam concurſus cruris S M cum aliqua rectarum C N, N R, R T, T Q *&c.* *ex. gr.* cum R T, perpetuo tanget.

In Serie rectarum C N, N R, R T, T Q *&c.* denotet *s* numerum rectæ R T, cujus concurſu cum S M Curva eſt deſcribenda, à C N incluſive; qui in hoc caſu eſt ternarius: erit Curva ordinis quem exprimit numerus

rus $sm + s + n + 1$: unde in caſu quem figura deſignat, cum $s = m = n = 3$ erit Curva ordinis 16ti.

In his deſcriptionibus Rectas ſolummodo atque Angulos dari poſtulavimus; ſed facilius ſæpe ſimpliciorum Curvarum ope complexiores deſcribuntur; atque Propoſitiones his non minus Univerſales huc pertinentes inveſtigavi: Eas vero cum harum demonſtrationibus utpote prolixis impræſentiarum omitto; Eaſdem poſtea publici juris facturus, ſi luce non videantur hæc Geometris indigna.

VI. *Extract of a Letter of the Reverend Mr.* William Rice, *Rector of* Caerleon *upon* Usk, *to* Charles Williams *Eſq; giving an account of an ancient* Roman *Inſcription lately found there. With ſome Conjectures thereon, by the Reverend Dr.* John Harris, *S. T. P.* and *R. S. S.*

Sir,

A Perſon laſt Week being at Plow in a Cloſe near the Bank of the River *Uſk*, which the Ancients ealled *Iſca*, (which glides by us about a quarter of a Mile off, and in ſight of this Town) came thwart a Stone, and finding Letters thereon, took it up whole; 'tis about a Yard in length, and about three Quarters broad I went to the place, and took a true Copy thereof, which I here make bold to ſend you. There was underneath it ſome ſeeming Oblong Square Sepulcher of Stones, rude in order. A little further in that Cloſe, where that River wears out the Land, there was, ſome time before, a large Earthen Pot taken out of the Bank by the River-ſide, which had therein the Scull and Bones of ſome Perſon, by ſome thought to be a Child Murther'd; But I rather conjecture it a *Roman* Urn.

Caerleon, March 21. 1717.

Your humble Servant,
William Rice.

D M
G. VALERIVS. G. F
GALERIA. VICTOR
LVGDVNI . SIG · LEG . II AVG
STIP . XVII . ANNOR XLV . CV
RA. AGINT. AMNIO. PERPITVO. B

Sir,

This ancient and fair Inſcription confirms what others have found hereabouts; and what *Cambden* and other Hiſtorians ſhew us, *viz.* That the ſecond *Roman* Legion called *Auguſta*, brought into *Britain* by *Claudius Cæſar* under the Conduct of *Veſpaſian*, was placed here at *Iſca* or *Caer Legion*, by *Julius Frontinus*, in order to awe the *Silures*: And that General obtained ſeveral Victories over them and their Neighbours in ſeveral places hereabouts.

There ſeems to be nothing of Moment or of difficulty in this Inſcription; but *Victor Lugduni*: Which as I think we have no ground from Hiſtory to refer to *Lyons* in *France*, ſo I gueſs that Expreſſion may be thus accounted for. The River *Lugg* is famous in the Neighbouring Parts; and as *Dynas* or *Dyn* hath been ſaid to ſignifie a Town in the Ancient *Britiſh* Language; and that *Dun* doth alſo ſerve to expreſs a Hill or *Down* as we ſtill call it; (which I think is derived from the *Britiſh* alſo) probably *Lugduni* here may expreſs ſome Town or Hill near the River *Lugg*; and ſince there is a Place called to this Day *Luckton*, on the ſide of the River *Lugg* in *Herefordſhire*, perhaps that may bid fair to be the very place where *Valerius* obtained the Victory perpetuated by this Inſcription.

F I N I S.

PHILOSOPHICAL TRANSACTIONS.

For the Months of *March*, *April* and *May* 1719.

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Jos: Banks

I. *De* Maximis & Minimis *quæ in motibus Corporum Cœleſtium occurrunt.*

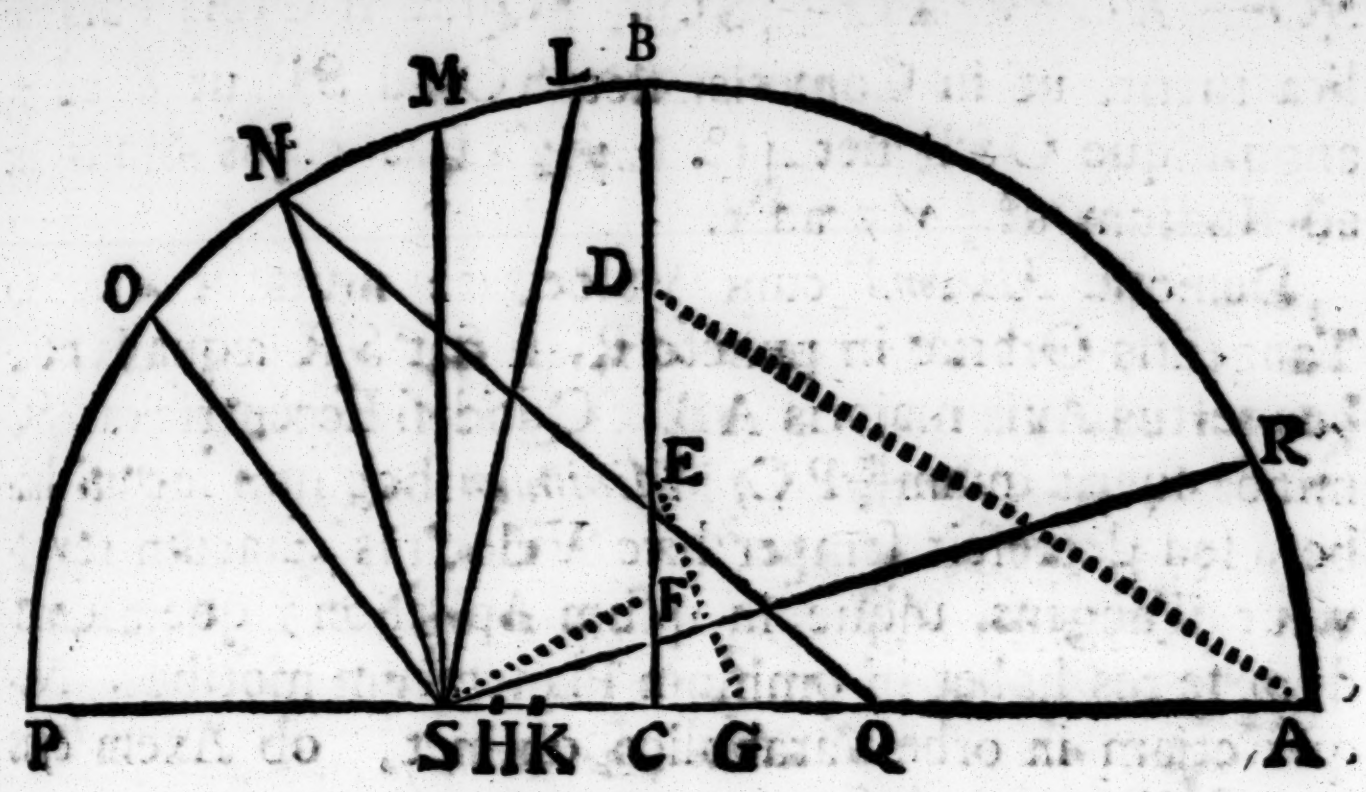
ANTE *Keplerum* Aſtronomi univerſi, per tot retro ſecula, Planetarum motum circularem non auſi ſunt in dubium vocare, ex præconceptâ, ut videtur, in figura Circuli neſcio quâ perfectionis Ideâ. *Keplero* autem Inventori debetur ea qua nunc utimur Theoria, nempe quod Corpora cœleſtia Solem ambiunt in communi orbium Ellipticorum Foco ſitum, eâ lege ut Areæ Temporibus proportionales radiis ad Solem ductis deſcribantur. Sublimiorem vero poſtulat Geometriam, ad oſtendendum quam ob cauſam hoc ita ſe habeat, quodque aliter eſſe non poſſit. Hoc in ſempiternam celeberrimi D. *Newtoni* Præſidis noſtri gloriam reſervatum eſt.

Hujus veſtigiis inſiſtens, Corollaria quædam exhibuit eximius Mathematicus D. *Abr. de Moivre* R. S. S. in *Philoſ. Tranſact.* N° 352 edita; Theoremata ſcil. parata, quibus determinantur Velocitates ſive Momenta Motûs tam veri quam apparentis circa Solem, ſicut etiam acceſsûs vel receſsûs à Sole, in dato quovis datorum Orbium puncto. Deinde ut Theoriam ſyſtematis Planetici penitius excoleret, ope eorundem Theorematum, dictorum Momentorum Momenta perſcrutatus eſt, oſtenditque quibus in orbium punctis fiant *Maxima* harum Velocitatum mutationes, idque Solutionibus facilitate & concinnitate præſtantibus.

Sit A B P Orbis Planetæ Ellipticus, A P Axis Tranſverſus, C B Semiaxis conjugatus, S Sol, Q Focus alter Ellipſeos. Per S ducatur S M ipſi C B parallela: & erit punctum M in quo *Maxima* cum velocitate creſcit

ſcit vel decreſcit diſtantia à Sole, & $SM = AC - \frac{SC^2}{AC}$.

Si vero capiatur SL media proportionalis inter Semiaxes AC, CB, erit punctum L in quo *Maxima* fit æquatio Centri, ut vocant; ſive ubi motus angularis fit æqualis medio Motui: Quod ſi Eccentricitas non major ſit quam in pleriſque Planetis, $BL = \frac{1}{4} BM$ quam proximè: Eſt vero $SL = \sqrt{\sqrt{AC^4 - AC^2 SC^2}}$.



Si quæratur punctum N, in quo fit *Maxima* mutatio Velocitatis motûs realis in Curvâ, Problema Solidum eſt. Eſt enim $2NS = 4AC - 2NQ$ ad $3NQ - AC$ ut $AC^2 - CS^2 = CB^2$ ad NQ^2; adeoque ſi ponatur $AC = a$, $CB = c$ & $NQ = y$, habebitur æquatio $y^3 - 2ayy + \frac{3}{2}ccy - \frac{1}{2}acc = 0$. Quâ reſolutâ erit y ſive NQ diſtantia puncti quæſiti N ab altero Ellipſeos foco. In Orbibus autem parum Eccentricis, quales ſunt Planetarum, ſi fiat $CD = SQ$, & junctæ AD æqualis ponatur AK, erit reliqua pars Axis $KP = NS$ diſtantiæ puncti N à Sole quamproxime. Si vero Orbis fuerit Parabolica erit SN ad SP ut 5 ad 4, anguluſque NSP erit 53°. 8′ fere, cujus Sinus eſt $\frac{4}{5}$ Radii.

At Punctum O, in quo motûs apparentis ſive angularis acceleratio Planetæ deſcendentis, vel retardatio

aſcenden-

aſcendentis *Maximæ* fit, hoc modo obtinebitur. In AC capiatur $CG = \frac{1}{6}AC$, ac fiat angulus CSF 30 *gr.* ductæque SF æqualis ponatur CE, ipſique GE ſit GH æqualis. Dico, ſi diſtantia SO fiat æqualis ipſi PH, quod in puncto O proveniet *Maximæ* mutatio motus angularis Planetæ in Orbe Elliptico ABOP gyrantis; eo ſcilicet in Orbis loco ſecundæ differentiæ æquationum centri Planetæ reperientur *Maximæ*. Eſt autem $SO = \frac{7}{6}AC - \sqrt{\frac{1}{36}AC^2 + \frac{1}{3}SQ^2}$. Quod ſi Orbis Parabolica fuerit, ut in Cometis, fiet SO ad SP ut 8 ad 7, anguluſque OSP fiet 41°. 24′ $\frac{1}{2}$, ſive cujus Sinus ſit ad Radium ut $\frac{1}{4}\sqrt{7}$ ad 1.

Denique *Minimâ* cum Velocitate mutatur directio Tangentis Orbitæ in puncto R, ſi fiat SR æqualis duabus tertiis Axis majoris AB. Quod ſi Eccentricitas SC minor fuerit quam $\frac{1}{3}PC$, *Minimum* hoc non locum habet, ſed decreſcit ſemper hæc Velocitas quacum revolvitur Tangens, uſque in ipſum Aphelion; quemadmodum ſe res habet in omnium Planetarum motibus. Neque etiam in orbe Parabolico obtinet, ob Axem ejus in infinitum protenſum.

Hæc omnia demonſtrantur, juxta præcepta Doctrinæ de *Maximis* & *Minimis*, ex Theorematis prædictis in N° 352 exhibitis, quæ quidem hac occaſione reviſere Lectorem curioſum non pigebit.

II. *Apologia*

II. *Apologia D.* Brook Taylor, *J V. D. & R.S.Soc. contra V.C* J. Bernoullium, *Math. Prof.* Baſileæ.

PACIS & concordiæ ſtudioſo ſatius eſſet injurias vincere ferendo, quàm odioſas contentiones obire ulciſcendo. Verum cum patientia noſtra pro ignaviâ habetur, ſilentium pro confeſſione criminis, & nuperam calumniam jam nova ſequitur contumelia, omnino reſpondendum eſt, ne nobis ipſis deeſſe videamur. In *Epiſtola pro eminente Mathematico D.* J Bernoullio, Actis Lipſienſibus An. 1716. inſertâ, plagii reus ſiſtor ſequentibus verbis: " Hoc nihil novi eſt in qui-" buſdam Anglis, qui ſibi ſolis licere putant, *aliorum* " *inventa tanquam ſua* impunè *uſurpare*; quando ipſi Ho-" mineſque Deoſque invocant, ubi vident, vel ſaltem " videre arbitrantur. Extraneos in ſuorum Inventa ma-" nus inferre. Exempla ſunt quorundam, ut Chey-" næi, Des Hayes, *Taylori*, aliorumque, *qui paſſim in-" ventis Bernoullii ſunt uſi alieniſque, vel nullâ prorſus " factâ mentione Autoris*, vel ".——Palam eſt ab ipſo Bernoullio promanaſſe hanc accuſationem. Nam in Actis Lipſienſibus An. 1718. [1] per filium ſuum fatetur ſe *res ipſas in illa epiſtola contentas quoad maximam partem amico alicui perſcripſiſſe.* Invidiam equidem prædictæ calumniæ à ſe amovere ſollicitè ſtudet, atque transferre in vicarium illum ſuum, cum ipſe profitetur, *ſe non approbare quæ in alios durius dicta cenſeri poſſunt* [2]. Sed admodum imperfecta eſt hæc purgatio. Nam calumniæ ſunt quæ duriùs dicta vocat. Ait ſe *dicta* illa non approbare: Sed improbaſſe neceſſe fuit. Teſtimoniũm

1 Pag. 261. 2 Pag. 262.

denique eſt pro ſe teſtantis: Autorem illum anonymum citaſſe oportebat, ut cum ipſo agere liceret: Sed is adhuc latitat. Quam verè autem & ex animo *ſe durius dicta non approbare* videatur, conſtare quodammodo poteſt ex ſequentibus, quæ de me ipſe profert, proprio ſuo nomine, nullâ uſus perſonâ: " Taylorus Geometra " inſignis & acutus, qui ad *profundiora noſtra* feliciter " penetravit, teſte ipſius libro de Methodo incremen- " torum, probè ſentiens impeditam nimis Analyſeos " fraternæ prolixitatem, eamque in compendium contra- " here, ac ſimul generaliorem nonnihil reddere volens, " tantam rei affudit obſcuritatem (quâ in aliis quoque " brevitatem affectans impensè delectari videtur) ut du- " bitem quenquam fore etiam inter perſpicaciores, qui " ubique & hic imprimis mentem viri aſſequatur, imo " etiamſi aliunde rem cognitam habeat. Ut jam nihil " dicam de ipſo calculo, pro more ejus, conciſo qui- " dem & contracto, ſatis tamen adhuc longo & intri- " cato, ſi quis ſingula ejus capita minutim perſequi " velit; præterquam quòd cum Fratre meo ad tertias " quoque fluxiones excurrat " [3]. Sit ſanè liber ille meus nonnihil obſcurus: Difficile eſt in re ferè nova, & ab uſu communi aliquantulum remota, non eſſe obſcurum. Sed maximè obſcurum oportet eſſe librum, in quem illa omnia verè dicantur. Et ſi verè dicantur, tamen ſine ullâ omnino causâ talia dixiſſe, ab ingenuis moribus prorſus alienum eſt, & mera contumelia.

Sed audio Bernoullium de exordio conquerentem quo nuper uſus ſum, in ſolutione problematis Leibnitiani in Tranſactionibus Philoſophicis editâ. Stylum acriorem reprehendit quàm virum benè moratum deceat, item nimium contemptum Extraneorum. Quæ liberius effatus ſum, hæc ſunt: " Si nondum viderint [fautores

3 Act. Leipſ. An. 1718. M. Jan. p. 18.

Leibnitii]

Leibnitii] quomodo ex illa " [ex anteriori nempe ſolutione generali] " æquationes ſint deducendæ, id pro" fecto illorum imperitiæ tribuendum erit " [4]. Hæc fateor paulò durius ſonant; ſed ſi ad cauſam attendas contumelia vacant. Fautores Leibnitii (non omnes intelligo, ſed Bernoullium tantum, & Socios ejus anonymos nobis infenſos,) univerſos Anglos indignè tractarunt. Solutionem illam generalem cum non intelligerent, deriſui habebant: In injurioſos & deriſores me liberius explicui; contumelia non eſt. Sed ubi ille contemptus Extraneorum? Neminem ego nominatim citavi: De Fautoribus Leibnitii ſum ſolùm locutus. Sed abſit ut omnes deſignatione illa omnino intelligerem quocunque modo cauſæ Leibnitii faventes; tanquam ipſe cauſæ Neutonianæ eſſem tam pertinaciter addictus, ut alios omnes odio habeam. Sed controverſia iſta Neutonum inter & Leibnitium nihil ad me. Solos intellexi Fautores illos qui in Anglos eſſent infenſi, qui me nominatim calumnia provocarunt; Bernoullium iterum dico quem Principem agnovi cauſæ iſtius, ſocioſque ejus anonymos vel veros vel fictos. Hæc apertius dico, ne alii de noſtra in alios contumelia immerito querantur. In immerentes injuria eſſet, in Bernoullium non eſt. Sed ad ſuperiora illa rede

Plagii accuſor, tanquam inventa Bernoullii, aliorum, uſurpaſſem ut mea. Exempla proferat, dabitur reſponſum. Plura ſanè tractavi cum aliis communia; ſed inventis alienis ſum minime uſus ut meis. Propria ubique ſum uſus Analyſi, (ſi Iſoperimetrum excipias, de quo poſtea dicetur,) ut nullo modo dici poſſit me alios fraudaſſe. At Autores nominaſſe oportebat, unde artem hauſeram. Tanta me quidem tenet reverentia illuſtrium nominum, Hugenii, Hoſpitalii, Varignonii, Leib

4 Tranſ. Phil. N° 354.

nitii, aliorum, ut neſciam an ex hac parte non peccaverim, cum mihi ipſi deeſſe videar, cui tantos viros citaſſe ſemper fuiſſet ornamento. Nimia fortaſſe ignavia erat, quod de rebus cum eſſem maxime ſollicitus, hiſtorias rerum penitus neglexerim. Sperabam tamen me in tantæ fraudis ſuſpicionem incidere non potuiſſe, cum illuſtriſſima tantorum virorum opera eam facilè detegerent. Quæ cum Bernoullio communiter tractavi problemata, ſunt, de Funiculariâ, de Centro Oſcillationis, & de Iſoperimetris. In duobus primis ſum propria omnino uſus analyſi; in Iſoperimetro uſus ſum analyſi Autoris Jacobi Bernoullii, Viri à rebus Mathematicis optimè meriti, cui debitos nunc perſolvo honores. Solutio noſtra problematis de Centro Oſcillationis, cum amicis meis communicata eſt uſque ab initio Anni 1712. ut teſtes poſſum citare epiſtolas autographas Keillii noſtri; Liber item noſter erat penes Societatem Regiam, & cum omnibus fere noſtris Mathematicis communicatus, uſque à menſe Aprilis Anni 1714. quod hic monitu neceſſarium duxi, ne & Solutionem illam ſibi vindicet Bernoullius; cujus Solutiones [5] duæ extant eodem Anno editæ; quarum poſterior cum noſtra, quoad principia, tam mirè conſentit, ut jurares ab eodem homine eſſe utraſque inventas. Materia de Iſoperimetris excogitata primum eſt à Jacobo Bernoullio, ſicut jam innuimus. Ejus extat Solutio cum Analyſi, in Actis Lipſienſibus Anni 1701. Extat Analyſis fratris in Commentariis Regiæ Scientiarum Academiæ Anni 1706. Extat & Solutio in Libro noſtro. De eadem materia Commentarium nuper edidit Bernoullius in Actis Lipſienſibus Anni 1718. proximi [6].

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5 Altera in Act. Lipſ M. Jun. In Comm. Reg. Sc. Acad. M. Aug. altera.

6 P. 16. & ſeq. Has igitur aliaſque ob rationes, actum agere minimè videbor, &c. p. 18.

Ibi,

lbi, ne actum agere videatur, non meis ſolummodo, verum etiam fraternis ſolutionibus malevolus detrahere aggreditur; fratri prolixitatem [7], mihi obſcuritatem [8] objiciens. De novis illis inceptis nihil non magnum [9] pollicetur; & *ope cujuſdam principii, ab uniformitatis lege, quam nemo hucuſque obſervavit, petiti,* rem totam pene ſine calculo, nullo labore abſolvet. Sed neſcio quo fato fit, ut in hac materia de Iſoperimetris, Bernoullius Deos omnes ſemper offendat iratos. Nam primo, priſtina illa Analyſis ejus à capite ad calcem quaſi unum aliquod vitium maximum conſtituit: Secundo, quod tantum jactitat Principium, à lege uniformitatis, quam nemo hucuſque obſervavit (ſic enim ſtrenuus affirmat) petitum, à me olim obſervatum eſt: Denique quam hic tanquam novam exhibet Analyſin, tota mera fraterna eſt. Analyſin enim conſtituunt Præcepta, juxta quæ deinde inſtituitur calculus; qui non Analy-

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7 Nullos hic offendet Lector *ſcopulos*, quos objicit *operoſa* Fratris analyſis, atque differentiarum tertiarum *tricas ac ſpinas*, quibus *undique obſeptum* ibi ſentit viam, in noſtra methodo nullas percipiet.——Nec fratris calculi prolixitatem, nec Taylori obſcuritatem æque ingratam ac moleſtam ſibi metuendam habeat, *p.* 18 ——quam Frater per *operoſiſſimam* ſuam analyſin elicuit, *p.* 23.——non tantum ea, quæ à fratre meo quonda[m] propoſita *magna pompa*, *nec minori conatu & labore* ſoluta fuere, ego ex ſola lege uniformitatis ſolvi citra calculum analyticum, &c.

8 *Vide Not. præced.*——*item quæ ex* p. 18. *jam ſunt deſcripta.*

9 ——ætus confidam, *publicum ei gratiam habiturum*, quod occaſio mihi extiterit, talia nunc divulgandi, quæ fortè cum multis aliis in ſchedis meis perpetuò manſiſſent ſepulta, quamvis *reconditæ Geometriæ fines non parum prolatura*, p. 17. quod ibi ex incuria prætervifum reparabo hic *novo ſolvendi modo*. qui ſingulari facilitate expedit problemata, non tantùm omnia quæ de Iſoperimetris propoſuerat Frater, ſed & innumera alia illis affinia, *ib.*——ope cujuſdam principii ab uniformitatis lege, *quam nemo hucuſque obſervavit*, petiti, ex ſola Figuræ inſpectione, ac ſine ullo pene calculo æquationes pro curvis quæſitis ſponte velut ſe offerentes ſtatim eliciam, &c. ut in *Not.* 7. actum agere minimè videbor ſi in hoc argumento per ſe difficili viam monſtrem & rationem *brevem, planam, claram, & facilem*. qua quiſque mediocri quoque ingenio præditus ad veritates illas abſtruſiores (non fide aliorum, ſed) propriis oculis ſpectandas pervenire poſſit, ita nempe, ut, &c. *ut in Not.* 7.

fis

ſis, eſt, ſed inſtrumentum Analyſeos. Præceptis ſemel poſitis, quivis facilè calculum inſtituit, more quiſque ſuo, hic prolixius, ille magis concinne, prout unicuique faverit Minerva. Negandum non eſt, Bernoullium calculum tandem concinnaſſe, & reddidiſſe elegantiorem; ſed tamen in Analyſi fraterna fecit, non ſua: Nec dubitandum quin frater, adhuc ſi vixiſſet, rem reddidiſſet non minus illuſtrem Analyſin diximus in præceptis contineri; præcepta verò ſunt omnia fraterna. Nam quod curvæ quæſitæ arculum minimum, tanquam ex tribus lineolis elementaribus compoſitum contemplatur, vel ipſo fatente [10] à fratre eſt: quod ex data longitudine arculi iſtius minimi quærit rationem differentiarum Ordinatarum in Lemmatis ſuis, à fratre eſt: quod rationem eandem denuo quærit, faciendo ut ſit areola naſcens, ex Functionibus (ut vocat) compoſita, vel maxima vel minima, à fratre eſt: quod denique ex duplici illa expreſſione ejuſdem iſtius rationis æquationem colligit qua curvæ quæſitæ natura definiatur, à fratre eſt. Sed hæc Solutionem conſtituunt. Ergo Solutio mera fraterna eſt. Dixi me olim uſum eſſe Principio illo, quod tanta cum oſtentatione ſibi arrogat Bernoullius: Ex eadem una pagina, en duo exempla. In pagina 113, Libri mei hæc ſunt " $\frac{m'}{R'} = \frac{m}{R}$ .. Sed " eſt $\frac{m'}{R'}$ novus valor ipſius $\frac{m}{R}$, unde eſt $\frac{m}{R}$ quantitas " data ". Luce clarius eſt me hoc loci ex obſervata

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10 Utar pro hoc, ut ipſe fecit in ſua Analyſi, contemplatione arculi minimi, &c. *p.* 18.

uniformi-

uniformitate inter formulas, $\frac{m'}{R'}$, $\frac{m}{R}$, concluſiſſe quod ſit $\frac{m}{R}$ quantitas data. Idem feci in ſequentibus. "Pone "$\frac{m''n}{nR'} = \frac{m}{R}$, hoc eſt $\frac{m''n'n}{R'} = \frac{m'n'n}{R}$, &c". ubi ut uniformitas appareret inter formulas $\frac{m''n'n}{R'}$, $\frac{m'n'n}{R}$, æquationem

transformavi Videtis, credo, quàm feliciter penetraverim ad *profundiora* Bernoullii. An hæc obſcura dicet?

Ad primam jam partem promiſſi pervenio, ut oſtendam priſtinam illam Analyſin Bernoullii eſſe omnino corruptiſſimam. Primo per ſubſtitutionem ſatis ridiculam, ex *profundioribus ſuis* neſcio quibus petitam, æquationem $F\omega \times \Delta RO = \varphi\omega \times \Delta\rho\omega$ transformat in hanc $FO \times \Delta^{D} F = \varphi\omega \times \Delta\pi\varphi$; quod in caſu particulari (nempe quando functiones ſunt ut quadrata ordinatarum) huc redit, ut ſint ſimul $FO \times RO = \varphi\omega \times \rho\omega$ & $FO \times PF = \varphi\omega \times \pi\varphi$; unde conſit $PF : RO :: \pi\varphi : \rho\omega$ Sed hoc impoſſibile eſt, quoniam eſt vel $PF \sqsupset RO \sqsupset \rho\omega \sqsupset \pi\varphi$, vel $PF \sqsubset RO \sqsubset \rho\omega \sqsubset \pi\varphi$; quorum neutrum cum analogia expoſita conciliari poteſt. Nam ſi $PF \sqsupset RO \sqsupset \rho\omega \sqsupset \pi\varphi$, per Analogiam etiam erit $\pi\varphi \sqsupset \rho\omega$ (propter $PF \sqsupset RO$) contra hypotheſin; vel ſi $PF \sqsubset RO \sqsubset \rho\omega \sqsubset \pi\varphi$, per Analogiam etiam erit $\pi\varphi \sqsubset \rho\omega$, contra hypotheſin Secundo parum ſcienter fingit curvaturam in F eſſe ad curvaturam in φ ſicut eſt φO ad FO; cum nihil in hac tota Analyſi ſit quod privilegium illud vindicet puncto O potius, quam alio cuilibet puncto ω in arculo minimo $FO\omega\varphi$ ubivis ſumpto. Nec ſanè Curvedo tam ridicule

dicule vult æſtimari. Tertio nimis imperitè facit $mn = ddx$, $nl = ddy$, & $ml = \frac{dtddy}{dx}$, cum ſint $mn = \frac{1}{2}ddx$, $nl = \frac{1}{2}ddy$, & $ml = \frac{dtddy}{2dx}$. Denique quod omnium peſſimum eſt, vitioſiſſimis hiſce principiis perfectiſſimam alligavit concluſionem. In problemate primo dico; nam in ſecundo eſt talium parentum magis digna proles. Errores Bernoullii veteres & exoletos me expoſuiſſe putatis. Non ita eſt; ipſe enim hæc habet: " *Omnia dudum ſepoſita accuratè rurſus excutiendo ad ſe-*
" *veri examinis trutinam revocavi* [11]. Notandum au-
" tem Solutionem *primi* problematis in ſchediaſmate
" meo Commentariis Acad. *p.* 215 inſerto. *rectiſſime ſe*
" *habere* " [12]. Errores ergo ſuos jam denuo adoptavit. Unde fortaſſe nunc quæret aliquis, Quo jure hic primas ſibi in ſublimiori illa Analyſi tam obſtinata ambitione arroget? Ut nemo ſit qui in illa aliquid profecerit, quin continuo accuſetur *ad profundiora Bernoullii penetraſſe* [13]: Unde conſtet verum eſſe, quod quidam nuper affirmavit, regulas extantes in libro de Analyſi infinitè parvarum à Bernoullio emanaſſe [14]? Quod laudes Excellentiſſimi Marchionis Hoſpitalii ſint ſuo Præceptori tribuendæ? An hic ſit idoneus qui alios docuerit *regulas differentiandi differentias* [15]? Cum multis aliis, quæ ſigillatim enumerare non eſt opus. Sed iſtis reſpondeat qui volet: nos in hiſce diutius non moramur.

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11 Pag. 16. 12 Pag. 17. 13 Pag. 18. *vide etiam Ep. pro Em. Math. & ſcripta ipſius Bernoullii paſſim.*

14 Concedit Dn. Marchionem de l'Hopital calculum iſtum intellexiſſe, nec ignorat, illuſtriſſimum hunc virum eundem à Cel. Bernoullio didiciſſe: atque minime ipſum fugit, regulas in dicto libro [*de Analyſi infinitè parvarum*] extantes à Cl. Bernoullio promanaſſe. *Act. Lipſ An.* 1718. *p.* 464.

15 Dum interea conjici poteſt, illum cum Dn. Newtono ab initio in iſto errore hæſiſſe, donec tandem liberati fuiſſent uſu *calculi differentialis, & regulas differentiandi differentias à Cel. Bernoullio edocti eſſent.* ib. p. 465.

Res ipsas exposui. peroratione non utor; harum enim tædet. Nec si quidquam regesserit Bernoullius, ulterius respondere necesse habebo. A contumeliis nos semel vindicare & jus & ratio postulat; ulterius non expedit.

III. *An Account of the Impression of the almost Entire Sceleton of a large Animal in a very hard Stone, lately presented the* Royal Society, *from* Nottinghamshire. *By Dr.* William Stukely, *M. D. and* R. S. Soc.

HAVING an Account from my Friend, *Robert Darwin*, Esq; of *Lincoln's-Inn*, a Person of Curiosity, of an human *Sceleton* (as it was then thought) impress'd in Stone found lately at the Rev. Mr. *John South's*, Rector of *Elston* near *Newark*, *Nottinghamshire*, I was desirous of a Description of it, for the Entertainment of the ROYAL SOCIETY, and have at length procured the Stone it self for their Repository, where such remarkable Appearances are best preserv'd, and deservedly valued. It cannot but be matter of Regret, that so considerable a Rarity, the like whereof has not been observ'd before in this Island (to my knowledge) should be maim'd and imperfect, yet we may content our selves if enough be still visible to favour a Conjecture of what it has been. The Stone it self is a blue Clay Stone, the same as (and undoubtedly came from) the neighbouring Quarries of *Fulbeck*, or thereabouts, upon the Western Cliff of the long Tract of Hills extending quite through the adjacent County of *Lincoln*. It lay, time out of mind,

at the ſide of a Well near the aforeſaid Mr. *South*'s Parſonage-Houſe, where it had ſerv'd for a Landing-place to thoſe that drew Water; but upon removal, the Under ſide exhibited this unuſual Form, and was accordingly taken notice of by that worthy Gentleman, and laid up in his Garden for Curioſity-ſake. Where the remaining part of the Stone, which contain'd the Upper part and Continuation of the *Sceleton*, or that which was the other ſide, and tally'd with it, may be, is now utterly unknown: but upon view, I am perſuaded, it cannot be reckon'd Human, but ſeems to be a *Crocodile* or *Porpoiſe*. There are Sixteen *Vertebræ* of the Back and Loyns very plain and diſtinct, with their Proceſſes and intermediate Cartilages, Nine whole or partial Ribs of the Left-ſide, the *Os Sacrum*, *Ileum in ſitu*, and two Thigh-Bones diſplac'd a little; the Beginnings of the *Tibia* and *Fibula* of the Right-Leg; on one Corner there ſeem to be the *Veſtigia* of a Foot with four of the five Toes, and a little way off an entire Toe, now left perfect in the Stone: there are no leſs than Eleven Joints of the Tail, and the Cartilages between them of a White Colour diſtinguiſhable from the reſt. We ſhould impoſe upon our Senſes, to queſtion, whether theſe be the real Reliques of an Animal; for the very Bones themſelves are now to be ſeen as plainly, as if preſerv'd in an *Egyptian* Mummy; a very little while ago, the Society had a Draught of a Crocodile, tho' a ſmall one, found after the like manner inclos'd in Stone, from a Quarry in the Mountains of upper *Germany*. I ſuppoſe the ſame Reaſon accounts for both and all the reſt of theſe kind of Foſſils; and I pleaſe my ſelf in an ocular Evidence, and ſo great a Confirmation of what I had the Honour to preſent to the *Royal Society*, in a late Diſcourſe, where I hinted at a Solution of ſome obvious

vious and remarkable Phænomena, in the external Face of the Globe, conſequent to its Formation, as ſet forth in the *Moſaic* Account; and of ſome Changes it ſuffer'd at the univerſal *Cataclyſm*, and Proofs of that great *Cataſtrophe* of the animal and vegetable World in Plants, Shells, and Parts of living Creatures found in Rocks and Quarries.

Its remarkable, that all the Stone Pits about the Country whence this came, abound with prodigious Quantities of Shells, and the like, and the greateſt part of the Subſtance of the Stone is a Compoſition of them. There are many Accounts of them in the *Tranſactions*, and this Stone has many Shells of different kinds in it. Sir *Hans Sloan* has a Fiſh-Sceleton, amongſt his immenſe Treaſure of Curioſities, found near this Place, given by the Duke of *Rutland*. If we look upon a Map of the Country, and obſerve the *Lincolnſhire Alps* which I ſpoke of before, how they run 50 Miles North and South, and on the Weſt ſide are ſteep and rocky, we may ſee the Reaſon why theſe Quarries ſhould be ſo ſtuft with them; for it is juſt to conceive, that upon retiring of the Waters of the Deluge from the Superficies of this Country, into the Eaſtern Seas, theſe heavy Bodies met a full ſtop, and were intercepted by this Cliff, which has retain'd ſuch vaſt Quantities of them ever ſince: whilſt thoſe which fell upon common Mold are moſtly rotten, and now loſt.

Sir *Iſaac Newton*'s Doctrine of the Attraction of the Particles of Matter, according to the Quantity of its Solidity, Proximity, and Surface, eſpecially that it is infinitely greater in the point of Contact, upon which depends its Coheſion and all the Varieties of Phyſical Action, will eaſily direct us to a Notion of Petrifaction. We learn how a proper Degree of Heat or Cold,

Moiſture, Motion, Reſt and Time, promote this Principle, from the common Experiments of Chryſtallization and Freezing even before the Fire, and in many Chymical Mixtures. Whence we cannot be ignorant of Stone growing in the Quarries gradually, not by any fancied Vegetation, tho' there is ſomething like it in Corals, but generally by Appoſition of Parts to Parts, as is notorious in the *Fluors* of ſubterraneous Grotts and Caverns. So that we have no reaſon to doubt but what was Clay, Sand, or Earth 3000 Years ago, may now be Stone or Marble, according to the Proportion of Concurrence of ſuch mentioned Cauſes. This will perſuade us that the now barren and rocky Plains of the Countries of *Syria*, *India*, and *Arabia*, are owing to Natural Cauſes, as well as an immediate Curſe of God for the Diſobedience of its ancient Poſſeſſors his peculiar People, becauſe the ſame is obſervable of the famous Countries of *Greece* and *Africa*, warm Regions ſo renowned for Fertility in antient Authors. Wherefore there may be ſome likelyhood in the Opinion of thoſe who think that in many Ages the whole Face of the Globe may become one great Rock. Dr. *Plott*, in his Natural Hiſtory of *Oxfordſhire*, gives an Account of a *Tumulus*, now a perfect Mount of Stone: and upon St. *Vincent*'s Rock near *Briſtol* are Fortifications now become ſolid Cliff. I remember, about ſix Years ago, Mr. *Ralph Widdrington*, Brother to the Earl of that Name, ſhew'd me many human Bones taken from whole Sceletons, with Brittiſh Beads, Chains, Iron Rings, Braſs Bitts of Bridles, and the like, which were dug up in a Quarry, near the Seat of the Family, at *Blankney*, *Lincolnſhire*; which very probably was plain Mold when theſe old Corpſes of the *Britons* were interr'd; and ſince then I ſaw many human Bones and Armour, with *Roman* Coins, *Fibulæ*, *&c.* found in

a Stone-

a Stone-pit in the Park at *Hunſtanton, Norfolk*; belonging to Sir *Nicolas L'Eſtrange*, in whoſe Cuſtody they now are, which were conjectured to have been buried in Earth after a Battle. From whence we may judge it a vulgar Miſtake, when in the Ruins of old Caſtles and Walls, we admire the Tenacity of the Mortar, and are apt to praiſe our Anceſtors, for an Art which we ſuppoſe now loſt; when doubtleſs the Strength of the Cement is owing to the Length of Time: and in future Ages our Modern Buildings may obtain the ſame Judgment.

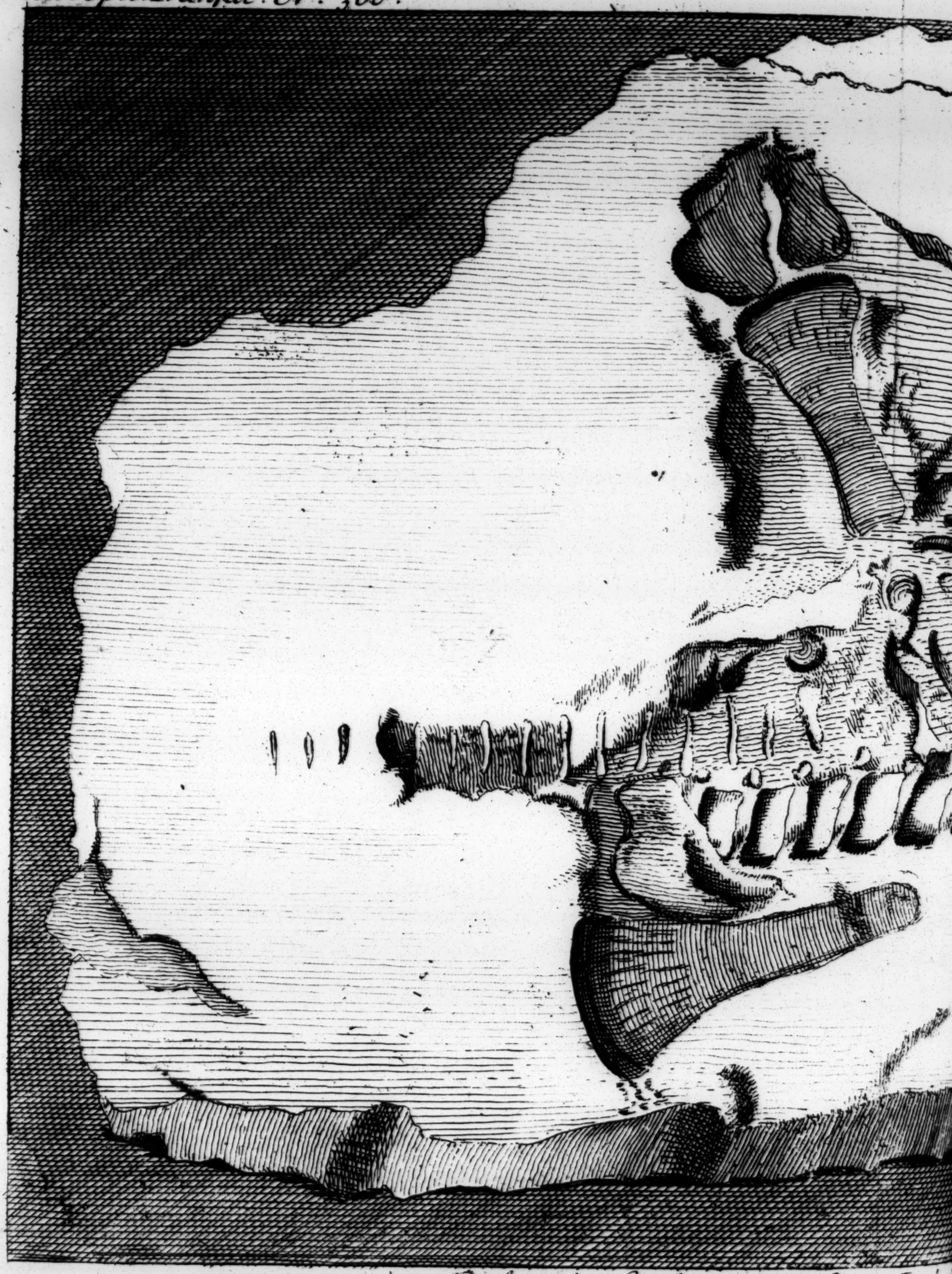
From all which Inſtances, I only deſire to infer the antient ſtate of theſe Cliffs, where this Sceleton was, and Shells are daily found, intimately mixt in the Subſtance of the Stone, to have been formerly of a ſofter Conſiſtence, capable of admitting them into its Bowels, and to have immur'd them as part of it ſelf; and that Earth which is now manageable by the Plough, may poſſibly in time aſſume the ſame Denſity, at leaſt very little below the Surface; for in this very Cliff the upper *Strata* are yet Clay, growing harder as deeper. What Creature this has been, for want of a Natural Hiſtory of Sceletons, well worthy the Endeavours of this Society, we cannot poſitively determine; but generally find the like to be amphibious or marine Animals. Why ſuch rather than many others, ſhould chance to be thus entomb'd, may be thought, becauſe they were able much longer than Terreſtrial Animals to live in that World of Waters, even till they began to abate and fall away into their deſtin'd Receptacles; ſo that while the Bodies of the reſt ſoon periſhing, were corrupted, and their Bones ſeparated and diſpers'd much earlier; this Sceleton, with others of its like, fell entire into the Fiſſures of this Bed of Clay, which has ſince turn'd into Stone, and made this noble Monument and

pregnant

pregnant Token of that general Inundation, durable as the vain-glorious *Egyptian* Monarchs Pyramids at *Memphis*; to be perpetuated in the lasting Records of this Society. *See the Figure of this Impression, in Tab.* I.

IV. *A curious Description of the* Strata *observ'd in the* Coal-Mines *of* Mendip *in* Somerlet-shire; *being a Letter of* John Strachey *Esq; to Dr.* Robert Welsted, *M. D. and* R. S. Soc. *and by him communicated to the Society.*

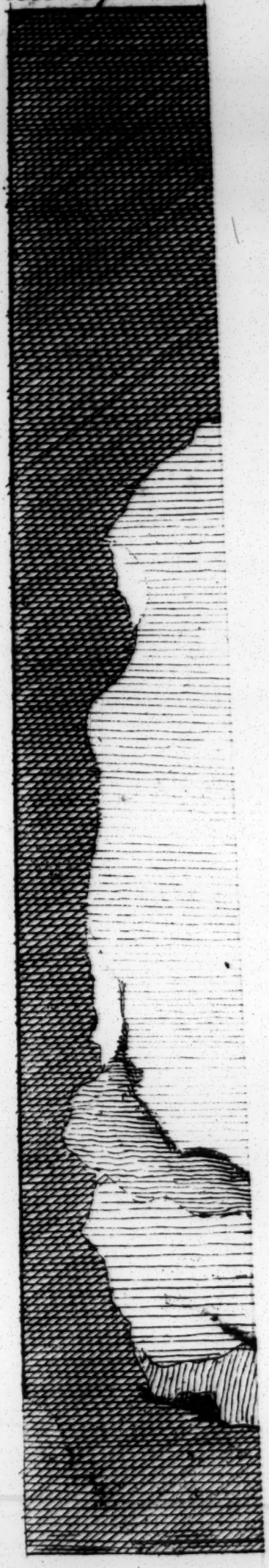
I Now send you the Observations which I some-time since promised you, relating to the different *Strata* of *Earths* and *Minerals* found principally in the Coal-Mines in my Neighbourhood. For the better Illustration whereof, I have inclosed a Draught, which you must suppose the Section of a Coal-Country, and to take in about Four Mile from the North-West to South-East, and may be applied to the Veins of Coal as they lye at *Faringdon-Gourny*, and likewise at *Bishop-Sutton*, which last Place is near *Stowy*, but in the Parish of *Chew Magna* in this County of *Somerset*. For Discovery of Coal, they first search for the *Crop*, which is really Coal, tho' very friable and weak, and sometimes appears to the Day, as they term it; or else for the *Cliff*, which is dark or blackish Rock, and always keeps its regular Course as the Coal does, lying obliquely over it. For all Coal lies shelving like the Tyle of a House, not perpendicular nor horizontal, unless it be broken by a *Ridge*, which is a parting of Clay, Stone, or Rubble; as if the Veins by some violent Shock were disjointed and broken, so as to let in



The Stone is 3 foot long and 2 foot 2 Inche

Tab: I: page 968.

e Inchi[illegible]ead



in Rubble, *&c.* between them. The Obliquity or *Pitch,* as they term it, in all the Works hereabout, is about 22 Inches in a Fathom; and when it riſeth to the Land is called the *Crop,* but in the North *Baſſeting.* In the Works near *Stowy,* and likewiſe at *Faringdon* it riſeth to the North Weſt, and pitcheth to the South Eaſt; but the farther they work to the South Weſt, the *Pitch* enclines to the South; and *è contra,* when they work towards the North Eaſt. So likewiſe they obſerve as they work to the South Weſt, when they meet with a *Ridg* it cauſeth the Coal to *trap up,* that is, being cut off by the *Ridg,* they find it over their heads, when they are thro' the *Ridg*: but on the contrary, when they work thro' a ridg to the *North Eaſt,* they ſay it *traps down,* that is, they find it under their feet.

Coal is generally dug in Valleys or low Grounds. The Surface in theſe parts is moſtly a red Soyl, which under the firſt or ſecond Spitt degenerates into *Malm* or *Loom,* and often yields a Rock of Reddiſh *Fireſtone;* till you come to four, five, and many times to twelve or fourteen Fathom depth, when by degrees it changeth to a Gray, then to a Dark or Blackiſh Rock, which they call the *Coal Clives.* Theſe always lye ſhelving and regular as the Coal doth. But in theſe parts they never meet with *Fireſtone* over the Coal, as at *Newcaſtle* and in *Staffordſhire.* Theſe *Clives* vary much in Hardneſs, in ſome places being little harder than *Malm* or *Loom,* in others ſo hard as that they are forced to ſplit them with Gunpowder: So likewiſe in Colour, the top inclining to red or grey, but the nearer to Coal the blacker they grow; and whereſoever they meet with them they are ſure to find Coal under them. But to their diſappointment 'tis not always worth the the digging. The firſt or uppermoſt Vein at *Sutton*

is

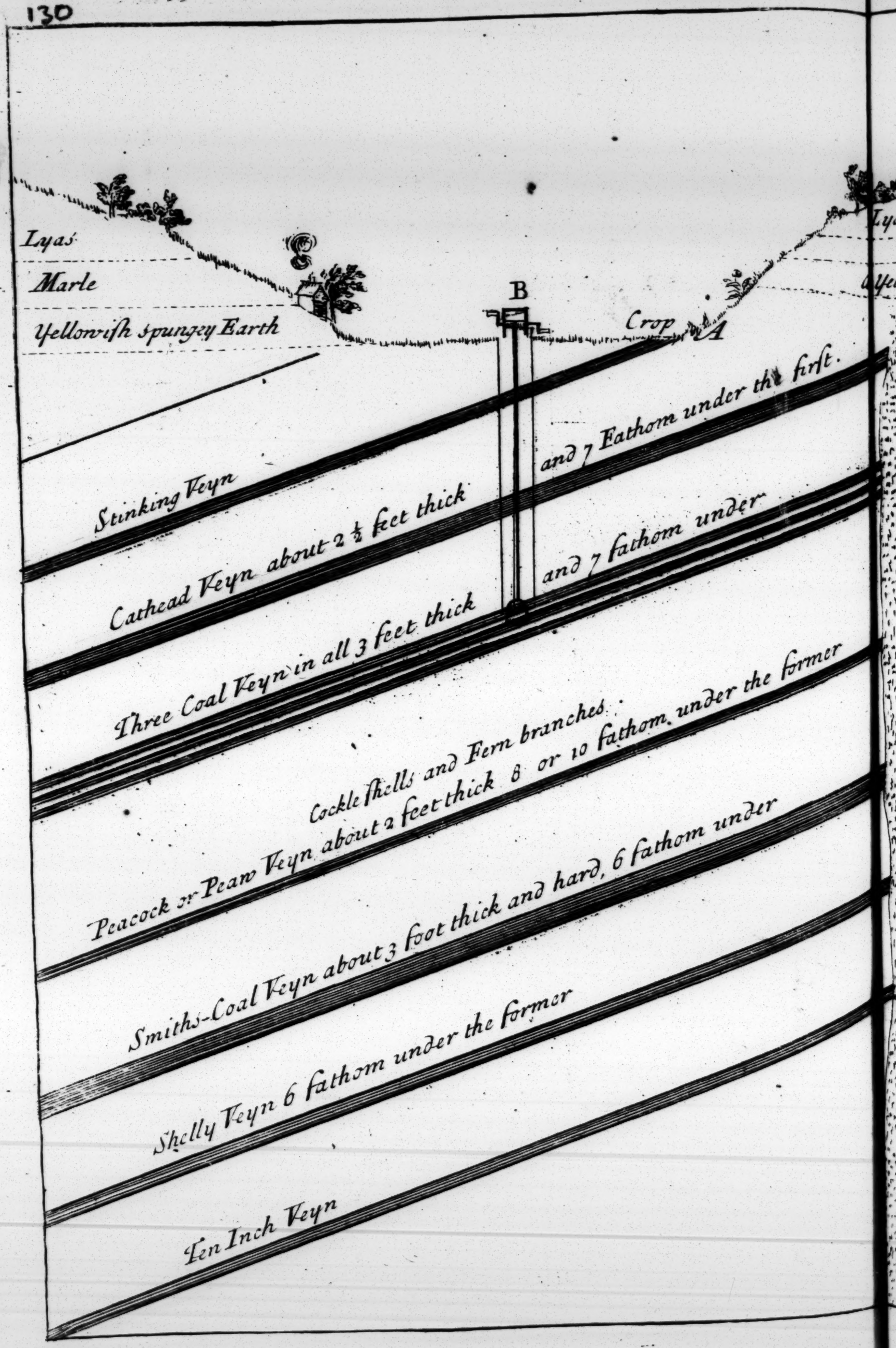
is called the *Stinking Vein.* It is hard Coal fit for Mechanick uſes, but of a ſulphurous Smell. About five Fathom and half, ſeldom more than ſeven Fathom under this, lyes another Vein, which from certain Lumps of Stone mixt with it like a *Caput mortuum* not Inflamable, called *Cats-head*, they call the *Cathead Vein.* About the ſame Depth under this again lyes the *Three Coal Vein*, ſo called becauſe it's divided into three different Coals; Between the firſt and ſecond Coal is a Stone of a foot, in ſome places two feet thick; but the middle and third Coal ſeem placed looſe on each other, without any ſeparation of a different Matter. Theſe three Veins before-mentioned are ſometimes work'd in the ſame Pit: But the next Vein which I am going to mention is generally wrought in a ſeparate Pit; for tho' it lyes the like depth under the other, the *Cliff* between them is hard and ſubject to Water; wherefore I have repreſented a Pit ſunk thro' the three Upper Veins at A. and another ſunk upon the *three Coal Veins* only at B. and ſo if they ſink on any of the lower Veins they go more to the North Weſt. *See Fig. Tab.* II.

Next under the *three Coal Veins* is the *Peaw Vein*, ſo denominated becauſe the Coal is figured with Eyes reſembling a Peacock's Tayl, gilt with Gold, which Bird in this Country Dialect is called a *Peaw.* The *Cliff* alſo over this Vein is variegated with *Cockle-ſhells* and *Fern Branches*, and this is always an Indication of this Vein, which, as I before hinted, is always ſearched for about 15 Fathom to the North Weſt of the former.

Under this again between five and ſix Fathom lies the *Smith's Coal Vein*, about a yard thick; And near the ſame depth under that again the *Shelly Vein:* And under that a Vein of 10 Inches thick, which being little valued, has not been wrought to any purpoſe.

Some ſay there is alſo another under the laſt, but

that


Lyas
Marle
Yellowish spungey Earth
B
Crop
A
Stinking Veyn
Cathead Veyn about 2 ½ feet thick
and 7 Fathom under the first.
Three Coal Veyn in all 3 feet thick
and 7 fathom under
Cockle shells and Fern branches
Peacock or Peam Veyn about 2 feet thick 8 or 10 fathom under the former
Veyn about 3 foot thick and hard, 6 fathom under
Smiths-Coal Veyn
Shelly Veyn 6 fathom under the former
Ten Inch Veyn
Lyas or
Yellowish
first.
A Rid which breaks

North West

A

yas or Lime-stone

Marle

ellowish spungey Earth

Red Earth or Malm or Loom

Lyas

Marle

Yellowish spungey Earth

Stinking Veyn

Cathead Veyn

Three Coal Veyn

Peav Veyn

Smith's-Coal Veyn

Shelly Veyn

Ten Inch Veyn because ten Inches thick

A Ridg which breaks off the Keyas and makes them Trap down or Trap up from their regular Course.

that has not been proved within Man's Memory. At *Faringdon* they have the ſame Veins, which, as I am informed, agree in all Parts with thoſe of *Biſhop-Sutton* before-mention'd. But as *Faringdon* lies four Miles South-Eaſt from *Biſhop-Sutton*, ſo, in the regular Courſe, they would lye a Mile and $\frac{1}{3}$ deeper than thoſe at *Sutton*. But as in fact they are dug near the ſame Depth, it follows there muſt be a *Trap*, or ſeveral *Traps down*, which in all muſt amount to that Depth between the ſaid Works.

Between *Faringdon* and *High-Littleton* the ſame Veins ſeem to retain their regular Courſe; but at *Littleton* their undermoſt and deepeſt Vein is the beſt Coal, which at *Faringdon* proves ſmall.

On the other hand, in the Pariſh of *Stanton-Drew*, to the North-Eaſt of the Coal-Works at *Sutton* aforeſaid, about a Mile diſtant, and in the true Courſe with thoſe at *Sutton*, the ſame Veins are found again. But here they wind a little, and their Courſe or Drift runs almoſt North, and they dip to the Eaſt; which Winding is attributed to *Ridges*, which the Workmen have met with on both Sides, and have occaſion'd them to diſcontinue the Work that way. At *Stanton* they have little of the Red Earth or *Malm* on the Surface, but come immediately to an *Iron-Gritt* or *grey Tile-Stone*, which is a Fore-runner of the *Coal-Clives*; in all other Matters they agree with the Works near *Stowy*.

In the ſame Pariſh of *Stanton-Drew*, a little to the Eaſtward, they have another Coal-work, but the Veins are in all reſpects different from the former. Their Drift or Courſe is to the Eleven a-Clock Sun, as they term it, they *Pitch* to the Five a Clock Morning, and riſe to land; conſequently to the Five a-Clock Evening-Sun. They have ſeveral Veins, but as yet only three are thought worth working. The uppermoſt about three

Feet thick ſmall *Lime Coal.* The next is about three Fathom under it, about two Feet and an half thick, fit for culinary Uſes: the undermoſt is about the like Depth under the former, only 10 Inches thick, but good hard Coal.

At *Clutton*, about two Mile from theſe latter, in the ſame Drift, *viz.* almoſt to the South East and by South, theſe laſt Veins appear again. The Surface here is red, and ſo continues to ten, and ſometimes to fourteen Fathom, and in other reſpects agree with the laſt-mention'd Works at *Stanton-Drew*.

At *Burnet*, *Queen-Charlton*, and *Briſleton*, they have Four Veins which Pitch to the North nearly, and conſequently the Drift lies almoſt Eaſt and Weſt. The Surface is red land generally to the Depth of four or five Fathom. The uppermoſt is from three to ſix Feet thick at *Briſleton*, but leſs at *Charlton* and *Burnet*. The next, call'd *Pot-Vein*, is ſix Fathom under the former, eighteen Inches thick, all hard Coal. *Thirdly*, The *Trench-Vein*, 7 Fathom under the other, which is from two Feet and half to three Feet thick, all ſolid Coal. *Fourthly*, *Rock-Vein*, always diſtinguiſh'd by a Rock of Paving-Stone, call'd *Penant*, lying over it, which Rock is ſometime twenty Feet thick, or more, and therefore this Vein is never wrought in the ſame Pit with the former Vein, but about 200 Yards more to the South, or to Land, as they term it. It's computed ſeven Fathom under the former.

This is all I can ſay in relation to the different Veins of Coal and Earth in the Coal works in theſe Parts; wherein all agree in the Oblique Situation of the Veins; and every Vein hath its *Cliff* or *Clives* lying over it, in the ſame oblique manner. All of them Pitch or Riſe about Twenty two Inches in a Fathom, and almoſt all have the ſame *Strata* of Earth, Malm,

and

and Rock over them, but differ in reſpect to their Courſe or Drift, as alſo in Thickneſs, Goodneſs, and Uſe.

Now as Coal is here generally dug in Valleys, ſo the Hills, which interfere between the ſeveral Works before mentioned, ſeem alſo to obſerve a regular Courſe in the *Strata* of Stone and Earth found in their Bowels: For in theſe Hills (I mean thoſe only that are diſpers'd between the Coal-Works above mention'd) we find on the Summits a ſtony Arable mixt with a ſpungy yellowiſh Earth and Clay; under which are Quarries of *Lyas*, in ſeveral Beds, to about eight or ten Feet deep, and ſix Feet under that thro' yellowiſh *Loom*, you have a blue Clay enclinable to *Marle*, which is about a Yard thick: Under this is another Yard of whitiſh *Loom*, and then a deep blue Marle ſoft, fat, and ſoapy, ſix Feet thick; only at about two Feet thick, it is parted by a *Marchaſite* about ſix Inches thick. But as this ſwells beyond the Bounds of a Letter, I muſt defer the farther Deſcription of theſe and ſome *Lead-Mines* to another Opportunity; only 'tis to be noted, that theſe Beds of *Stone* and *Marle*, different from *Coal*, lie all Horizontal.

Your humble Servant,

John Strachey.

V. *Some Inſtances of the very great and ſpeedy Vegetation of* TURNIPS. *Communicated by the Rev. Dr.* J. Theoph. Deſaguliers, *R. S. S.*

AT *Sutton Coldfield* in *Warwickſhire*, a peaty Ground near a Pool (of which it was formerly a part) was ſown with *Turnip-Seed* on the 2*d* Day of *July* 1702. In leſs than Three Days Time the Turnips were ſeen above Ground. At Three Weeks end the Roots were in Bigneſs equal to Walnuts. Within leſs than Five Weeks after the Sowing, the Gardener drew great Quantities of Turnips to ſell, they then being as big as large Apples. At the end of Six Weeks, *viz.* on the 12th Day of *Auguſt*, a large Turnip was plucked up (though probably not ſo big as ſeveral others then growing upon the ſame Ground) which, together with its Top and long deſcending part of the Root, weighed above Two Pounds and Fourteen Ounces. At the ſame time alſo was weighed an Ounce of the ſame ſort of Turnip-Seed, that the Gardener had ſown his Ground with; and afterwards a Thouſand of the Grains were counted ſingly out of the Ounce ſo weighed; and the reſt of the Ounce was divided into Heaps, as near as could be gueſſed, equal to the 1000 Seeds firſt ſevered and laid together: And it was found that the whole Ounce contain'd above 14600 ſingle Grains; which Number multiplied by 46 (*viz.* the Number of Ounces that the Turnip weighed) produceth 671,600, *viz.* the Number of ſingle Grains of Seed required to equal the Weight of the Turnip. From whence may be gathered, that (upon ſuppoſition, that the Increaſe of the Turnip was all along uniform and equal, from the Time

Time it was ſown till it was pluck'd up) the Grain of Seed which it ſprung from, weighing when it was ſown but $\frac{1}{14600}$ of an Ounce, was increaſed in Weight according to the following Proportions, *viz.*

In Six Weeks time	—	671,600	
Every Week	—	111,933 $\frac{1}{3}$	times its own Weight.
Every Day	—	15,990 $\frac{1}{2}$	
Every Hour	—	666 $\frac{1}{4}$	
Every Minute of an Hour	—	11	

Some time after, another Ounce of the ſame ſort of Seed was exactly weighed, and the Grains were found to be in Number 14673.

Another Turnip of the ſame Crop was plucked up on the 21*ſt* Day of *October*; and being put into a Scale, was found to weigh above Ten Pounds and an half; which unuſual and truly wonderful Bulk it acquired (it being ſuppoſed, as above, that the Growth was all along alike) by increaſing the Weight of the Seed it was raiſed from, 15 times in every Minute of an Hour from the Sowing to the Drawing of it.

The Gardener neglected to thin his Turnips in due Time, elſe probably their Growth had been more conſiderable.

At another Time, in two other ſorts of Turnip-Seed, it was found by counting, that an Ounce of one ſort contained 14702 Grains; and an Ounce of the other ſort no fewer than 14905 Grains.

It's credibly reported, that of late Years, Turnips have been pretty frequently found growing in ſeveral Counties of this Kingdom, that have weighed above twice as much; one of which was ſeen at *Birmingham* about the Year 1710.

VI. *An Account of ſome Experiments tried with Monſ.* Villette's *Burning Concave, in* June 1718. *By the Rev. Dr.* J. Harris *and Dr.* J. T. Deſaguliers, *Reg. Soc. SS.*

THIS Miroir is a Concave 47 Inches wide, and ground to a Sphere of 76 Inches *Radius*; ſo that its *Focus* is about 38 Inches diſtant from the *Vertex* of the Glaſs. The Metal of which it is made is a Mixture of Copper, Tin and Tin-Glaſs, whoſe Reflexion has ſomething of a yellow Caſt. The Concave-Surface has ſcarce any Flaws, and thoſe very ſmall; but the Convex ſide, which is alſo poliſh'd, has ſome Holes in it.

Having held ſeveral Bodies in the *Focus* of this Miroir, we obſerv'd what happen'd to them whilſt expos'd to this great Heat; and with an half ſecond Pendulum took notice of the Time in which any material Change happen'd to them.

The Experiments were as follow, and made from Nine till Twelve in the Morning.

N° 1. A red piece of a *Roman Patera*, which began to melt in 3 Seconds, was ready to drop in 100.
2. Another black Piece melted at 4, and was ready to drop at 64 Seconds.
3. Chalk taken out of an *Echinus Spatagus* fill'd with Chalk only, fled away in 23 Seconds.
4. A Foſſile-Shell calcin'd in 7 Seconds, and did no more in 64.
5. A piece of *Pompey*'s Pillar at *Alexandria*, was vitrified in the Black Part in 50 Seconds, and in the White part in 54. 6. Cop-

6. Copper-Oar, that had no Metal in it visible, vitrified in 8 Seconds.
7. Slag, or Cinder of the ancient Iron-work said to have been wrought by the *Saxons*, ready to run in 29 Seconds and an half.

Here the Glass growing hot, burn'd with much less Force.

8. Iron-Oar fled at first, but melted in 24 Seconds.
9. Talk began to calcine at 48 Seconds, and held in the *Focus* 64.
10. *Calculus humanus* in 2 Seconds was calcin'd, and only dropp'd off in 60.
11. An anonymous Fish's Tooth melted in 32 Seconds and an half.
12. The *Asbestos* seem'd condens'd a little in 28 Seconds; but it was now something cloudy: Monsr. *Villette* says that the Glass usually calcines it.
13. A Golden Marchasite broke to pieces, and began to melt in about 30 Seconds.
14. A Silver Sixpence melted in 7 Seconds and an half.
15. A King *William*'s Copper Halfpeny melted in 20 Seconds, and ran with an Hole in it in 31.
16. A King *George*'s Halfpenny melted in 16 Seconds, and ran in 34.
17. Tin melted in 3 Seconds.
18. Cast Iron in 16 Seconds.
19. Slate melted in 3 Seconds, had an Hole in 6.
20. Thin Tile melted in 4 Seconds, had a Hole and was vitrifi'd thro' in 80.
21. Bone calcin'd in 4 Seconds, and vitrifi'd in 33.

An Emerald was melted into a Substance like a Turquois Stone.

A Diamond weighing 4 Grains lost $\frac{1}{4}$ of its Weight.

VII. *An Account of the Extraordinary* METEOR *ſeen all over* England, *on the* 19*th of* March 171$\frac{8}{9}$. *With a Demonſtration of the uncommon Height thereof. By* Edm. Halley, *LL. D. and Secretary to the* Royal Society.

THIS wonderful luminous *Meteor* which was ſeen in the Heavens on the 19th of *March* laſt, as it was matter of Surprize and Aſtoniſhment to the Vulgar Spectator, ſo it afforded no leſs Subject of Enquiry and Entertainment to the ſpeculative and curious in Phyſical things: Some of its *Phænomena* being exceeding hard to account for, according to the Notions hitherto received by our Naturaliſts; ſuch are the very great Height thereof above the Earth; the vaſt Quantity of the Matter thereof; the extravagant Velocity wherewith it moved; and the prodigious Exploſions thereof heard at ſo great a Diſtance, whoſe Sound, attended with a very ſenſible Tremour of the ſubject Air, was certainly propagated through a *Medium* incredibly rare and next to a *Vacuum*.

In *Num.* 341. of theſe *Tranſactions*, I have collected what I could find of ſuch-like *Meteors*, and ſince, turning over the *Ephemerides* of *Kepler*, I accidentally hit upon another, prior to all thoſe there deſcribed, and which was ſeen all over *Germany*. Of this the Words of *Kepler* are: *Die* $\frac{7}{17}$ *Nov.* 1623. *Meteorum ignitum, Globus ardens ab occaſu in ortum volans totâ paſſim* Germaniâ *fuit conſpectus. In* Auſtria *etiam fragorem exauditum affirmarunt quaſi à fulmine; quod vanum tamen puto: nihil enim tale confirmant deſcriptiones quæ extant.* Yet

neither

neither this, nor any of the other hitherto deſcribed, ſeem to come up in any Circumſtance to this late Appearance; of which I am in hopes to give a ſatisfactory Account, being enabled by the very many Relations thereof communicated to the *Royal Society*, from moſt parts of the Kingdom; tho' it was not my good Fortune to ſee it my ſelf; and tho' very few of our Countrymen who beſt know the *Stars*, had better luck. Some of the moſt perfect Deſcriptions we have receiv'd are the following:

Firſt, Our very worthy Vice-Preſident Sir *Hans Sloan*, Baronet, being abroad at that time, happen'd to have his Eyes turned towards it, in its very firſt Eruption; and the next Day he was pleaſed to give me in Writing what he had with great Exactneſs noted about it, in the Terms following: " On *Thurſday*, *March* 19. $17\frac{18}{19}$. " paſſing along Eaſtward by the N. E. Corner of *Sou-* " *thampton-ſtreet* in *Bloomsbury-Square*, *London*, at about " a Quarter after Eight at Night, I was ſurpriz'd to " ſee a ſudden great Light, much beyond that of the " Moon, which ſhone then very bright. I turn'd to " the Weſtward where the Light was; which I appre- " hended at firſt to be artificial Fire-works or Rockets. " The firſt place I obſerv'd it in, was about the *Plei-* " *ades* Northerly, whence it moved after the manner, " but more ſlowly than a falling Star, in a ſeeming " direct Line, deſcending a little beyond, and withal " below, the Stars in *Orion*'s *Belt* then in the S. W. " The long Stream appear'd to me to be branched about " the middle, and the *Meteor* in its way turn'd Pear- " faſhion'd or tapering upwards. At the lower end it " came at laſt to be bigger and Spherical, tho' it was " not ſo big as the Full Moon. The Colour of it was " whitiſh, with an eye of Blue, of a moſt vivid daz- " ling Luſtre, which ſeem'd in Brightneſs very nearly

" to reſemble, if not ſurpaſs that of the Body of the " Sun in a clear Day, beheld by the naked Eye. This " Brightneſs obliged me to turn my Eyes (which had " their Pupils adapted to the Light of the Moon) from " it ſeveral times, as well when it was a Stream, as " when it was Pear-faſhion'd and a Globe; tho' I had " a great Curioſity to obſerve it with Attention. It " ſeem'd to move in about half a Minute or leſs, about " the Length of 20°, and to go out, as I gueſs'd, about " as much above the Horizon. There was left behind " it, where it had paſs'd, a Track of a cloudy or faint " reddiſh Yellow Colour, ſuch as red-hot Iron or glow- " ing Coals have, which remained more than a Minute, " ſeem'd to ſparkle, and kept its Place without falling. " This Track was interrupted, or had a Chaſm towards " its upper end, at about two Thirds of its Length. I " did not hear any Noiſe it made, but the place where " the Globe of Light had been, remain'd after it was " extinct, of the ſame reddiſh Yellow Colour with the " Stream for ſome time, and at firſt ſome Sparks ſeem'd " to iſſue from it, ſuch as come from red-hot Iron bea- " ten on an Anvil. The Surprize, Brightneſs of the " Light, and Noiſe of the People upon the Variations " of the Appearance, calling to one another to obſerve " what they never had obſerv'd in their Days, and " thought to be prodigious, hinder'd me from taking " notice or remembring any thing farther about it.

It were to be wiſht that Sir *Hans* had more eſpecial- ly regarded the Situation of the Track of this Meteor among the fixt Stars, and let us know how much it paſt above the *Pleiades*, and how much under the *Belt* of *Orion*, that ſo we might with more Certainty have determin'd its Poſition in reſpect of the Horizon of *London*; for which purpoſe the whole Number of Spe- ctators there has not furniſhed us with one ſufficient

Obſerva-

Obſervation. But all the Relations, however otherwiſe differing, agree in this, that the Splendour was little inferior to that of the Sun; that within doors the *Candles* gave no manner of Light, and in the Streets not only all the *Stars* diſappear'd, but the *Moon* then Nine Days old, and high near the Meridian, the Sky being very clear, was ſo far effaced as to be ſcarce ſeen, at leaſt not to caſt a Shade, even where the Beams of the Meteor were intercepted by the Houſes: ſo that for ſome few Seconds of Time, in all reſpects it reſembled perfect Day.

The Time when this happen'd was generally reckoned at a quarter paſt Eight; but by the more accurate Account of the Rev. Mr. *Pound* (who only ſaw the Light) agreeing with what has been ſent us from the *Pariſian* Obſervatory, it appears to have been at $8^h\ 8'$ apparent Time at *London*. And the Sun being then in $9\frac{1}{2}$ *gr.* of *Aries*, the Right Aſcenſion of the Mid-Heaven was 130 *gr.* 45', whereby the Poſition of the Sphere of fixt Stars is given. Hence the *Lucida Pleiadum* will be found at that time to have been $25\frac{1}{4}$ *gr.* high, in an *Azimuth* 6 *gr.* to the Northward of the Weſt, and conſequently the Arch the Meteor moved in, was inclined to the Horizon with an Angle of about 27 *gr.* having its Node or Interſection therewith, nearly *South South Weſt*; as will be more evident by what follows.

At *Oxford* five Minutes earlier, Mr. *John Whiteſide*, *R. S. Soc.* Keeper of the *Aſhmole Muſeum*, and very skilful in both Mathematical and Phyſical Matters, immediately after the Extinction of the Meteor, made haſte out to ſee what it might be, and well conſider'd the Situation of the Track it had left in the Sky: He found it to have paſt about $1\frac{1}{2}$ Degree above the preceding Shoulder of *Orion*, and about $3\frac{1}{2}$ *gr.* above

the middle of his *Belt*, where there appear'd a luminous *Nubecula* of a reddiſh Light, being a Dilatation of the Track, ſeeming to have been occaſion'd by ſome Exploſion there; and by what he could learn from thoſe that ſaw it, it was thereabout that it broke out, and firſt began to efface the Stars. Hence it proceeded as to ſenſe in an Arch of a great Circle, and paſſing in the middle between the Tail of *Lepus* (θ *Bayero*) and β in the Fore-Foot of *Canis major*, it terminated about ξ in the Breaſt of the ſame, nearly in 95 *gr.* of Right-Aſcenſion, with 23 *gr.* South Declination: and at the place of its Extinction there remained a large whitiſh *Nebula*, much broader and of a ſtronger Light than the reſt of the Track, which he took for a certain Indication of a very great Exploſion made there. By Computation it will be found that the Angle this Track made with the Horizon of *Oxford* was neareſt 40 *gr.* and its Interſection due *S S W*; and that the place of its Extinction was about 9 *gr.* above the Horizon, in the Azimuth of 32 *gr.* to the *Weſt*.

At *Worceſter* Mr. *Nicolas Fatio*, a Perſon greatly skill'd in Aſtronomical Affairs, ſaw this Meteor deſcend obliquely towards the *South*, making an Angle with the Horizon of about 65°, and interſecting it about *S S W* ½ *S*, as may be collected from a Scheme thereof ſent up by him, and communicated to the *Royal Society*, ſeeming to be deſign'd with ſufficient Exactneſs. By this the Track left all *Orion* and *Canis major* to the Weſtward, and divided the Diſtance between *Sirius* and *Procyon*, ſo as to be almoſt twice as far from *Procyon* as *Sirius*. The Time here was one Minute before Eight, this City being about 9′ of Time to the Weſt of *London*, and conſequently the Right-Aſcenſion of the Mid-Heaven 128½ *gr.*

Now

Now the Situation of the three Cities *London*, *Oxford*, and *Worceſter* being nearly on the ſame *W.N.W.* Point, whereon the Track of the Meteor had its greateſt Altitude above the Horizon, equal to the Angle of its viſible Way; if we ſuppoſe it at *London* to have been 27 *gr.* high, and at the ſame time at *Worceſter* to be 65 *gr.* high, in the Plane of the Vertical Circle paſſing through *London* and *Worceſter*; ſuppoſing likewiſe the Diſtance between them to be 90 Geographical Miles, or one Degree and half of an Arch of a great Circle of the *Earth*, we ſhall by a Trigonometrical *Calculus*, too obvious to be here inſerted, find the perpendicular Height to have been 64 ſuch Miles; and the Point over which it was then perpendicular to have been 30 ſuch Miles *W.N.W.* from *Worceſter*. And the Geographical Mile to the *Engliſh* Statute Mile being as 23 to 20, this Height will be no leſs than 73 $\frac{1}{7}$ *Engliſh* Miles. The place alſo directly under it, will be found to be about *Preſtain* on the Confines of *Hereford* and *Radnor*-Shires. Nor can we be much out in this Determination, the *Oxford* Obſervation concurring nearly in the ſame Concluſion.

This Altitude being added to the Semidiameter of the Earth as *Radius*, becomes the *Secant* of Eleven Degrees, ſo that the Meteor might be ſeen above the Horizon in all Places not more than 220 Leagues diſtant from it. Whence it will not be ſtrange that it ſhould be ſeen over all Parts of the Iſlands of *Great Britain* and *Ireland*, over all *Holland* and the hither Parts of *Germany*, *France* and *Spain*, at one and the ſame inſtant of Time

This ſuggeſts a very great uſe that might be made of theſe momentaneous *hænomena*, to determine the *Geographical Longitudes* of Places. For if in any two Places two Obſervers, by help of *Pendulum Clocks* duly corrected by

by Cœleſtial Obſervation, do exactly note at what Hour, Minute and Second ſuch a Meteor as this blows up and is extinguiſht, the Difference of thoſe Times will be the Difference of Longitude of the two Places, as is well known. Nor does it require ſo much as the Uſe of a *Teleſcope*, as in the Methods hitherto put in practice for that purpoſe: ſo that if theſe Appearances could be predicted, and Notice given of their coming, that we might know when to expect them, I ſhould make no Difficulty to prefer this way of ſettling the *Geography* of a Country before all others.

Having thus fixt one Point in the Line of its Motion, let us now conſider what courſe the Meteor took from thence. And firſt at the Town of *Kirkby-Stephens*, on the Borders of *Yorkſhire* and *Weſtmoreland*, in a Meridian very little to the Weſtward of *Worceſter*, but about $2\frac{1}{2}$ *gr.* more to the North, it was obſerved to break out as from a dusky Cloud, directly under the Moon, and from thence to deſcend, nearly in a Perpendicular, almoſt to the Horizon. Now the Moon, being at that time in the third Degree of *Leo*, was about half an hour paſt the Meridian, and conſequently much about a point to the Weſt, or *S b W*: and the Situation of *Preſtain* from *Kirby-Stevens* being ſufficiently near upon the ſame Point, it follows that the Direction of the Track of the Meteor was according to the Great Circle paſſing over thoſe two Places.

And this is further confirm'd by the Obſervation of *Sam Crunys*, Eſq; *Reg. Soc. Soc.* who at *Tiverton*, about twelve Geographical Miles nearly due North from *Exeter*, obſerved the firſt Exploſion of this Meteor exactly in his Zenith, as he was aſſured by applying his Eye to the ſide of his Door, which he took to be perpendicular, and looking upwards: And from thence he ſaw it deſcend to the Southwards directly in the ſame

Azi-

Azimuth, without declining either to the Right or Left: Hence it is plain, that the Track likewiſe paſs'd over this place, which by our beſt Maps is found to lie in a Line with *Preſtain* and *Kirby-Stevens* with ſufficient Exactneſs; ſo that we ſhall take it for granted that this was the very Courſe it held.

On this Suppoſition, that the firſt Exploſion attended with the reddiſh *Nubecula*, was directly over *Tiverton*, we have the *Oxford* Obſervation to compare with it, in order to determine more nicely the perpendicular Altitude there. At *Oxford* this *Nubecula* was found to be $3\frac{1}{2}$ *gr.* above the middle Star of *Orion's Girdle*, at $8^h\ 3'$, and was therefore $26\frac{1}{2}$ *gr.* above the Horizon; and the Diſtance between *Oxford* and *Tiverton*, being 1°. 55′ or 115 Geographical Miles, it will be as the Sine of 61°. 35′ to the Sine of 63° 30′ So the Semidiameter of the Earth being $3437\frac{3}{4}$ ſuch Miles, to 3498 Miles the Diſtance of the Meteor from the Center of the Earth; from which deducting the Semidiameter, there remains $60\frac{1}{4}$ Geographical Miles for the Height of the Meteor above *Tiverton*: And that this was ſo is confirmed by the Obſervation of the Rev. Mr. *Will. Derham*, who at *Windſor* ſaw the aforeſaid *Nubecula* about two Degrees above the moſt Southerly of the Seven Stars in the Shield of *Orion*; that is (the Time being $8^h\ 6'$) in the Altitude of $23\frac{1}{2}$ *gr.* whence, the Diſtance between *Tiverton* and *Windſor* being 150 meaſured Miles, or 130 Geographical by a like Proportion we ſhall find the ſame Height of the Meteor 60 ſuch Miles wanting only one Quarter. So that in a round Number we may conclude it to have been juſt 60 Geographic or 69 Statute Miles above the Earth's Surface. Nor is it poſſible to come at a preciſe Determination of this matter, by reaſon of the Coarſeneſs and Inaccuracy of our *Data*, which were only

only the Notes of Perſons under the Surprize of the ſuddenneſs of the Light, and no ways pretending to Exactneſs; however, ſuch as they are, they abundantly evince the Height thereof to have exceeded 60 *Engliſh* Miles, not to ſay 38 or 40, as ſome would fain have it.

I was unwilling to leave off, till I had pitcht upon ſome Hypotheſis that might ſubject the Motion of this Meteor to a *Calculus*, that the Curious might be able to compute the viſible way thereof, either in reſpect of the Horizon, or among the fixt Stars: This I found might be done with tolerable Exactneſs, ſuppoſing that it mov'd in the Arch of a Circle concentrick with the Earth, but 60 Geogr. Miles without it; and that the Point of the firſt Exploſion was over the Lat. of 50° 40′ and 3° 40′ to the Weſt of *London*; and that of the laſt Extinction over Lat. 47°. 40′ with 4°. 50′ *Weſt* Longitude: The Time being fixt to 8 Minutes paſt Eight at *London*. Hence it will be eaſy, by a Trigonometrical Proceſs, to obtain the viſible Altitude and Azimuth of the Meteor at either of its Exploſions, as ſeen from any Place whoſe Longitude and Latitude is known; and from the Time given, the Points in the Sphere of Stars anſwering to thoſe Azimuths and Altitudes are readily deduced. Let thoſe that contend for a much leſs Height of this Meteor try if they can on ſuch their Suppoſition reconcile the ſeveral *Phænomena* before recited with one another, and with the Obſervation of the Rev. Mr. *William Ella*, Rector of *Eampton* in *Nottinghamſhire*, between *Gainsborough* and *Redford*, which for its Exactneſs I muſt not omit. Here at 8^h 5′ the Meteor was ſeen to paſs preciſely in the middle between *Sirius* and the Fore-Foot of *Canis major*, moving obliquely to the Southward, in a Line whoſe Direction ſeem'd to be from the middle between

the

the two Shoulders of *Orion*. The Latitude of the place being nearly 53°. 20′, and Longitude *West* from *London* 0°. 45′. Let them try how they can account for its being ſeen five Degrees high at *Aberdeen* in *Scotland*, and near as much at *Peterhead* half a Degree more Northerly: and then they will be better able to judge whether it did not exceed the reputed Limits of our Atmoſphere. Laſtly, if the apparent Altitude of the Meteor at *Paris* was not 5½ but 11 *gr.* on the *W b N* Point, when it muſt have been in its greateſt Luſtre, there will be no pretence to bring it lower than I have made it, eſpecially if it be allowed to have follow'd the Track I have aſſign'd it, over *Preſtain*, *Cardiff*, *Minhead*, *Tiverton*, and *Breſt* in *Bretany*.

Allowing this to have been the Path it mov'd in, it would be eaſy to aſſign the real Magnitude and Velocity of this Meteor, if the ſeveral Accounts of its apparent Diameter, and of the Time of its Paſſage from one of its Exploſions to the other, were conſiſtent with themſelves. But ſome of them making its viſible Appearance nearly equal to the Sun's, which in the Opinion of many it far exceeded, we may ſuppoſe with the leaſt that, at the time when it firſt broke out over *Tiverton*, its Diameter was half a Degree. And its Horizontal Diſtance being 150 Geogr. Miles from *London*, and its Altitude 60, the Hypothenuſal or real Diſtance from the Eye will be more than 160 ſuch Miles; to which Radius the Subtenſe of half a Degree will be above an Engliſh Mile and half, being about 2800 Yards *quamproximè*. After the ſame manner it is difficult to aſſign its due Velocity, whilſt ſome make it half, others leſs than a quarter, of a Minute, in paſſing from its firſt Exploſion to its laſt Extinction: But the Diſtance it moved in that time being about 3 *gr.* or 180 Geogr. Miles, we may modeſtly compute

it to have run above 300 ſuch Miles in a Minute; which is a Swiftneſs wholly incredible, and ſuch, that if a heavy Body were projected horizontally with the ſame, it would not deſcend by its Gravity to the Earth, but would rather fly off, and move round its Center in a perpetual Orb, reſembling that of the *Moon*.

Of ſeveral Accidents that were reported to have attended its Paſſage, many were the effect of pure Fancy; ſuch as the hearing it hiſs as it went along, as if it had been very near at hand: others imagined they felt the Warmth of its Beams; and ſome there were that thought, at leaſt wrote, that they were ſcalded by it. But what is certain, and no way to be diſputed, is the wonderful Noiſe that follow'd its Exploſion. All Accounts from *Devon* and *Cornwal* and the neighbouring Counties are unanimous, that there was heard there, as it were the Report of a very great Cannon, or rather of a Broad ſide, at ſome diſtance, which was ſoon follow'd by a rattling Noiſe, as if many ſmall Arms had been promiſcuouſly diſcharged. What was peculiar to this Sound was, that it was attended with an uncommon Tremour of the Air, and every where in thoſe Counties, very ſenſibly ſhook the Glaſs-Windows and Doors in the Houſes, and according to ſome, even the Houſes themſelves, beyond the uſual Effect of Cannon, though near; and Mr. *Crumys* at *Tiverton*, on this occaſion, loſt a Looking-Glaſs, that being looſe in its Frame, fell out on the ſhock, and was broken. Nor do we yet know the Extent of this prodigious Sound, which was heard, againſt the then Eaſterly Wind, in the Neighbourhood of *London*, as I am inform'd; and by the Learned Dr. *Tabor*, who diſtinctly heard it beyond *Lewis* in *Suſſex*: So that I cannot help thinking, that ſuch a Meteor as this might have occaſion'd that famous Ode of *Horace*: *Parcus Deorum cultor*, &c.

* ——— *Namque*

——*Namque Dieſpiter*
Igne coruſco nubila dividens
Plerumque, per purum tonantes
Egit equos volucremque currum,
Quo bruta tellus, &c. Concutitur.——

But whether the Report heard near *Lewis* were of that Exploſion right over *Devonſhire*, or rather of that latter and much greater at the Extinction over *Britany*, I ſhall not undertake to determine, till we have ſome further Accounts from *France*, whence hitherto we have only had, that at *Paris* the Time of the Appearance was at 17 Minutes paſt Eight.

It remains to attempt ſomething towards a Solution of the uncommon *Phænomena* of this Meteor; and by comparing them with things more familiar to us, to ſhew at leaſt how they might poſſibly be effected. And firſt the unuſual and continu'd Heats of the laſt Summer in theſe Parts of the World, may well be ſuppos'd to have excited an extraordinary Quantity of Vapour of all ſorts; of which the aqueous and moſt others, ſoon condens'd by Cold, and wanting a certain Degree of Specifick Gravity in the Air to buoy them up, aſcend but to a ſmall Height, and are quickly returned in Rain, Dews, *&c.* whereas the inflammable ſulphureous Vapours, by an innate Levity, have a ſort of *Vis centrifuga*, and not only have no need of the Air to ſupport them, but being agitated by Heat, will aſcend in *Vacuo Boileano*, and ſublime to the top of the Receiver, when moſt other Fumes fall inſtantly down, and lie like Water at the bottom; the Experiment whereof was firſt ſhewn me by the Reverend Mr. *Whiteſide* at *Oxford*, and was very lately made before the *Royal Society*. By this we may comprehend how the matter of the Meteor might have been raiſed from a large Tract of the Earth's Surface, and aſcend far above the reputed Limits of the *Atmoſphere*; where, being diſengaged from all other Particles, by that principle of Nature that congregates *Homogenia* viſible in ſo many Inſtances, its

Atoms

Atoms might in length of time coaleſce and run * *fortuitouſly* together, as we ſee Salts ſhoot in Water; and gradually contracting themſelves into a narrower compaſs, might lie like a Train of *Gunpowder* in the *Ether*, till catching fire by ſome internal Ferment, as we find the Damps in Mines frequently do, the Flame would be communicated to its continued parts, and ſo run on like a Train fir'd.

This may explain how it came to move with ſo unconceivable a Velocity; for if a continu'd Train of Powder were no bigger than a Barrel, it is not eaſy to ſay how very faſt the Fire would fly alongſt it; much leſs can we imagin the Rapidity of the Accenſion of theſe more inflammable Vapours, lying in a Train of ſo vaſt a Thickneſs. If this were the Caſe, as it is highly probable, it was not a Globe of Fire that ran along, but a ſucceſſive kindling of new Matter: and as ſome parts of the Earth might emit theſe Vapours more copiouſly than others, this Train might in ſome parts thereof, be much denſer and bigger than in others, which might occaſion ſeveral ſmaller Exploſions, as the Fire ran along it, beſides the great ones which were like the blowing up of Magazines. Thus we may account for the rattling Noiſe like ſmall-Arms, heard after the great Bounce on the Exploſion over *Tiverton*; the Continuance of which for ſome time, argues that the Sound thereof came from Diſtances that encreaſed.

What may be ſaid to the Propagation of the Sound thro' a *Medium*, according to the receiv'd Theory of the Air above 300000 times rarer than what we breath, and as I ſaid before, next to a *Vacuum*, I muſt confeſs I know not. Hitherto we have concluded the Air to be the Vehicle of Sound; and in our artificial *Vacuum* we find it greatly diminiſh'd: but we have this only Inſtance of the effect of an Exploſion of a Mile or two diameter, the immenſity of which may perhaps compenſate the extream Fineneſs of the *Medium*.

F I N I S.

* Dele *fortuitouſly*.

PHILOSOPHICAL TRANSACTIONS.

For the Months of *June*, *July* and *Auguſt*, 1719.

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III. *An Account of ſome Experiments relating to the Specifick Gravity of Human Blood. By* James Jurin, M. D. *Fellow of the* College *of* Phyſicians, *and* F. R. S.

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V. *A Way for* Myopes *to uſe* Teleſcopes *without Eye-Glaſſes, an Object-Glaſs alone becoming as uſeful to them, and ſometimes more, than a Combination of Glaſſes. Communicated to the* Royal Society, *by the Reverend* J. T. Deſaguliers, LL. D. *and* F. R. S.

VI. *New and accurate Tables for the ready Computing of the Eclipſes of the* firſt Satellite *of* Jupiter *by Addition only. By the Reverend Mr.* James Pound, R. S. S.

Jos: Banks

I. *An Observation of the end of the Total Lunar Eclipse on the 5th of* March 1718. *observed near the Cape of* Good Hope, *serving to determine the Longitude thereof. With Remarks thereon. By* E. Halley, *R. S. Secr.*

'TIS now better than thirty Years since I had a Dispute with some of the *French Geographers* about the Longitude of the Cape of *Good Hope*, said to have been observ'd by the Religious Missionaries sent to *China* in the Year 1685. By an Emersion of the first Satellite of *Jupiter*, they determined that Cape to be $1^h.\ 11'$ or $17\frac{3}{4}$ *grad.* more Easterly than *Paris*, that is 20 *grad.* from *London*; which for the reasons I then gave, I concluded could not be more than 17 *grad.* See *Phil. Transact.* N° 185. Very lately I have fallen upon an Observation which I believe will determine the Controversy in my favour: for I had accidentally a Journal of an Officer of the Ship *Emperor* put into my Hands, who in his return from *India*, on the fifth of *March* 1718. observ'd the End of a Lunar Eclipse, when the visible altitude of the Moons Centre was 13°. 25'. he being then in the Latitude of 34°. 23' *South*, and as they found afterwards, just 180 Leagues to the Eastwards of Cape *Bonne Esperance*. By Calculation I find that in that Latitude the Moon had that height at $7^h.\ 17'\frac{1}{2}$ *P. M.* and by comparing this Eclipse with that we observ'd with great exactness on *Febr.* 11°. 1682. (which agrees perfectly well with our Numbers) I conclude the middle of this to have been at *London* at $3^h.$ 48' *P. M.* To which adding $1^h.$ 46' for the

the Semiduration (this being very certain from the obſerv'd Continuance of the Eclipſe of 1682.) the End will be found to have been at *London* at $5^h.\ 34'$. The Ship was therefore in a Meridian 26° to the Eaſtwards of *London*: But ſhe was at that time 180 Leagues to the Eaſtwards of the *Cape*, which diſtance in that Latitude gives eleven Degrees of Longitude; this therefore being deducted from the Longitude of the Ship, leaves juſt 15 *grad.* or one Hour, for the difference of Meridians between *London* and the *Cape*. So that by this account the *Cape* is yet nearer our Meridian than I had formerly made it, and near ſix Degrees nearer than M. *De la Hire* places it in his *Tables*.

This Eclipſe was attended with all the Circumſtances requiſite to make the Concluſion as certain as the nature of the thing will admit of: For the Moon was nearly *in Perigæo*, and the Eclipſe almoſt central; for which reaſons ſhe emerged out of the Shadow as ſwiftly as poſſible: The Sea was very ſmooth, there having been little Wind for above 30 Hours before; and the Moon was not too high to be well obſerved with a Foreſtaff. Nor were they long at Sea before they made the Land, for in leſs than five Days, on the tenth of *March* at Noon, they had paſt Cape *d'Agulhas* the moſt Southerly Promontory of *Africa*, which then bore from them *North Eaſt*, about ſeven Leagues diſtant. The End of this Eclipſe, though not viſible here, might have been ſeen in *Germany*, both at *Nurenburg*, *Leipſick* and *Berlin*, but we cannot learn that it was any where obſerved there; however our Numbers in this Caſe may be ſecurely relied on.

On this occaſion it may not be amiſs to inſert an Obſervation or two I procured to be made at the *Cape*, by Mr. *Alexander Brown* a *Scotch* Gentleman, who went to reſide in *India* on our Companies account. He carried

with

with him a very good *Braſs Quadrant* of above two Foot Radius, and at the *Dutch* Settlement at *Table Bay*, having rectify'd his *Pendulum-Clock* by correſpondent Altitudes, on the 4th of *Auguſt* 1694, at $5^h.$ 59′ *Manè*, the diſtance of the bright Limb of the Moon from the right Shoulder of *Orion* was obſerv'd to be 25° 3′. And the next Morning *Aug.* 5. at $5^h.$ 21′. 12″, the ſame Limb was diſtant from *Procyon* 25°. 57′, and at $5^h.$ 36′. 48″ from the *Lucida Arietis* 58°. 29′.

It were much to be wiſht that the Moon had, either of theſe Mornings, been accurately obſerv'd at *Greenwich* or *Paris*, or at ſome Place in *Europe*, whoſe Longitude from them is well known. But that failing us, I had recourſe to the Period of the Lunar Motions, which is perform'd in 18 Years and ten or eleven Days, after which the Errors of our Lunar Computations return very nearly the ſame; and I found among my own old Obſervations, one that tallyed well with that of the 4th of *Auguſt*. *Viz.* Anno 1676. *July* 23°. $13^h.$ 11′. 35″. at *Oxford*, I obſerv'd the Moon to apply to the Star *in medio Collo Tauri*, by *Bayer* markt *A*. The Star at that time was diſtant from the Southern and neareſt Cuſp of the Moon by the Micrometer 20′. 32″. and at $13^h.$ 17′. 15″. when it ſeem'd to immerge upon the bright Limb of the Moon, it was diſtant from the Northern Cuſp 23′. 20″; but this leſs certain by reaſon of the hazey Air. The Star at that time was in ♉ 28°. 56′. with 1°. 13′. 20″. *North* Lat. whereby I found that our Lunar Tables, founded on Sir *Iſaac Newton*'s correct Theory of her Motion, gave her place at that time only two Minutes too ſlow; which Error being allowed on the 4th of *Auguſt* 1694. the reſult was, that $5^h.$ 59′. at Cape *Bonne Eſperance* was at *London* $4^h.$ 53′. whence the difference of Longitude $16\frac{1}{2}$ degrees, ſufficiently near what we had before determin'd.

II. JACOBI KEILL, *M. D.*

De Viribus Cordis Epiſtola.

Viro Celeberrimo

RICHARDO MEAD, *M. D.*

S. P. D.

Jacobus Keill.

EPiſtolam D. *Jurin*, Tibi, Vir Clariſſime, inſcriptam, in Actis Philoſophicis nuper publicatam legi; in quâ, ea quæ à me traduntur de viribus Cordis, infirmare conatur vir Doctiſſimus. Cum æſtimatio virium, quibus Cor Sanguinem expellit, à *Borellio* facta, fere omnibus valde incredibilis videbatur, non me temerarium, non *Borelli* nomini injurioſum, non orbi literato ingratum facturum exiſtimavi, ſi ad verum propius accedere tentarem. In quo Tentamine, non accuratam virium Cordis definitionem mihi propoſitum erat dare, ſed potius methodum, quâ hæ vires forte inveniri poſſent, indicare; & Geometriæ peritiores ad Problematis valde deſiderati inveſtigationem incitare. Quo animo tentamen illud primùm ſuſcepi, eodem ea, quæ in eo reprehendit vir Doctiſſimus, nunc defendam. Ne-

quaquam enim mihi honorem quæro (utcunque parva mea exiſtimatio ſit, eſt certe debito major) ſed Genti Medicæ lucem undecunque illatam gaudeo. Idcirco non ut Decreta mea ſuſtineam, ſed ut vir huic negotio pluſquam par, ſuas demonſtrationes ſecum reputare, & Reipublicæ Literariæ correctiores denuo reddere dignetur, hanc Tibi ſcripſi literam. Quem enim Adverſarius Patronum ſibi ambivit, ego Te judicem Controverſiæ intelligentem & æquiſſimum mehercule exopto.

Præcipuum quod *Borellio*, D. *Moreland*, mihi objicit vitium eſt, quod in Potentiâ Cordis æſtimandâ, quam rationem ad pondus iners, vel corporis gravitatem obtineat, determinare ſuſcepimus. " Sed Cor, inquit, " cum & ipſum inter contrahendum movetur, & corpora " oppoſita, Sanguinem nempe & arteriarum tunicas " in motum impellit, patet ejus *Potentiam* non aliâ " ratione ſciri poſſe quanta ſit, quam ut motus hujus " quantitatem cognitam teneamus. *Motus* autem qui- " libet cum pondere quieſcente comparari non magis " poteſt, quàm linea cum rectangulo." At à nemine certè noſtrûm, quod ſcio, eſt *Motus* Cordis cum pondere quieſcente comparatus. Potentiam autem Cordis, ſeu vim Cordis motricem & Sanguinem impellentem cum pondere conferre, quid prohibet non video. Quamquam enim inter *Pondus* & *Motum* corporis ſolidi nulla ſit relatio, vis tamen motrix, ſi in fluidum agit, ad vim gravitatis quandam certe rationem habet. Et revera vis corporis motrix, certam in fluido motûs quantitatem in dato tempore efficiens, æqualis eſt ponderi, quod vi gravitatis cadendo, in eodem tempore, eandem motûs quantitatem ſibi acquirit. Hinc vis, quâ ex orificio aliquo aqua exprimitur, certo ponderi æqualis eſſe dicitur: quia pondus datum, & vis aquam exprimens æquales motus in temporibus æqualibus ge-

nerant.

nerant. Hic genuinus Corollarii *Newtoniani* ſenſus mihi videtur eſſe, nec ab hoc ſenſu diſcrepant, quæ de Cordis viribus explicui. Verba *Newtoni* ſunt, *Vis, qua totus aquæ exilientis motus generari poteſt, æqualis eſt ponderi &c.* quæ non ſatis attendiſſe videtur *Jurinius*, cum dicit *Pondus autem illud quo motus aquæ ex vaſe effluentis generari poteſt*, &c.

Sed ſi hâc re à nobis peccatum eſt, cum ſummis certe hujus ſæculi Geometris *Hugenio & Newtono* peccavimus, quorum uterque vim fluidorum per vim gravitatis exponit. Nec in *Corol.* prædicto id ſolummodo facit *Newtonus*, ſed in aliis etiam locis oſtendit Methodum, quâ ratio reſiſtentiæ Medii, id eſt, actionis fluidi in corpus ſolidum, ad vim Gravitatis vel centripetam inveniri poteſt, ut videri licet in *Prop. 4ta* & *5ta* Libri ſecundi, eorumque Corollariis. Alia profecto eſt actio fluidorum in ſolidum, & alia ſolidorum in ſe invicem. Fluidum datâ velocitate motum, datum pondus ſuſtinere poteſt, cum fluidi partes ſibi mutuo continuò ſuccedentes in pondus impingunt, adeoque vis fluidi eſt revera ponderi æqualis; ſed cum Solidorum non par eſt ratio, eorum Vis cum Gravitate comparari nequit.

Me inſuper reprehendit Vir ingenioſiſſimus, quòd velocitatem Sanginis è corde detruſi, per totam ſyſtolem æqualem poſui, quam ille valde inæqualem eſſe demonſtravit. Verum à me nuſquam Sanguini æqualis data eſt velocitas, ſed pro ſummâ omnium velocitatum mediam poſui. Sed utrum æqualis vel inæqualis eſt Sanguinis è Corde ejecti celeritas, nondum ſatis mihi conſtat; certe quæ pro æquali velocitate ſtat ratio, ea mihi in præſens firmior videtur.

Sic emaculatis vitiis quæ in primâ noſtrâ methodo reprehendit vir Cl: quid in alterâ quæ ſubjuncta eſt, illi diſplicet, videamus. Et eſt certe aſſumptio illa quæ à *Borellio*, aliisque viris doctis fæpius uſurpata eſt,

nempe, quod ſimilium muſculorum vires ſunt in ratione ponderum. Aliam virium rationem in Theoremate 5to ſtabilire conatur *Jurinius*: ſed cum ex communi omnium ſuorum Theorematum Principio oritur demonſtratio, communi etiam eorum fato involvetur: Si enim principium illud fallax eſt, (ut mihi videtur) nec ad caſus ad quos adhibetur, congruit; corruunt certe omnia, quæ hâc baſi innituntur. Supponit Vir Clar: Vaſorum tunicas in Sanguinem intùs contentum impetu irruere, & motûs ſui partem Sanguini ictu communicare: & hîc in motu Cordis, vult Ventriculum tanquam ſolidum Corpus, datâ velocitate motum, in Sanguinem impingere, & ictu motûs ſui partem illi impertire: quæ ſuppoſitio nec Sanguinis nec Cordis, nec Aeris è Pulmone expreſſi motui competit, nec ullâ minimorum ictuum reiteratione, horum motibus ita accommodari poteſt, quin quæ inde deducentur concluſiones pro incertis & omnino falſis haberi debeant.

Cum inter Sanguinem & Cordis intimum nullum intercedit ſpatium, ſed eſt alter alteri contiguus, non ictu hoc in illum, ſed preſſu agit: nec ullam in initio ſuæ contractionis celeritatem ventriculi habent, ſed ſe contrahendo velocitatem tempore acquirunt, tanquam gravia cadendo, vel ut fluida rareſcendo, ex quo forte omnis vis Cordis oritur. Adeoque non æquabilis eſt motus contractionis, ut vult Vir Doctiſſimus, ſed eſt motus inſtar cadentis acceleratus. Idem igitur eſt diſcrimen inter ictum quo Cor Sanguinem ferire vult *Jurinius*, & preſſuram quâ Cor revera in Sanguinem agit, quod eſt inter actionem corporis ſolidi moti, & vim gravitatis: ſed ipſo fatente, hæc comparari nequeunt, adeoque preſſura ſeu actio cordis in Sanguinem per ictum nec à Viro laudato expoſita eſt, nec unquam exponi poteſt. Hanc ſententiam confirmat ipſa Cordis potentia à Viro Cl. inventa. Si enim pondus datâ velocitate motum,

cordis

cordis potentiæ æquale eſſet, tunc Sanguis omni vi Cordis in pondus illud directe impulſus motum ponderis temporis momento deſtrueret: ſed quocunque magno impetu ponderi occurrat Sanguis, nunquam illi omnem motum in inſtanti eripiet, adeoque eſt hoc pondere potentia Cordis minor, nec recte per motum ponderis vires Cordis exponuntur.

Fluidorum vires in corpora ſolida, ubique eodem prorſus modo quo ſolidorum vires in ſe invicem, *Jurinius* æſtimat & perpendit, cum tamen maxima interſit differentia; & ab hoc capite fluit quicquid eſt in illius Propoſitionibus erroris. Ubi enim corpus ſolidum, cujus partes firmiter inter ſe cohærent, in aliud impingit, unaquæque corporis particula ſimul & ſemel ſuam alteri vim impertit; at res aliter ſe habet in fluidis, in queis nulla eſt partium cohærentia, nulla fluidi pars, niſi in ipſo tactu, in corpus ſibi oppoſitum agit: idcirco cum columna aquæ adverſus corpus ſolidum ſurſum vertitur, partes columnæ à corpore remotiores nullam illi vim imprimunt. Corpus etiam ſolidum unicum ſolummodo ictum alteri communicat; at columna fluidi in corpus ſibi oppoſitum continue agit, & minima columnæ pars minimo temporis momento, ictum infinite parvum illi imprimit, eodem prorſus modo quo gravia cadendo agunt, quibus igitur fluidorum motus recte comparatur. Porro omnis motus corporis ſolidi in alterum directe impingentis in temporis momento deſtrui poteſt: ſed motus ſolidi vim fluido imprimentis non niſi gradatim imminuitur, & in dato tempore evaneſcit, pari ratione, quâ Gravitas in corpus ſurſum miſſum vim ſuam exerit. Ex quibus ſatis abunde conſtat, inter vim fluidi in motum acti, & vim gravitatis magnam eſſe affinitatem, & unam per alteram recte exponi poſſe; vim autem corporis ſolidi ad vim gravitatis referri non poſſe. Cumque hanc differentiam

rentiam non ſatis attendiſſe videtur Doctiſſimus *Jurinius*, à vero multum aberraſſe mihi videtur. Si igitur ſepoſitâ ſuâ, de Vaſorum ictu, hypotheſi, & vi preſſuræ, quâ Natura utitur, pro Principio adhibitâ, alia Theoremata de Cordis & Sanguinis motu & viribus, elegante ſuâ demonſtrationis methodo, conſtruere dignabitur, ſeſe dignum, mihi certe gratum, nec eruditis inutile præſtiterit. Tu, qui Rei Medicæ principatum tenes, Vir Ampliſſime, diſſentientium diſputationes tuâ prudentiâ ita moderari digneris, ne Indoctis ludibrio, ſed ut Doctis emolumento eſſe poſſint. Dabam *Northamptoniæ* 23. die *Junii* 1719.

III. *An account of ſome Experiments relating to the Specifick Gravity of Human Blood.* By James Jurin, M. D. *and* F. R. S.

IT is well known from the Obſervations of Mr. *Leeuwenhoek* and others, that Human Blood conſiſts of red globular Particles, ſwimming in a pellucid *Lympha*, or *Serum*. Which two different Subſtances, tho' of unequal Specifick Gravities, yet ſo long as they continue to circulate in the Veins and Arteries, are prevented from ſeparating by their Motion and Warmth. But when the Blood comes to ſtagnate and cool in a Porringer, the globular Particles uniting together by their attractive Power, and ſinking by their Weight, which is greater than that of the *Serum*, form the *Coagulum*, or *Craſſamentum*, at the bottom of the Porringer, the *Serum* ſwimming above it.

Things always happen in this manner, when the *Craſſamentum* is at liberty to ſubſide: but it often falls out, that, either by its adheſion to the ſides of the Veſſel, or by the bubbles of Air, which the Blood gathers

gathers upon falling into the Porringer, and which ſtick to it's Surface, the *Craſſamentum* is kept from ſinking, and ſeems to float upon the top of the *Serum*.

Theſe Accidents ſeem to have given the firſt occaſion to that Opinion, which, I think, has been generally entertain'd by thoſe who have writ upon this Subject, namely, that the globular part of the Blood is ſpecifically lighter than the *Serum*, in which it ſwims.

But that which has ſo fully eſtabliſht this perſuaſion, is the Authority of the late excellent Mr. *Boyle*, who, among the many valuable and curious Experiments he has given us in his Natural Hiſtory of Human Blood, has left the following ones upon this Subject.

The ſpecifick Gravity of *Serum* of Human Blood was found by weighing a piece of Sealing Wax firſt in *Serum*, and afterwards in Water, to be to the ſpecifick Gravity of Water, as 1024 to 1000.

In a ſecond Experiment, which for greater accuracy was made with an Inſtrument contriv'd on purpoſe, the ſpecifick Gravity of *Serum* was found to be to that of Water, as 1194, to 1000.

In a third Experiment made by the ſame Inſtrument, and with *Serum* from the Blood of another Perſon, it's ſpecifick Gravity appear'd to be 1186.

The Medium between theſe two laſt Experiments is 1190, which has ſince been univerſally receiv'd for the ſpecifick Gravity of *Serum* of Human Blood, the firſt Experiment being declar'd by Mr. *Boyle* himſelf to be leſs exactly made than the other.

The ſpecifick Gravity of Human Blood was found by Mr. *Boyle*, to be to that of Water, as 1040 to 1000; though on account of difficulties by him mention'd, he was far from being ſatisfy'd with this Experiment, and recommended the thing to farther tryals.

Theſe

These Experiments however having hitherto paſt uncontroverted, and it appearing from them, that the ſpecifick Gravity of *Serum* was greater than that of Blood in the proportion of 1190 to 1040, or of 8 to 7 nearly; it was a neceſſary conſequence of this, that the Blood Globules were ſpecifically lighter than the *Serum*, and that in a very great degree, conſidering the ſmall proportion, that the bulk of the *Craſſamentum* was found to bear to that of the *Serum*, from other Experiments.

From this it was not improbably conjectured, that theſe Globules were thin Veſicles fill'd with an Aereal ſubſtance: and this Opinion ſeem'd to receive a great confirmation, upon it's being obſerv'd, in viewing the Circulation by a Microſcope, that a Blood Globule, in paſſing through a very narrow Veſſel, would change its ſhape from a Globular to an Oval Form, and would recover it's former Figure, as ſoon as it was got thro' the narrow Paſſage; which appearance ſeem'd to be naturally accounted for from the Elaſticity of the included *Aura*.

Upon this conjecture have been built a great many Solutions of the Phænomena obſervable in the Animal Oeconomy, and the diſorders of it; particularly a late ingenious account of Muſcular Motion. It it not my buſineſs at preſent to examine any of theſe, nor is it my deſign to caſt any reflection upon their Authors, who were led into this miſtake by the natural conſequence of a matter of Fact, for the truth of which they had ſo great an Authority, as that of the excellent Perſon above-mentioned. But I hope, I ſhall eaſily be pardon'd for enquiring into the ſoundneſs of the Foundation, when the Superſtructure erected thereupon is ſo conſiderable; and the following Experiments, however trivial in themſelves, will not appear unwo thy

the conſideration of the *Royal Society*, if it be found, that they may prevent us from running into Errors of the greateſt conſequence.

Exp. I. I have ſeveral times cut off a ſmall part of the *Craſſamentum*, when by its adheſion to the ſides of the Porringer it has ſeem'd to ſwim upon the Surface of the *Serum*, and have put it into another Veſſel fill'd with *Serum*: upon which it has immediately ſunk to the Bottom.

Exp. II. When the *Coagulum* has been buoyd up in the *Serum* by the bubbles of Air adhering to its Surface, I have ſeparated a ſmall part of it, where thoſe Bubbles have been thickeſt, and put it into a Glaſs of *Serum*, in which it has ſwom, as before. Then ſetting the Glaſs upon the Air-Pump, thoſe Bubbles burſt after one another, as the Receiver was exhauſting, and the Air being again let into the Receiver, the lump of *Craſſamentum* ſunk to the bottom of the Glaſs.

Exp. III. I have often placed a drop of *Serum* upon a clean Glaſs before a Microſcope, in which I had diſſolv'd a very ſmall quantity of Blood, and obſerv'd, that when the Glaſs was held in a perpendicular Poſture, the Blood-Globules ſubſided to the bottom of the Drop; and inverting the Glaſs, the Globules again deſcended thro' the *Serum* to the Bottom. I had the ſame ſucceſs with a ſmall quantity of *Serum* and Blood in a Capillary Tube. And the ſame thing has been long ſince obſerv'd by the famous Mr. *Leeuwenhoek*.

Theſe Experiments undeniably demonſtrate, that the *Craſſamentum*, or globular part of the Blood, is ſpecifically heavier than the *Serum*; and conſequently it is by no means probable, that the Blood Globules are Veſicles fill'd with Air, or any other Fluid lighter than *Serum*. And that they are not fill'd with any ſort of

Elaſtick Fluid, will appear from the following Experiment.

Exp. IV. In a ſmall quantity of *Serum* of Human Blood, I diſſolv'd ſo much Blood, as that the Globules might not lye too thick together, to hinder their being ſeen diſtinctly. Then having lodged a ſmall drop of this Liquor on the inſide of a thin Glaſs Tube, I fitted the Tube on to the Air-Pump, and placed a Microſcope by it, ſo that I could ſee the Blood-Globules through the Tube. This being done, I caus'd the Tube to be exhauſted, keeping my Eye upon the Globules all the time, in order to obſerve whether they dilated themſelves, as the Air was withdrawn; but could not perceive the leaſt alteration, they appearing exactly of the ſame bigneſs in the *Vacuum*, as they had done before. Whereas if they had been fill'd with an Elaſtick Fluid, they would either have burſt, or have been dilated to at leaſt 70 or 80 times their former Magnitude. The Stop-cock being afterwards turn'd, and the Air ſuffer'd to re-enter the Tube, the Blood-Globules ſtill retain'd the ſame bigneſs, as in *Vacuo*.

To this it will perhaps be objected, that a learned Member of this Society, in a Book lately publiſh'd, has aſſerted the direct contrary to what I here affirm, and has aſſur'd us, that the Blood-Globules in an exhauſted Receiver, inſtantly ſwell, and dilate themſelves ſo, as to become incredibly large. But as that Gentleman does not tell us, upon what Experiment this aſſertion is grounded, it may not be unreaſonable to ſuppoſe, that he was miſled by the common Hypotheſis, which he there maintains, of the Blood-Globules being fill'd with Air, and by what he has heard or ſeen of the bubbles of Air, which ariſe from Blood in the Air Pump in the ſame manner as from other Liquors, and which not eaſily breaking out from ſo viſcid a Fluid, occaſion

the

the appearance he mentions. However this may be; to prevent any diſpute, and avoid the coming to *Utri creditis, Quirites?* I ſhall offer a Teſtimony, that every body will be ſatisfy'd with, namely that of the learned and ingenious Mr. *Machin*, Profeſſor of *Aſtronomy* in *Greſham Colledge*, and one of our Secretaries, who having honour'd me with his Company at a repetition of this Experiment, in order to be witneſs to the Event, was fully ſatisfied upon repeated tryals, that there was no perceivable difference between the Magnitude of the Blood Globules in the Air, and in *Vacuo*. Upon this occaſion the two firſt Experiments were likewiſe repeated in his preſence, with the ſame Succeſs, as above related.

Though what has been already ſaid is a ſufficient proof of the Opinion above-mention'd, yet however to prevent the Objections, which may ariſe for want of Experiments made in the ſame manner with Mr. *Boyle*'s, as well as for the ſatisfaction of the Curious, who may be deſirous to know the true Specifick Gravities of *Serum* and Blood, I ſhall proceed to demonſtrate the ſame thing by Hydroſtatical Experiments.

Exp. V. *Novemb.* 13. 1713. Having ſuffer'd a quantity of my own Blood to ſtand about 24 Hours in the Porringer, and then drawing off the *Serum* carefully with a ſmall *Siphon* into a convenient Glaſs, I found by the Hydroſtatical Balance it's Specifick Gravity to be to that of Water, as 1029,8 to 1000.

Exp. VI. *Feb.* 21. 1716-7. I examin'd the *Serum* from the Blood of another Perſon in the ſame manner and found it's Specifick Gravity to be 1028, 6.

Exp. VII. VIII. and IX. *April* 8th, 1717. I obtain'd three ſeveral quantities of *Serum* from the Blood of different Perſons. The firſt of theſe was of a deep

Colour, inclining ſomething to red, and a little Turbid. It's Specifick Gravity was 1029, 7.

The ſecond was likewiſe a little Turbid, and of a pale whitiſh Colour. The Specifick Gravity of this was 1030, 2.

The third quantity of *Serum* was perfectly clear, and of the colour of Canary. It's Specifick Gravity was found to be 1030.

Though theſe five ſeveral Experiments were all carefully made, and with a Balance whoſe accuracy I was well aſſur'd of, yet for farther Evidence, I thought it proper to make that which follows, after another manner.

Exp. X. *Jan.* 15th. 1718-9. I drew off all the *Serum* from five or ſix ſeveral Porringers, containing the Blood of different Perſons. This I found to be a little tinged with Blood, which was occaſion'd by my being oblig'd to draw it off pretty near to the bottom of the Porringers, in order to obtain a quantity ſufficient for my purpoſe. For this reaſon I ſuffer'd it to ſtand about two Days, in which time the Globular part of the Blood was entirely precipitated to the Bottom, and the *Serum* was become perfectly fine and tranſparent. I then drew it off with a *Siphon* into a Glaſs Vial with a narrow Neck, which I fill'd to a certain mark made in the Neck for that purpoſe. This done, I plac'd my Vial in a nice pair of Scales, in which I had a counterpoiſe for the weight of the Vial, and found that quantity of *Serum* to weigh 2284$\frac{1}{4}$ Grains.

Then pouring out the *Serum*, I fill'd the Vial with common Water to the ſame mark, and found the weight of the Water to be 2219 Grains.

From which it follows, that the Specifick Gravity of this *Serum* was 1029, 4.

Exp.

Exp. XI. *July* 14. 1719. I procur'd a quantity of Blood taken from the temporal Artery, from which I drew off the *Serum* the next Day, and weighing it in the ſame manner found it's Specifick Gravity to be 1028, 8.

Theſe Experiments agree ſo nearly together, that the little difference between them may very well be attributed to that which is between the *Serum* of different Perſons; or to the variations occaſion'd by heat and cold in the ſeveral Seaſons of the Year, in which they were made. So that from them we may ſafely determine the Specifick Gravity of *Serum* of Human Blood at a Medium to be 1029, 5, or in a round number 1030. From which the greateſt Variation in any of theſe Experiments is little more than one in 1000; whereas the difference between Mr. *Boyle*'s Experiments and mine amounts to 160 in 1000.

Exp. XII. *April* 6. 1717. In order to find the Specifick Gravity of Human Blood, which, by reaſon of it's tenacity, and ſudden alterations upon ſtanding, cannot be determin'd by the Hydroſtatical Balance; I took a narrow-neck'd Vial, and fill'd it to a Mark, with Blood pour'd immediately out of the Porringer, as ſoon as the Perſon was blooded. This I weigh'd, as I had done the *Serum* before, and found it's Specifick Gravity to be 1051.

Exp. XIII. *Aug.* 5th. 1717. Having fill'd the ſame Vial with the Blood of another Perſon, running immediately out of the Vein through a Funnel, it's Specifick Gravity was determin'd at 1053.

Suffering this to ſtand till it was cold, I found the Blood was ſunk a ſmall matter below the Mark in the neck of the Vial. This being fill'd up with the Water, which in ſo ſmall a quantity could make no ſenſible

ble difference from Blood, I found the Specifick Gravity of cold Blood to be 1055.

Exp. XIV. *Aug. 6th.* 1718. The laſt Experiment being repeated in the ſame manner as the Year before, the Specifick Gravity of cold Blood was again found to 1055.

Exp. XV. *July* 14th. 1719. The Arterial Blood, from which the *Serum* was afterwards drawn off for the 11th Experiment, being weigh'd in the ſame manner, it's Specifick Gravity was 1052, 5.

As this Arterial Blood and it's *Serum*, differ no more in Specifick Gravity from Venal Blood and *Serum*, than the ſeveral Portions of theſe do from one another, it's plain, that the difference in this reſpect between Arterial and Venal Blood is wholly inconſiderable. The Animal Oeconomy indeed teaches us, that the *Serous* Liquor is perpetually drawing off from the Arterial Blood by the ſeveral Secretions, but as the quantity ſeparated in one Circulation is very ſmall, the Blood muſt arrive in the Veins nearly of the ſame denſity, as when it runs through the Arteries.

In the 13th Experiment we obſerv'd, that the Blood alter'd it's Specifick Gravity upon cooling from 1053 to 1055; from which we may infer, that if the Blood made uſe of in the 12th Experiment had been ſuffered to ſtand till it was cold, it's Specifick Gravity would have been 1053; wherefore, taking a Medium between the four laſt Experiments, we may allow the Specifick Gravity of cold Human Blood to be 1054.

The difference of 14 Parts in a 1000, between this and the Specifick Gravity determined by Mr. *Boyle*, is eaſily accounted for, if we conſider, that that Gentleman did not make uſe of a Veſſel with a narrow Neck, as plainly appears from the circumſtances men-

tioned

tioned in his Experiment; and consequently a small error in the height of the Liquor would make a considerable alteration in the Specifick Gravity.

Since therefore the Specifick Gravity of Human Blood is 1054, and that of its *Serum* 1030, it is plain, that Blood is heavier than *Serum* by about one part in 43. From which it manifestly follows, that the Globular part of the Blood is specifically heavier than the *Serum*, since the Globular part being separated from the Blood leaves the remainder, or the *Serum*, specifically lighter than the intire Mass.

But in order to determine the exact Specifick Gravity of the Blood Globules, it is first necessary to know the Proportion, which the whole quantity of the *Crassamentum* contained in Blood bears to the *Serum*. To this end Mr. *Boyle* has given us two several Observations of the weights of the *Crassamentum* and *Serum*, after they have separated one from another in the Poringer. But besides the difficulty of making this Experiment with any tolerable exactness, it is to be consider'd, that there is a great deal of *Serum* contain'd in the interstices of the Globules, that compose the *Crassamentum*.

This difficulty however is in some measure answer'd by two other Experiments, which Mr. *Boyle* made for this purpose, after the following manner. He put a quantity of the *Crassamentum*, already separated from the *Serum*, into an Alembick, and distill'd off the remaining *Serum* to dryness, but without drawing off the Oil, or Volatile Salt; after which he weigh'd the distill'd Liquor, and the dry Mass left behind.

By comparing these Experiments with the two former, it will be found that the entire weight of *Serum* contain'd in Blood is nearly $\frac{13}{15}$ of the whole, and consequently

conſequently the weight of the dry'd *Craſſamentum* is only two fifteenths of the Blood.

But for farther ſatisfaction, an Analyſis was made at my deſire with a large quantity of Blood, amounting to four Pounds fourteen Ounces, by that ingenious and skilfull Chymiſt, Mr. *John Brown*.

From this was obtain'd, with a very gentle heat, two Pounds, fourteen Ounces, and ſix Drachms of a Phlegmatick Liquor, that had ſcarce any thing of the fœtid Scent, which is uſual in the diſtillation of Animal Subſtances; and its Specifick Gravity was nearly the ſame with that of common Water, being but 1000, 8. This being mixt with a ſtrong ſolution of Alum, ſcarce afforded any *Coagulum*; but exhibited a conſiderable one upon mixture with a ſolution of *Roman* Vitriol.

The diſtillation being continued with the ſame Heat, we had ſeven Ounces more of Phlegm conſiderably impregnated with Volatile Salt, as was manifeſt from the Smell. The Specifick Gravity of this was 1007, and having mix'd it with *Tinctura Martis optima*, Solution of Alum, and of *Roman* Vitriol, a large *Coagulum* was precipitated. In diſtilling theſe there was loſt by Evaporation, two Ounces and two Drachms.

The third portion of Liquor, being rais'd with a ſtronger Fire, amounted to ſeven Ounces ſix Drachms. This was reddiſh, and turbid, and ſo ſtrongly charg'd with Volatile Salts, that it might very well deſerve the name of Spirit. Its Specifick Gravity was 1080, 1.

Beſides theſe we had ſeven Drachms of Volatile Salt, an ounce of Oil, and eight Ounces four Drachms of *Caput Mortuum*, which ſtill retain'd ſome ſmall remainder of the Oil, as was manifeſt from its taking Fire at the flame of a Candle. In this latter part of the Operation was loſt three Ounces, ſeven Drachms.

Upon making due allowance for the difference between the Specifick Gravities of the three firſt Portions of Liquor and that of *Serum*, as likewiſe for what was loſt in the two ſeveral parts of the Operation, which we may reaſonably conclude to have been of a Specifick Gravity nearly the ſame with that of the Liquor drawn off, it will be found, that the quantity of *Serum* contain'd in this Maſs of Blood was about $\frac{15}{17}$ of the whole Weight, and conſequently that the quantity of *Craſſamentum* was $\frac{2}{17}$ of the ſame Weight.

If we calculate therefore upon this Suppoſition, that the weight of the Globular part of the Blood is $\frac{2}{17}$ of the whole, we ſhall find the Specifick Gravity of a Blood Globule to be to that of Water as 1277 to 1000.

If we follow the proportion of $\frac{2}{15}$, which reſults from Mr. *Boyle*'s Experiments, the Specifick Gravity of a Blood Globule will be 1242.

But this computation is in all appearance a great deal too large; for we cannot be aſſur'd, that our whole quantity of aqueous Liquor was rais'd from the *Serum* of the Blood. On the contrary it is more than probable, that a conſiderable part of it was afforded by the Blood Globules themſelves, eſpecially in the latter part of the Operation, when their texture muſt of neceſſity have been broken and diſſolv'd by the ſtrong Fire that was made uſe of. To prove this, we need only conſider the condition of the dry'd *Craſſamentum*, after the Phlegm is drawn off, that being now a hard and brittle ſubſtance: whereas the Globules in their natural State are ſoft and yielding. For which reaſons it may perhaps be more ſatisfactory, if we attempt to find the quantity of the Globular part of the Blood after another manner.

It appears therefore from Mr *Boyle*'s Obſervations, that the quantity of *Serum*, which may be pour'd off

from the *Craſſamentum*, is about one half of the whole Maſs. The remaining *Craſſamentum* conſiſts of the Blood Globules, and a quantity of *Serum* filling up the Interſtices between them; which, if the Globules keep their Spherical Form, may eaſily be found by the principles of common Geometry, to be nearly one half of the bulk of the *Craſſamentum*: but if the Globules by their preſſure againſt one another change their Figure, the quantity of *Serum* will be ſomething leſs.

If this quantity of *Serum* lying between the Blood Globules be added to that pour'd off, it appears, that the *Serum* contain'd in Blood is about $\frac{3}{4}$ of the whole bulk, and conſequently that the Blood Globules make about $\frac{1}{4}$ of the whole. From which we ſhall find the Specifick Gravity of the Blood Globules to be to that of Water as 1126 to 1000.

If we ſuppoſe the Blood Globules to make $\frac{1}{6}$, $\frac{1}{5}$, $\frac{1}{3}$, or $\frac{1}{2}$ of the whole bulk, their Specifick Gravity will be reſpectively 1174, 1150, 1102, or 1078. So that upon any of theſe Suppoſitions, the Specifick Gravity of the Blood Globules will be conſiderably greater than that of the *Serum*, and conſequently they cannot be ſuppos'd to be Veſicles fill'd with an Aereal Subſtance.

It will therefore perhaps be askt, What do they really conſiſt of?

In order to come to a Solution of this Queſtion, it may be proper to take notice,

That Blood is compos'd of Phlegm, Oil, Volatile and fixt Salts, and Earth. For as to the Spirit, we look upon it with Mr. *Boyle*, to conſiſt of the Phlegm and Volatile Salt united together.

That the *Serum*, upon a Chymical Analyſis, exhibits a great deal of the firſt of theſe, and the others in a very ſmall quantity.

That

That on the contrary the *Craſſamentum* yields much leſs Phlegm, but the other Principles much more copiouſly than the *Serum*.

From which *Data*, I think, we may ſafely conclude, that the *Craſſamentum*, or Globular part of the Blood, conſiſts of ſome Phlegm united with the Oil and Salts, and a ſmall quantity of Earth.

But what is the exact proportion of theſe ſeveral Principles to one another; what alterations are produced in the Body by a change of this proportion; how, and in what part theſe Globules are form'd; by what means they preſerve their Figure, without diſſolving in the *Serum*, or uniting with one another; what variations are made in their Specifick Gravities by Heat and Cold; and what are the effects of thoſe Variations, are Queſtions not very eaſy to be ſolv'd, and yet of ſo much importance to the Animal Oeconomy, that it were greatly to be wiſht, we had a number of *Data* ſufficient to determine them.

P. S. Since this Paper was ſent to the Preſs, I made the following Experiments, which ſerving to confirm the Method laſt made uſe of, for finding the Specifick Gravity of the Blood Globules, it may not be improper to relate them.

Auguſt 6 1719. I took a lump of the *Craſſamentum* and waſh'd it gently in fair Water, to free it from the looſe Globules, which precipitating out of the *Serum*, after the *Coagulum* is form'd, do not unite into one Body with it. This done, I laid it on a ſpungy brown Paper, in order to drain off the ſuperfluous Moiſture. After which, weighing it firſt in Air, and then in Water, I found its Specifick Gravity to be $1085\frac{1}{2}$.

Another lump of the ſame *Craſſamentum* being weigh'd in the ſame manner, its Specifick Gravity was 1082.9.

Sept. 18. 1719. I found the Specifick Gravity of another piece of *Craſſamentum* to be 1082.1.

A ſecond piece from the Blood of a different Perſon gave me 1086,1.

A third from the ſame Perſon gave 1086,6.

From this it follows that the Specifick Gravity of the Blood Globules is at leaſt 1084, which is the *Medium* between theſe five Experiments.

But if we allow one half of the bulk of the *Craſſamentum* to conſiſt of *Serum*, filling up the Spaces between the Blood Globules, we ſhall find their Specifick Gravity to be 1138.

From this we muſt make a ſmall abatement, becauſe ſome part of the *Serum* muſt have been ſquees'd out from between the Globules, by their yielding to one anothers Preſſure, when the lump of *Craſſamentum* lay upon the Paper: and this will reduce their Specifick Gravity ſufficiently near to 1126, as we had before determin'd it.

IV. *An Acconnt of the* Sunk *Iſland in* Humber, *ſome Years ſince recover'd from the Sea. Being an Extract of a Letter Communicated to the* Royal Society *by* John Chamberlayne, *Eſq*; R. S. S.

THis Iſland goes by the name of the *Sunk Iſland*, ſo called I ſuppoſe from the ſinking Marſh Ground about it. As for its Original one may make pretty ſure Conjectures of that I believe, becauſe 'tis yet within

in the memory of Man, ſince it began to raiſe its Head above the Ocean, there being ſeveral old People here alive, who can remember when there appeared nothing of it but a waſt and barren Sand; and that only at Low-Water too, when for the ſpace of a few Hours it ſhewed its Head, and then was buried again till the next Tides Retreat: thus ſucceſſively it liv'd and died until the Year 1666, when it began to maintain its ground againſt the inſult of the Waves; about which time it began to be reſcued wholly from future danger, by the Care and Induſtry of Colonel *Gilby*, who having, as I am imform'd, a Leaſe or Gift of it from the Crown, did raiſe Banks about the riſing Grounds of it, and ſo defending it from the Encroachments of the Water, it became Firm and Solid; and in a ſhort time afforded good Paſturage for Sheep and other Cattel. The Expences at firſt to improve it to what it is, muſt needs have been very conſiderable; it being encompaſs'd with high Banks, and deep Canals for receiving and diſcharging the Liquid Element, which every now and then notwithſtanding threatens to re-poſſeſs it ſelf of its ancient Hereditament, but hitherto in vain; for I now acquaint you of its preſent Safety.

This Iſland is now about 9 Miles in Circumference, within the Banks, which ſeem to render it impregnable againſt all future attacks of the Sea, and is of a very fat and fertile Soil, affords good Graſs, Corn and Hay, and is repleniſhed with numerous flocks of Sheep, which are of a larger Size and finer Wool than thoſe in *Holderneſs*, from which it is divided by about two Miles in Water; and from *Lincolnſhire* by about four. It is ſtor'd with vaſt numbers of Rabits, that ſeem innumerable, they appearing through all Parts in prodigious Swarms; their Skins are counted the fineſt in *England*, of a dark Mouſe Colour, Shagg'd, and ſoft as Silk.

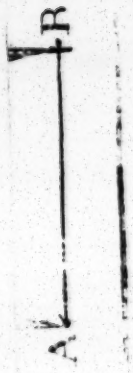
There

There are alſo Cows and Horſes feeding conſtantly in the Place, with great plenty of Wild Foul.

The Inhabitants are not ſo numerous, there being only three Families that live conſtantly upon the Place; however they are never too ſolitary, there being abundance of Workmen and Labourers that continually reſort thither, ſometimes I am told to the number of a Hundred and upwards, for the repairing of the Banks, *&c.*

The Yearly Income of the Proprietor Mr. *Gilby*, amounts to about 800*l.* and pays the King's Taxes to thoſe who Collect for the *Eaſt-Riding*, and is uſually upliſted by thoſe of the Liberty and Townſhip of *Ottringham*, from the Marſhes of which there is a Paſſage over the Sands to the *Sunk* at Low-water. But this Cuſtom of paying the King's Ceſs to them, proceeds only from the conveniency, not Neceſſity; for it never belong'd to that or any other Pariſh, ſo that I cannot reſolve you in what Dioceſe this Iſland lyes, unleſs it had been united to ſome neighbouring Pariſh, or converted to one of it ſelf; which if effected, the Tyth of Lambs, Wool and Rabits, *&c.* would make up a handſome Benefice. It lyes nearer indeed to the Dioceſe of *York*, by at leaſt two Miles, than to that of *Lincoln*, being two Miles South of *Holderneſs*, in the River *Humber*, and four Miles North of *Lincolnſhire*, *&c.*

Welwick, April 14.
1717.



at *a*, where they project the Image of the Point
from whence diverging, they fall on the Eye-
ſs *ll*, and having paſs'd through it, go on parallel
ach other, and enter the *Cornea* of a common Eye E,
ich unites thoſe parallel Rays upon its *Retina* R R R
a, where the Image of *a* is projected: The ſame
ay be ſaid of the Rays that come from B, and after
their

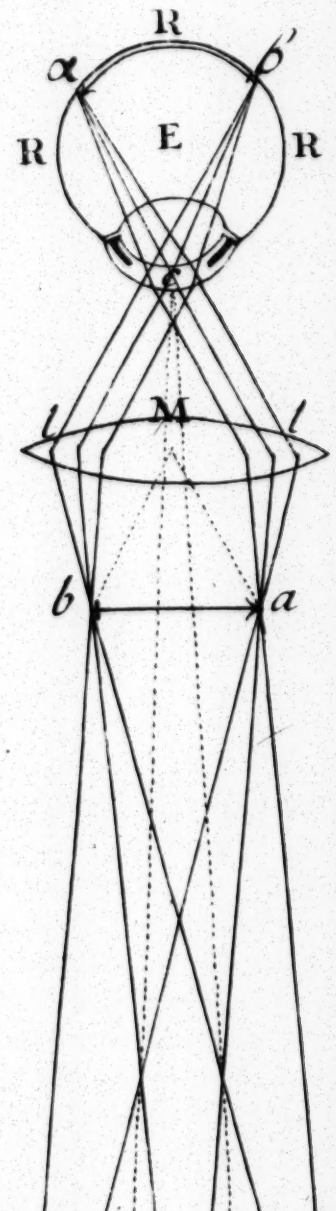


Fig. I.

Fig: II.

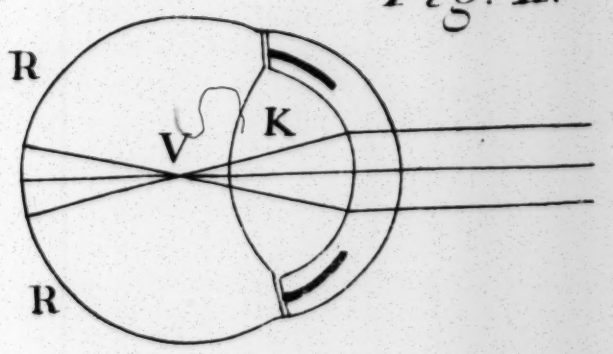


Fig: III.

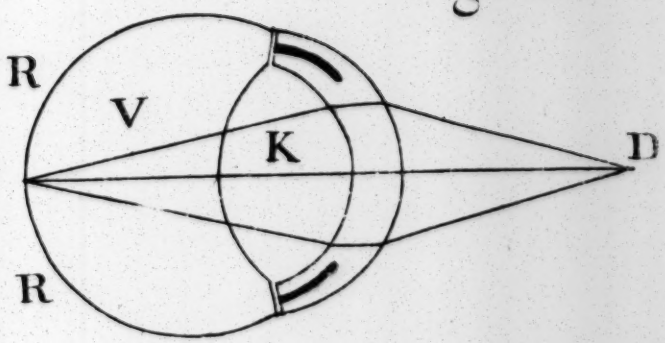


Fig: I.

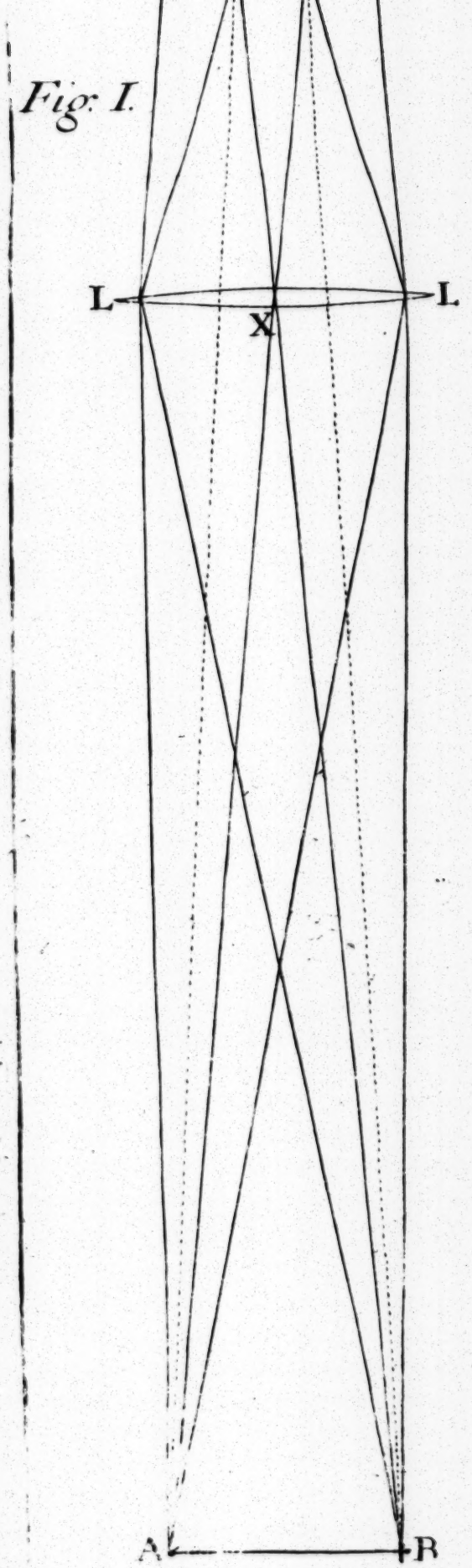


Fig: IV.

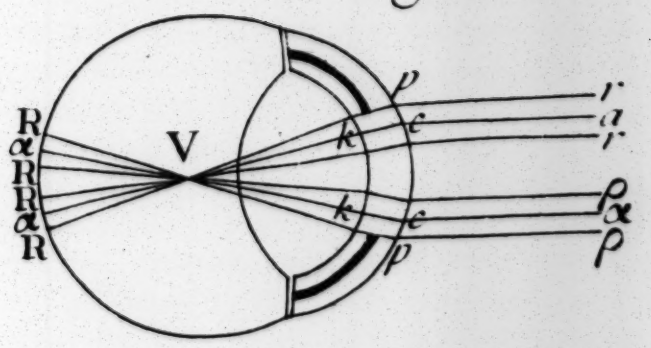
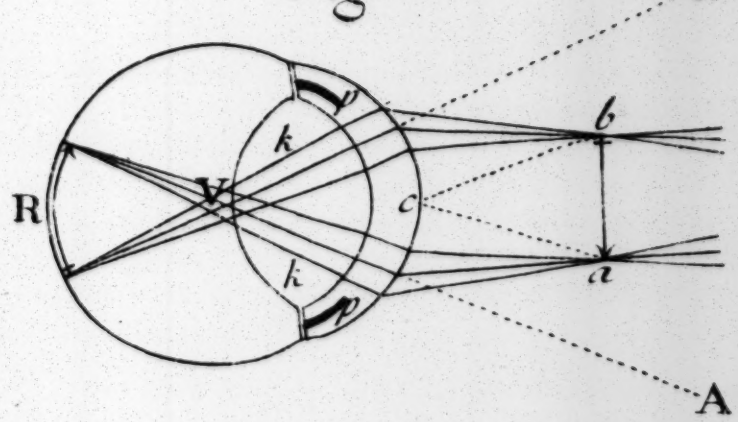


Fig: V.



V. A

V. *A Way for* Myopes *to uſe* Teleſcopes *without Eye-Glaſſes, an Object-Glaſs alone becoming as uſeful to them, and ſometimes more than a Combination of Glaſſes. Communicated to the* Royal-Society, *by the Reverend* J. T. Deſaguliers, LL. D. *and* F. R. S.

Lemma I.

WHat is requir'd of a Teleſcope is to give large, and diſtinct Viſion; that is, to make the Object (as in *Galileo's* Teleſcope) or its Image (as in the Teleſcopes made up of Convex *Lentes)* appear under a great Angle, and to have all the Rays of thoſe Pencils that enter the Eye, meet in a point upon the *Retina* of the Eye, on their reſpective Axes.

The firſt Figure repreſents the Combination of two Convex *Lentes* for the Aſtronomical or inverting Teleſcope; where the above-mentioned Requiſites are obtain'd. A B is the Object ſuppos'd at a vaſt diſtance from the Ojective *Lens* L L, ſo that Rays coming from the extremity A of the Object, will fall upon the *Lens* L L, in the ſame manner as if they were parallel to their Axis A X; and after paſſing the Glaſs unite at *a*, where they project the Image of the Point A; from whence diverging, they fall on the Eye-Glaſs *l l*, and having paſs'd through it, go on parallel to each other, and enter the *Cornea* of a common Eye E, which unites thoſe parallel Rays upon its *Retina* R R R at α, where the Image of *a* is projected: The ſame may be ſaid of the Rays that come from B, and after

their

their ſeveral refractions through the two Glaſſes and the Coats and Humours of the Eye, meet upon the *Retina* at β, where they project the diſtinct Image of the Point *b*. The Rays that come from all the Points of the Object A B being affected after the ſame manner, give a diſtinct Image of thoſe Points upon the *Retina*, and therefore the Object does appear diſtinct.

The Object will alſo appear magnified in the ſame proportion as the Angle *l* C *l* = to *b* M *a* (under which its Image is ſeen,) is greater than the Angle A C B under which the Object A B would be ſeen by the naked Eye; as is more at large demonſtrated by Dioptrical Writers.

Lemma 2.

If parallel Rays fall upon the *Cornea* of a *Myops*, or ſhort-ſighted Perſon, they will unite in the Eye, before they come to the *Retina*, the farther from it the more Convex the Eye is; but if the Rays which fall upon the *Cornea* diverge in proportion to the too great Convexity of the Eye, as from D, ſuch Rays will be ſo refracted by the Coats and Humours of the Eye as to meet in one point upon the *Retina* R R, ſee *Fig.* 2 and 3. Where I have in the Scheme neglected the Refraction of the Rays paſſing out of the *Cryſtalline* K into the *Vitreous* Humour V, as I do in the other Caſes.

This *Lemma* is alſo demonſtrated by Dioptrical Writers.

Lemma 3.

If two Pencils of Rays (in each whereof all the Rays are parallel to the Axis, as *a* C) fall upon different Parts of the *Cornea*, at the greateſt diſtance from one another that can be allow'd for thoſe Rays to enter the Pupil P P, their *Axes* will, after entring the

Aqueous

Aqueous Humour, converge, and meet either in the Vitreous, or Cryſtalline Humour, according to the Convexity of the *Cornea* thro' which they paſs'd, and diverge again before they come to the *Retina*; the Rays of each Pencil converging upon their reſpective Axes, to the place where the ſaid Axes croſs one another, *Fig.* 4.

Demonſtration.

The Axes *a* C *a*, *α* C *α*, falling obliquely upon the *Cornea* at C, C, and entring from *Air* into the *Aqueous* Humour, will be refracted towards the Perpendicular to K: where ſtriking more directly upon the *Cryſtalline*, they will go on to *α*, *a*, upon the *Retina* R R R R, decuſſating at V within the *Vitreous* Humour. The other Rays *r*, *r*; *ρ*, *ρ*, after their Refraction in the *Aqueous* Humour, fall more obliquely on the *Cryſtalline*, and therefore are refracted again ſo as to meet at V, where the Axes alſo meet, and thence go on to the *Retina* R R R R, *Fig.* 4.

Lemma 4.

But if the Axes of the above-mention'd Pencils are Parallel, the Rays that accompany them diverging from a Point ſo near the Eye, that the divergence may be proportionable to the too great Convexity of the Eye; then only the Axes will meet in the Eye before they come to the *Retina* (by *Lemma* 3.) but the other Rays will not unite upon their reſpective Axes, till they come to the *Retina*, (by *Lemma* 2.)

Propoſition.

I ſuppoſe the Eye of the *Myops* ſo Convex that he can ſee no farther than a common Eye, with the Eye-Glaſs of a Teleſcope before it: then the Eye of the

Myops being in the place of the Eye-Glaſs, will receive the Rays diverging from the ſeveral points of the Image (projected by the Object-Glaſs in its *Focus*,) in ſuch manner, that they will after their ſeveral refractions meet in reſpective Points on the *Retina*; and the Axes of the Pencils which come from the extremities of the Object, will, in the Eye, make the Angle B V A = to *b c a*, under which the Image *a b* is ſeen, by *Lemma* 4. The *Cornea* and *Aqueous* Humour here ſupply the place of the Eye-Glaſs, and the *Cryſtalline* and *Vitreous* Humours that of a common Eye, See the 5th *Fig.* wherein R is the *Retina*, V the *Vitreous* Humour, and K K the *Cryſtalline* Humour; and the Image *b a* is ſuppos'd to be brought down from the *firſt Fig.* which repreſents the *Aſtronomick Teleſcope:* the too great Convexity of the Eye here being in the place of an Eye-Glaſs.

An Objection may be made to this, *viz.* that P P the Pupil of the Eye being ſmall, will take in but a very little Image, or a ſmall part of the Object: But then if the Eye be mov'd ſucceſſively, to all the parts of the Space where the Eye-Glaſs was, it can take any part of the Object; and if the Object-Glaſs be large, which may more eaſily be made than a large Eye-Glaſs, and the Tube a Foot wide or wider, as much may ſucceſſively be taken in, as if an Eye-Glaſs might be had of a Foot Diameter. A little practice may make any *Myops* ſo ready, as to keep an Object when once found, though the place where he ſtands be ſhaken. It would not be amiſs to hold a *Lens* in one's Hand (for an Eye-Glaſs) to find the Object at firſt, till cuſtom has made it eaſy without it: when once the Object is found, it may be eaſily kept.

An Eye more ſhort-ſighted than I have ſuppos'd, will perform the Office of a more Convex Eye Glaſs, being

being brought nearer to the distinct Base of the Object Glass; and an Eye less Convex, the office of a less Convex Eye-Glass: but with this difference, that the more Convex the Eye is, the easier may any part of the Object be found, and the larger and more lucid it will appear.

I have seen *Saturn*'s Ring very plain with an Object-Glass of little more than six Foot *Radius*, without an Eye-Glass.

I have also found out a way for the *Presbytæ* to make use of an Object-Glass, by placing their Eye nearer the *Lens* than its *Focus*, by so much as their Eye is flatter than a common Eye, so as to make (as it were) the Telescope of *Galilæo*; the flat Eye serving as a common Eye arm'd with a Concave *Lens*. I have so fixed the Telescope, as to make a *Presbyta* read at a great distance a small Print. The truth of this may be easily demonstrated, if it be requir'd.

If this Experiment be made at Sea with a very large Tube, big enough to put in the Head and move it about, and the Object-Glass be also large, it may not perhaps be difficult to observe the Eclipses of the *Satellites* of *Jupiter*, which I would recommend to the Consideration of those that would try for the Longitude by such like Observations.

VI. *New and accurate Tables for the ready Computing of the* Eclipses *of the* first Satellite *of* Jupiter, *by Addition only. By the Reverend Mr* James Pound, R. S. S.

IN *Numb.* 214. of these *Transactions*, for the Months of *Novem* and *Decem.* 1694. we exhibited an Epitomy of Mr. *Cassini*'s curious Tables then newly published,

liſhed. for computing the Eclipſes of the *firſt Satellite* of *Jupiter*, without the help of any other Numbers. The eaſe of this *Calculus* gave great ſatisfaction to thoſe that delight in Teleſcope obſervations; and has been of good uſe to encourage Aſtronomers to aſcertain the Geographical Longitudes of many places, by help of theſe Eclipſes; whoſe frequency ſeems to afford us the propereſt means for that purpoſe.

But it being now 26 Years ſince thoſe Tables were publiſhed, length of Time has diſcovered that this *Satellites* motion is a ſmall matter ſwifter than M. *Caſſini* had ſuppoſed it; and the Reverend Mr. *Pound* being provided with all the Qualifications requiſite for ſuch a Work, has of late apply'd himſelf to rectify by frequent Obſervation what he found amiſs in the aforeſaid *Calculus*; and withal has put it into another Form yet much more eaſy and compendious, by bringing what M. *Caſſini* had given us in odd Numbers, to the Milleſimals of a Circle, both as to *Numb.* I. which he calls *Numb.* A. being the mean Anomalie of *Jupiter* in ſuch parts; as alſo to *Numb.* II. or our *Numb.* B. which is the diſtance of the mean place of *Jupiter*, from the true place of the *Sun*, and which with the addition of the Equation of *Numb.* B. gives the true angle of *Commutation* in the ſame Milleſimals of a Circle. And having deducted from the *Epoches* the greateſt Equations both of *Numb.* A and B. he reſtores them by adding as much to the Equations themſelves, by which means they all become Affirmative, ſo that the whole computation is performed by Addition only.

The Reader is ſuppoſed to be acquainted with the Method of M. *Caſſini*'s *Calculus*, which is at large explain'd in the aforeſaid *Tranſaction, Num.* 214. For which reaſon this ſhorter Deſcription may ſuffice at preſent.

EPOCHÆ

Epochæ Conjunctionum Primi Satellitis Cum Jove.

An. Jul. Curr.	Conjunct. D.	H.	′	″	Num. A.	Num. B.
1719	1	6	11	13	872	396
1720	0	20	22	40	956	310
21	1	5	2	44	40	229
22	0	19	14	11	125	143
23	0	9	25	38	209	57
1724	1	18	5	42	293	971
1725	0	8	17	10	377	889
26	1	16	57	13	462	808
27	1	7	8	41	546	722
28	0	21	20	8	630	636
1729	1	6	0	12	715	554
1730	0	20	11	39	799	468
31	0	10	23	7	883	382
32	0	0	34	34	967	296
33	0	9	14	38	52	215
1734	1	17	54	41	136	133
1735	1	8	6	9	220	47
36	0	22	17	36	305	961
37	1	6	57	40	389	880
38	0	21	9	7	473	794
1739	0	11	20	35	557	708
1740	0	1	32	2	642	622
41	0	10	12	6	726	540
42	0	0	23	33	810	454
43	1	9	3	37	895	373
1744	0	23	15	4	979	287
1745	1	7	55	8	63	205
46	0	22	6	35	148	119
47	0	12	18	3	232	33
48	0	2	29	30	316	947
1749	0	11	9	34	400	866

An. Jul. Curr.	Conjunct. D	H.	′	″	Num. A.	Num. B
1749	0	11	9	34	400	866
1750	0	1	21	1	485	780
51	1	10	1	5	569	698
52	1	0	12	33	653	612
53	1	8	52	37	738	531
1754	0	23	4	4	822	445
1755	0	13	15	32	906	359
56	0	3	27	0	990	273
57	0	12	7	3	75	191
58	0	2	18	30	159	110
1759	1	10	58	34	243	24
1760	1	1	10	1	328	938
61	1	9	50	35	412	856
62	1	0	1	2	496	770
63	0	14	13	0	580	684
1764	0	4	24	27	665	598
1765	0	13	4	31	749	517
66	0	3	15	58	833	431
67	1	11	56	2	918	349
68	1	2	7	29	2	263
1769	1	10	47	33	86	182
1770	1	0	59	0	171	96
71	0	15	10	28	255	10
72	0	5	21	56	339	924
73	0	14	2	0	423	842
1774	0	4	13	27	508	761
1775	1	12	53	31	592	675
76	0	3	4	58	676	589
77	1	11	45	1	761	507
78	1	1	56	28	845	421
1779	0	16	7	56	929	335

Revolutiones Primi Satellitis Jovis in menſibus.

Januarii				N. A.	Nu. B.
D.	h.	′	″		
1	18	28	36	0	5
3	12	57	12	1	9
5	7	25	48	1	14
7	1	54	24	2	18
8	20	23	0	2	23
10	14	51	36	2	27
12	9	20	12	3	32
14	3	48	48	3	37
15	22	17	24	4	41
17	16	46	0	4	46
19	11	14	36	4	51
21	5	43	12	5	55
23	0	11	47	5	60
24	18	40	23	6	64
26	13	8	59	6	69
28	7	37	35	7	73
30	2	6	11	7	78
31	20	34	47	7	82
Februarii					
0	20	34	47	7	82
2	15	3	23	8	87
4	9	31	59	8	92
6	4	0	35	9	96
7	22	29	11	9	101
9	16	57	47	9	105
11	11	26	23	10	110
13	5	54	59	10	114

Februarii.				N. A.	Nu. B.
D.	h.	′	″		
15	0	23	35	11	118
16	18	52	11	11	123
18	13	20	47	11	128
20	7	49	23	12	132
22	2	17	59	12	137
23	20	46	35	13	141
25	15	15	11	13	146
27	9	43	47	13	150
Martii.					
1	4	12	23	14	155
2	22	40	59	14	159
4	17	9	35	15	164
6	11	38	10	15	168
8	6	6	46	16	173
10	0	35	22	16	177
11	19	3	58	16	182
13	13	32	34	17	186
15	8	1	10	17	190
17	2	29	46	18	195
18	20	58	22	18	199
20	15	26	58	18	204
22	9	55	34	19	208
24	4	24	10	19	213
25	22	52	46	20	217
27	17	21	22	20	221
29	11	49	58	20	225
31	6	18	34	21	230

Revolutiones Primi Satellitis Jovis in mensibus.

Aprilis. D.	h.	′	″	N. A.	Nu. B.
0	6	8	34	21	230
2	0	47	10	21	235
3	19	15	46	22	239
5	13	44	22	22	244
7	8	12	58	22	248
9	2	41	34	23	252
10	21	10	10	23	257
12	15	38	46	24	261
14	10	7	22	24	265
16	4	35	58	25	270
17	23	4	33	25	274
19	17	33		25	279
21	12	1	45	26	283
23	6	30	21	26	287
25	0	58	57	27	292
26	19	27	33	27	296
28	13	56	9	27	300
30	8	24	45	28	304
Maii.					
0	8	24	45	28	304
2	2	53	21	28	309
3	21	21	57	29	313
5	15	50	33	29	317
7	10	19	9	29	322
9	4	47	45	30	326
10	23	16	21	30	330
12	17	44	57	31	335
14	12	13	33	31	339

Maii. D.	h.	′	″	N. A.	Nu. B.
16	6	42	9	31	343
18	1	10	45	32	348
19	19	39	21	32	352
21	14	7	57	33	356
23	8	36	33	33	361
25	3	5	9	33	365
26	21	33	45	34	369
28	16	2	21	34	373
30	10	30	57	35	378
Junii					
0	10	30	57	35	378
1	4	59	32	35	382
2	23	28	8	36	386
4	17	56	44	36	391
6	12	25	20	36	395
8	6	53	50	37	399
10	1	22	32	37	403
11	19	51	8	38	408
13	14	19	44	38	412
15	8	48	20	38	416
17	3	16	56	39	420
18	21	45	32	39	425
20	16	14	8	40	429
22	10	42	44	40	433
24	5	11	20	40	438
25	23	39	56	41	442
27	18	8	32	41	446
29	12	37	8	42	450

Revolutiones Primi Satellitis Jovis in menſibus.

Julii D.	h.	′	″	N. A.	Nu. B.
1	7	5	44	42	455
3	1	34	20	42	459
4	20	2	56	43	463
6	14	31	32	43	468
8	9	0	8	44	472
10	3	28	44	44	476
11	21	57	20	45	480
13	16	25	55	45	485
15	10	54	31	45	489
17	5	23	7	46	493
18	23	51	43	46	498
20	18	20	19	47	502
22	12	48	55	47	506
24	7	17	31	47	510
26	1	46	7	48	515
27	20	14	43	48	519
29	14	43	19	49	523
31	9	11	55	49	528
Auguſti.					
0	9	11	55	49	528
2	3	40	31	49	532
3	22	9	7	50	536
5	16	37	43	50	541
7	11	6	19	51	545
9	5	34	55	51	549
11	0	3	31	51	554
12	18	32	7	52	558
14	13	0	43	52	562

Auguſti. D.	h.	′	″	N. A.	Nu. B.
16	7	29	19	53	567
18	1	57	55	53	571
19	20	26	31	54	575
21	14	55	7	54	580
23	9	23	43	54	584
25	3	52	18	55	588
26	22	20	54	55	593
28	16	49	30	56	597
30	11	18	6	56	602
Septembris.					
1	5	46	42	56	606
3	0	15	18	57	610
4	18	43	54	57	615
6	13	12	30	58	619
8	7	41	6	58	624
10	2	9	42	58	628
11	20	38	18	59	632
13	15	6	54	59	637
15	9	35	30	60	641
17	4	4	6	60	646
18	22	32	42	60	650
20	17	1	18	61	655
22	11	29	54	61	659
24	5	58	30	62	663
26	0	27	6	62	668
27	18	55	42	62	672
29	13	24	18	63	677

Revolutiones Primi Satelliitis Jovis in menſibus.

Octobris. D.	h.	′	″	N. A.	Nu. B.
1	7	52	54	63	681
3	2	21	30	64	686
4	20	50	6	64	690
6	15	18	41	65	695
8	9	47	17	65	699
10	4	15	53	65	704
11	22	44	29	66	708
13	17	13	5	66	713
15	11	41	41	67	717
17	6	10	17	67	721
19	0	38	53	67	726
20	19	7	29	68	730
22	13	36	5	68	735
24	8	4	41	69	739
26	2	33	17	69	744
27	21	1	53	69	749
29	15	30	29	70	753
31	9	59	5	70	758
Novembris.					
0	9	59	5	70	758
2	4	27	41	71	762
3	22	56	17	71	767
5	17	24	53	71	772
7	11	53	29	72	776
9	6	22	5	72	781
11	0	50	41	73	785
12	19	19	17	73	790
14	13	47	53	74	794

Novembris. D.	h.	′	″	N. A.	Nu. B.
16	8	16	29	74	799
18	2	45	5	74	804
19	21	13	40	75	808
21	15	42	16	75	813
23	10	10	52	76	817
25	4	39	28	76	822
26	23	8	4	76	827
28	17	36	40	77	831
30	12	5	16	77	836
Decembris.					
0	12	5	16	77	836
2	6	33	52	78	840
4	1	2	28	78	845
5	19	31	4	78	849
7	13	59	40	79	854
9	8	28	16	79	859
11	2	56	52	80	863
13	21	25	28	80	868
14	15	54	4	80	873
16	10	22	40	81	877
18	4	51	16	81	882
19	23	19	52	82	886
21	17	48	28	82	891
23	12	17	4	82	897
25	6	45	40	83	900
27	1	14	16	83	905
28	19	42	52	84	909
30	14	11	28	84	914

Primæ Æquationes Conjunctionum Primi Satellitis cum Jove.

Num. A.	Æquat. Conjun. Adde.	Æq Nu. B	Num. A.	Æquat. Conjun. Adde.	Æq Nu. B	Num. A.	Æquat. Conjun. Adde.	Æq Nu. B.	Num. A.	Æquat. Conjun. Adde.	Æq Nu. B.
	′ ″			′ ″			′ ″			′ ″	
0	39 8	15	128	12 7	26	256	0 1	31	384	11 52	26
4	38 12	16	132	11 27	26	260	0 0	31	388	12 37	26
8	37 16	16	136	10 47	26	264	0 1	31	392	13 23	25
12	36 21	16	140	10 9	27	268	0 3	31	396	14 11	25
16	35 26	17	144	9 31	27	272	0 7	31	400	14 59	25
20	34 30	17	148	8 45	27	276	0 12	31	404	15 48	24
24	33 35	17	152	8 19	27	280	0 19	31	408	16 38	24
28	32 40	18	156	7 44	28	284	0 28	30	412	17 30	24
32	31 45	18	160	7 10	28	288	0 38	30	416	18 22	23
36	30 50	19	164	6 38	28	292	0 50	30	420	19 15	23
40	29 56	19	168	6 7	28	296	1 3	30	424	20 9	23
44	29 3	19	172	5 37	28	300	1 17	30	428	21 4	22
48	28 10	20	176	5 8	29	304	1 33	30	432	22 59	22
52	27 16	20	180	4 41	29	308	1 50	30	436	22 55	22
56	26 23	20	184	4 15	29	312	2 8	30	440	23 53	21
60	25 30	21	188	3 49	29	316	2 28	30	444	24 51	21
64	24 38	21	192	3 24	29	320	2 51	30	448	25 49	21
68	23 47	21	196	3 1	29	324	3 15	29	452	26 48	20
72	22 56	22	200	2 40	30	328	3 40	29	456	27 48	20
76	22 5	22	204	2 20	30	332	4 6	29	460	28 48	19
80	21 15	22	208	2 1	30	336	4 34	29	464	29 49	19
84	20 26	23	212	1 42	30	340	5 3	29	468	30 50	19
88	19 37	23	216	1 25	30	344	5 34	29	472	31 51	18
92	18 48	23	220	1 10	30	348	6 5	28	476	32 53	18
96	18 0	24	224	0 58	30	352	6 38	28	480	33 55	17
100	17 14	24	228	0 47	30	356	7 13	28	484	34 57	17
104	16 28	24	232	0 36	30	360	7 50	28	488	35 59	17
108	15 42	24	236	0 26	30	364	8 27	27	492	37 1	16
112	14 57	25	240	0 18	30	368	9 6	27	496	38 5	16
116	14 13	25	244	0 12	31	372	9 46	27	500	39 8	15
120	13 30	25	248	0 7	31	376	10 27	27	504	40 11	15
124	12 48	26	252	0 4	31	380	11 9	26	508	41 15	14
128	12 7	26	256	0 1	31	384	11 52	26	512	42 17	14

Primæ Æquationes Conjunctionum Primi Satellitis cum Jove.

Num. A.	Æquat. Conjunct. Adde.	Æq Nu. B.	Num. A.	Æquat. Conjun. Adde.	Æq Nu. B.	Num. A.	Æquat. Conjun. Adde.	Æq Nu. B.	Num. A.	Æquat. Conjun. Adde.	Æq Nu. B.
	′ ″			′ ″			′ ″			′ ″	
512	42 17	14	640	70 26	3	768	77 40	0	896	61 48	6
516	43 19	14	644	71 3	3	772	77 29	0	900	61 2	7
520	44 21	13	648	71 38	3	776	77 18	0	904	60 15	7
524	45 23	13	652	72 11	2	780	77 6	0	908	59 28	7
528	46 25	13	656	72 42	2	784	76 51	1	912	58 39	8
532	47 26	12	660	73 13	2	788	76 34	1	916	57 50	8
536	48 27	12	664	73 42	2	792	76 15	1	920	57 1	8
540	49 28	11	668	74 10	2	796	75 56	1	924	56 11	9
544	50 28	11	672	74 36	1	800	75 36	1	928	55 20	9
548	51 28	11	676	75 1	1	804	75 15	1	932	54 29	9
552	52 27	10	680	75 25	1	808	74 52	1	936	53 38	10
556	53 25	10	684	75 48	1	812	74 27	1	940	52 46	10
560	54 23	9	688	76 8	1	816	74 1	2	944	51 53	10
564	55 21	9	692	76 26	1	820	73 35	2	948	51 0	11
568	56 17	9	696	76 43	0	824	73 8	2	952	50 6	11
572	57 12	8	700	76 59	0	828	72 39	2	956	49 13	11
576	58 7	8	704	77 13	0	832	72 9	2	960	48 20	12
580	59 1	8	708	77 26	0	836	71 38	3	964	47 26	12
584	59 54	7	712	77 38	0	840	71 6	3	968	46 31	12
588	60 46	7	716	77 48	0	844	70 32	3	972	45 36	13
592	61 38	6	720	77 57	0	848	69 57	3	976	44 41	13
596	62 28	6	724	78 4	0	852	69 21	3	980	43 46	13
600	63 17	6	728	78 9	0	856	68 45	4	984	42 50	14
604	64 5	5	732	78 13	0	860	68 7	4	988	41 55	4
608	64 53	5	736	78 15	0	864	67 29	4	992	41 0	14
612	65 39	5	740	78 16	0	868	66 49	4	996	40 4	15
616	66 24	5	744	78 15	0	872	66 9	5	1000	39 8	15
620	67 7	4	748	78 12	0	876	65 28	5	1004	38 12	16
624	67 49	4	752	78 9	0	880	64 46	5	1008	37 16	16
628	68 30	4	756	78 4	0	884	64 3	5	1012	36 21	15
632	69 10	4	760	77 58	0	888	63 19	6	1016	35 26	17
636	69 49	3	764	77 50	0	892	62 34	6	1020	34 30	17
640	70 26	3	768	77 40	0	896	61 48	6	1024	33 35	17

Secundæ Æquationes Conjunctionum Primi Satellitis cum Jove.

Addendæ.

Num. B. Æqu.	0	100	200	300	400	500	600	700	800	900
	Æqu.	Æqu.	Æq	Æq.	Æq.	Æq.	Æq	Æq.	Æqu.	Æqu.
	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″
0	14 0	12 52	9 45	5 30	1 37	0 0	1 37	5 30	9 45	12 52
4	14 0	12 46	9 36	5 20	1 30	0 0	1 44	5 40	9 54	12 58
8	13 59	12 40	9 26	5 9	1 23	0 1	1 52	5 51	10 3	13 2
12	13 59	12 35	9 17	4 59	1 16	0 2	1 59	6 1	10 12	13 7
16	13 58	12 29	9 7	4 48	1 9	0 3	2 7	6 11	10 21	13 11
20	13 57	12 23	8 58	4 38	1 3	0 4	2 15	6 22	10 31	13 16
24	13 56	12 17	8 48	4 28	0 57	0 5	2 24	6 33	10 40	13 20
28	13 54	12 11	8 38	4 18	0 52	0 7	2 32	6 44	10 49	13 25
32	13 53	12 4	8 28	4 8	0 46	0 10	2 41	6 55	10 57	13 29
36	13 51	11 56	8 17	3 58	0 40	0 13	2 50	7 5	11 5	13 33
40	13 49	11 49	8 7	3 58	0 35	0 16	2 59	7 16	11 13	13 36
44	13 47	11 42	7 57	3 38	0 31	0 19	3 9	7 26	11 20	13 38
48	13 44	11 34	7 47	3 29	0 27	0 23	3 19	7 36	11 27	13 41
52	13 41	11 27	7 36	3 19	0 23	0 27	3 29	7 47	11 34	13 44
56	13 38	11 20	7 26	3 9	0 19	0 31	3 38	7 57	11 42	13 47
60	13 36	11 13	7 16	2 59	0 16	0 35	3 48	8 7	11 49	13 49
64	13 33	11 5	7 5	2 50	0 13	0 40	3 58	8 17	11 56	13 51
68	13 29	10 57	6 55	2 41	0 10	0 46	4 8	8 28	12 4	13 53
72	13 25	10 49	6 44	2 32	0 7	0 52	4 18	8 38	12 11	13 54
76	13 20	10 40	6 33	2 24	0 5	0 57	4 28	8 48	12 17	13 56
80	13 16	10 31	6 22	2 15	0 4	1 3	4 38	8 58	12 23	13 57
84	13 11	10 21	6 11	2 7	0 3	1 9	4 48	9 7	12 29	13 58
88	13 7	10 12	6 1	1 59	0 2	1 16	4 59	9 17	12 35	13 59
92	13 2	10 3	5 51	1 52	0 1	1 23	5 9	9 26	12 40	13 59
96	12 58	9 54	5 40	1 44	0 0	1 30	5 20	9 36	12 46	14 0
100	12 52	9 45	5 30	1 37	0 0	1 37	5 30	9 45	12 52	14 0

Tertiæ Æquationes Addendæ.			Semidurationes Eclipsium Primi Satellitis Jovis.												
Nu. A.	Æquationes.	Num. A.	Nu. A.	Semidurationes.		Nu. A.	Semidurationes.		Nu. A.	Semidurationes.		Nu. A.	Semidurationes.		
	° ′ ″			H. ′ ″			H. ′ ″			H. ′ ″			H. ′ ″		
0	3 30	1000	0	1 5	9	250	1 7	0	500	1 5	9	750	1 7	46	
20	3 29	980	10	1 4	56	260	1 7	15	510	1 4	53	760	1 7	57	
40	3 28	960	20	1 4	44	270	1 7	31	520	1 4	39	770	1 8	7	
60	3 25	940	30	1 4	33	280	1 7	45	530	1 4	26	780	1 8	15	
80	3 19	920	40	1 4	23	290	1 7	57	540	1 4	15	790	1 8	22	
100	3 12	900	50	1 4	13	300	1 8	7	550	1 4	7	800	1 8	26	
120	3 4	880	60	1 4	7	310	1 8	15	560	1 4	3	810	1 8	28	
140	2 56	860	70	1 4	4	320	1 8	22	570	1 4	1	820	1 8	30	
160	2 46	840	80	1 4	2	330	1 8	27	580	1 4	0	830	1 8	28	
180	2 34	820	90	1 4	0	340	1 8	28	590	1 4	3	840	1 8	26	
200	2 22	800	100	1 4	2	350	1 8	29	600	1 4	7	850	1 8	22	
220	2 10	780	110	1 4	3	360	1 8	27	610	1 4	13	860	1 8	16	
240	1 57	760	120	1 4	6	370	1 8	24	620	1 4	23	870	1 8	8	
260	1 44	740	130	1 4	12	380	1 8	17	630	1 4	35	880	1 8	0	
280	1 30	720	140	1 4	21	390	1 8	9	640	1 4	49	890	1 7	50	
300	1 17	700	150	1 4	31	400	1 7	58	650	1 5	4	900	1 7	37	
320	1 5	680	160	1 4	42	410	1 7	46	660	1 5	19	910	1 7	22	
340	0 53	660	170	1 4	55	420	1 7	31	670	1 5	36	920	1 7	8	
360	0 41	640	180	1 5	9	430	1 7	14	680	1 5	54	930	1 6	55	
380	0 31	620	190	1 5	23	440	1 6	58	690	1 6	10	940	1 6	40	
400	0 22	600	200	1 5	39	450	1 6	40	700	1 6	28	950	1 6	23	
420	0 14	580	210	1 5	55	460	1 6	20	710	1 6	46	960	1 6	8	
440	0 8	560	220	1 6	11	470	1 6	2	720	1 7	2	970	1 5	54	
460	0 4	540	230	1 6	26	480	1 5	45	730	1 7	17	980	1 5	37	
480	0 2	520	240	1 6	43	490	1 5	26	740	1 7	33	990	1 5	22	
500	0 0	500	250	1 7	0	500	1 5	9	750	1 7	46	1000	1 5	9	

The

The Uſe of the foregoing TABLES.

THE Eclipſes of the firſt Satellite of *Jupiter*, as has been already ſaid, afford the beſt means of determining the Longitude of places on the Land, where Teleſcopes of a convenient length may be uſed; thirteen of theſe Eclipſes happening every 2; Days; but it is requiſite that the Obſerver know near the matter when theſe opportunities offer themſelves, leaſt on the one hand he let them ſlip, or elſe grow weary by a too long attendance on them.

Thoſe therefore who are curious to obſerve them, may readily compute the times of the Immerſions or Emerſions of this Satellite, and that with great exactneſs, by the following very ſhort Precepts, which admit of no Exception or Caution, *viz.*

Out of the firſt Table take the *Epoche* for the Year, with its correſponding *Numb.* A and *Numb.* B; and to them add, out of the Tables of Months, the Day, Hour, Minute and Second, neareſt leſs than the time of the Eclipſe you ſeek for, together with its *Num.* A and B: the Sum of the times is the mean time of the middle of the Eclipſe. 2. With *Num.* A thus collected take out the firſt Æquation of the Conjunctions; as alſo the Æquation of *Num.* B. always to be added to *Num* B. before found. 3. With *Num* B ſo equated, take out the ſecond Æquation of the Conjunctions; and in the laſt Table, the third Æquation, as alſo the Semi-duration of the Eclipſe anſwering to *Num.* A. 4. To the mean time of the middle of the Eclipſe, add all thoſe three Æquations; the Sum ſhall be the true equated time of the middle of the Eclipſe ſought. 5. If *Num.* B. equated be leſs than 500, ſubtract

the

the Semiduration, and you will have the time of the Immerſion, or if it be more than 500, adding the ſame, it will give the time of the Emerſion.

But Note, the times thus found are equal time, ſtill to be reduced to the Apparent: and that in the *Biſſextile* Year, after *February*, one Day is to be deducted from the Day of the Month.

The leſs skilful may perhaps be pleas'd with an Example or two, which may ſerve them to imitate. Let it be required to find the time of the Immerſion of this Satellite into *Jupiter*'s ſhadow, *November* the 9*th* 1719, in the Morning. The Work ſtands thus,

	D. h.	′	″	Nu. A.	Nu. B.	
1719.	1. 6.	11.	13	872	396	
Novemb.	7. 11.	53.	29	72	776	
Conj. Med.	8. 18.	4.	42	944	172	
Æquat. I.		51.	53		10	Æq. B.
Æquat. II.		10.	26		182	B. Æquat.
Æquat. III.		3.	26			
	8. 19	10.	27			
	1. 6.	33		*Semidur. Subſt.*		
Novemb.	8. 18.	3.	54			

So that by this *Calculus*, on the ninth of *Novemb.* at 4 Minutes after 6 in the Morning, equal Time, may be ſeen the *Immerſion* of this Satellite into *Jupiter*'s ſhadow.

Another Example ſhall be of the *Emerſion* on the fifth of *April* 1720. *viz.*

1720.

	D. h. ′ ″		Nu. A.	Nu. B.	
1720.	0.20.22.40		956	310	
April	4.13.44.22	Biſs.	22	244	
Conj. Med.	5.10.07.02		978	554	
Æquat. I.	44.13			13	Æq. B.
Æquat II.	0.45				
Æquat. III.	3.29			567	B. Æquat.
	1.5.40	*Semidur. Add.*			
April	5.12.01.09				

Hence it appears that at one Minute after Midnight following the fifth of *April*, equal Time, will happen the Emerſion required. Nor do we doubt but that the Event will very nearly anſwer.

Laſtly, it may not be amiſs here to inform the Reader, that we have learnt, by the experience of many Years Obſervation, that the ſecond inequality of this Satellite proceeds from the progreſſive Propagation of Light, and is common to all the reſt of the Satellites: Light being found to proceed in about ſeven Minutes of time as far as from the Sun to the Earth, whether with an equable motion or otherwiſe is ſtill a queſtion. For this reaſon we have added a *Third Æquation*, whereby to account for the greater diſtance of *Jupiter* from the *Earth* in *Aphelio* than in *Perihelio*, as the *Second Æquation* anſwers to the greater diſtance of the Planet when near the Conjunction of the Sun, than when near his Oppoſition.

FINIS.

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PHILOSOPHICAL TRANSACTIONS.

For the Months of *September* and *October* 1719.

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II. *Epiſtola D^ni.* Jacobi Jurin, M. D. *&* R. S. Socii, *quâ doctrinam ſuam* De Potentia Cordis, *contra nuperas Objectiones Viri Clariſs. D.* Jacobi Keillii, M. D. *in* Num. 361. Philoſ. Tranſact. *editas, defendit.*

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Jos: Banks

I. *A Letter of the curious Mr.* Henry Barham, R. S. Soc. *to Sir* Hans Sloan, *Bart.* Vice-Preſident *of the* Royal Society; *giving ſeveral Experiments and Obſervations on the production of* Silk-Worms, *and of their* Silk *in* England, *as made by him laſt Summer.*

Worthy Sir,

AS you are the Patron of Induſtry and encourager of Natural Experiments, I think you juſtly claim the firſt View of theſe ſmall ones, I made upon *Silk Worms*, the laſt Summer.

And altho' they may have been done before by others in ſome other parts of the World; yet in all the Authors I have Read I do not find they make uſe of the ſame Method; and I dare be bold to ſay, that theſe following Obſervations and Experiments were never made in *England* with that Nicety, as I have done, and ſhall do if I live. It being the firſt Attempt of this kind, it may come ſhort of that compleat Methodical Manner it may be brought to hereafter; the which I hope you'll excuſe.

After you have peruſed it your ſelf, if you ſee any thing in it worthy the Communicating to the *Royal Society* (it being deſign'd for the Publick) you may do me the Honour. But I wholly ſubmit to your Judgment and Opinion in this, as I do in all other things whatſoever. *I am,*

Your humble Servant.

Henry Barham.

EXperiments made in *Chelſea Park*, in the Months of *May June* and *July* 1719.

April 27. I receiv'd a ſmall parcel of *Silk-Worms* Eggs from *Languedoc.*

May 6. Early in the Morning I found them Hatcht of themſelves, the Wind ſhifting in the Night from *Eaſt Northerly* to the *Weſt Southerly*, changing the Air of a ſudden to Warm, two Days before the change of the Moon.

After Feeding and Managing them according to Art, through the whole Courſe of their four Sickneſſes, they were come to their State of Perfection, being then as thick as a Man's little Finger, and from 4 to 5 Inches long, of a yellowiſh Colour, and when held againſt the Light, they

might

might be ſeen through as you may an Egg, being of the ſame Colour and Conſiſtence (fill'd with the matter that makes the Silk) This is a certain Sign that they will begin to Spin in 24 Hours or leſs. They then forſake their Food (being very Voracious before) and hunt about for a convenient Place to fix their firſt Hold-faſts, for ſupporting the Balls or Cones that they are to make, which they do in a moſt wonderful Mathematical Manner, with a Mixture of a Gummy Subſtance that tyes all together; and when the looſe furzy Subſtance is taken off, and ſome of the Silk is wound off, the remainder is ſo Smooth and Compact, ſhining like *Sattin*, that they are made uſe of for Artificial Flowers, and eſteemed the beſt of any thing yet known for that purpoſe, for which (only) they are generally kept in *Boarding Schools*. I weighed many hundreds of theſe Silk-Balls or Cones, which I found to weigh from 35 to 40 Grains, with their *Aurelia*'s or *Chryſalis* within them.

June 27. They begun to Spin, having been Hatcht 7 Weeks and 3 Days; and in 4 or 5 Days finiſhed their laborious and curious Work: but their Balls were not fit to be removed until 8 or 10 Days.

July 7. *Monſ. Lachivre* began to wind off their Silk-Balls with a Machine that made great diſpatch, winding much fine Silk in a Day: I found that an Ounce of Silk-Balls would make about a Dram of fine Silk; but to be more certain, I weighed out to the Winder 12 Pounds of Silk-Balls at 4 times, and told the Balls in every 3 Pound as followeth, *viz.*

The firſt 3 Pound contained	812 Balls
The ſecond 3 Pound contained	842
The third 3 Pound contained	797
The fourth 3 Pound contained	868
So that the whole 12 *lib.* Weight contained	3319 Balls.

Which when wound off, was found to yield and make one Pound and one Ounce, or 17 Ounces of fine Silk, and about 7 Ounces of coarſe Refuſe unwound, in all a Pound and half of *Averdupois* Weight, or 2 Pounds *Troy*; which is as great or greater making or yielding as in any part of the World, and the Silk as fine. I ſhewed it to a noted *Silk Broker*, who ſaid it was *Italian* Silk, (not knowing it was made in *England*) and worth about 20 *Shillings per* Pound, if I had never ſo many Bales of it, *&c.*

Now

Now upon this Experiment finding that 3319 Silk-Balls would make one Pound and one Ounce of fine Silk. I was desirous to know what quantity of Silk might be expected from the Worms Hatched from one Ounce of Eggs.

Of which to obtain the Knowledge, I made use of the following Method: by often weighing and telling I found that one hundred Eggs weighed but one Grain, so that if one Grain contains 100, a Scruple must contain 2000, and a Dram 6000, and an Ounce at 8 Drams to the Ounce, must contain 48000 Eggs. Now if every Egg hatch a Worm, and every Worm makes a Silk-Ball, there must be from one Ounce 48000 Silk-Balls; and if 3319 Balls will make one Pound and one Ounce of fine Silk, (which by Experience I found they did) then 48000 Silk-Balls will make 15 Pounds and 6 Ounces of *Averdupois* Weight in fine Silk, or 18 Pounds and eight Ounces of *Troy* Weight, which is very considerable. And in the same Proportion one Pound of *Silk Worms* Eggs, will produce Worms sufficient to make above 180 Pounds of Silk. But allowing for Casualties, and supposing but 12 Pound of fine Silk made from the Worms and their Silk-Balls produced from an Ounce of *Silk Worms* Eggs; it will be found much to exceed most Countries, according to *Augustino Gallo*'s Computation: For he sayeth, that in the *Southern* parts of *France, viz. Languedoc* and *Provence*, they make but 7 or 8 Pound of Silk from *Silk Worms* hatched from an Ounce of Eggs; and in *Brescia* in *Italy*, but 8, 9, or 10 Pound of Silk from an Ounce; only in *Calabria*, where the *Silk-Worms* and their Eggs are larger, they make 11 or 12 Pounds of Silk from an Ounce of Eggs; which still doth not exceed, nay hardly comes up to, what we make in *England*.

As to the Charge and Expences of making the aforesaid quantity of Silk in *England*, different from that of other Places, I shall be able to give you a more particular Account in my next Experimental Observations.

I have only this to add, that Experience hath taught me how to hatch *Silk Worms* twice in a Year, so as to have two good Crops of Silk in one Year. And that the *Mulberry* Trees will have Leaves in *England* twice in a Year, without prejudice to either Tree or Fruit, is most certainly true. But more in my next.

Fig. 2.

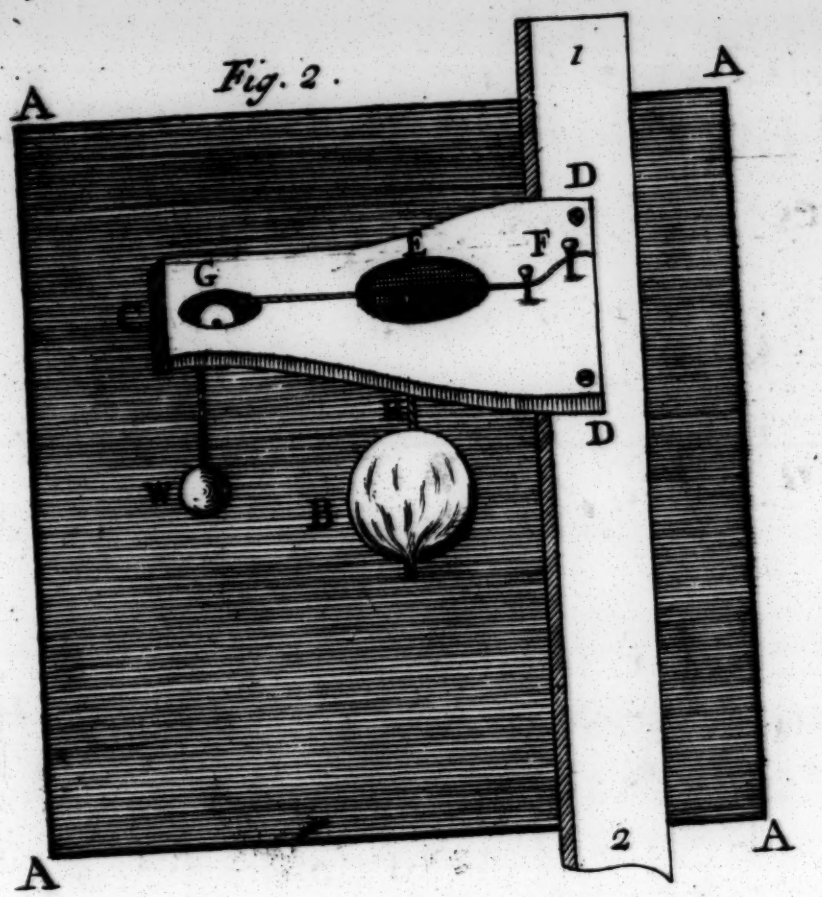
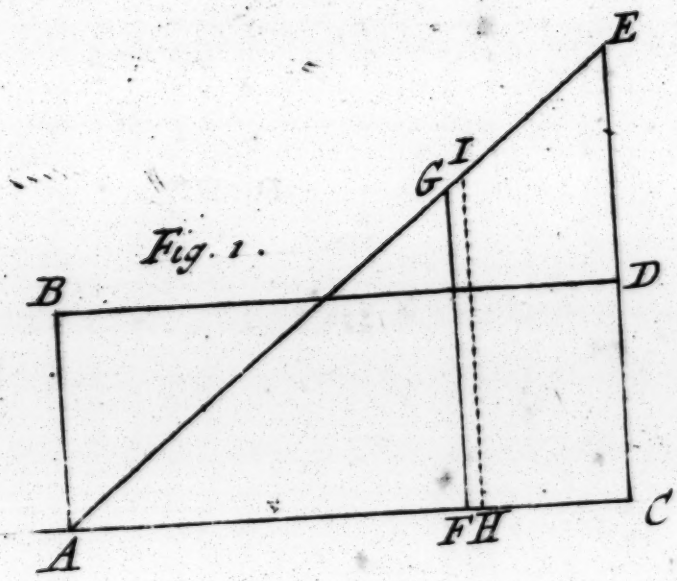


Fig. 1.



Viro Celeberrimo,

RICHARDO MEAD, *M.D.*

Collegii Medicorum *Londinenſium* & Societatis Regiæ Socio. S. P. D.

Jacobus Jurin, M.D. & Reg. Soc. S.

APologiam Præſtantiſſimi Viri, *Jacobi Keilii*, qui acerbâ nuper & immaturâ morte præreptus magnum ſui deſiderium Eruditis reliquit, ſtudioſe pervolvimus. Quam inter legendum ſingularem ſimul candidiſſimi Ingenii humanitatem, quâ noſmet, utut à placitis ſuis diſſentientes, excipere dignatus eſt, ſimul Animi magnitudinem, & ſtudium in Rempublicam Literariam tanto Viro dignum, magnâ admiratione proſequebamur. Huic enim ille non ſolum per omnem vitæ curſum diligentem & ſtrenuam operam navavit, ſed etiam pulcherrimo Exemplo, confectus jam atroci morbo & ſe perire ſentiens, eruditam illam Epiſtolam tanquam ſupremi amoris pignus, eidem legavit. Cui tamen neceſſario nobis reſpondendum eſt, non ſane quod acerrimo tuo Judicio diffidamus, ſed ne aliis Lectoribus minus idoneis impedimento eſſe poſſit, ad rectam ſententiam ferendam, Viri illius Doctiſſimi Auctoritas. Accipe igitur, Vir Clariſſime, quæ in ejus defenſione minus recte tradita cenſemus, & tuum ſimul Arbitrium eſto, utrum contentionis abrepti ſtudio iniquiores

ſimus ipſius Manibus, an ita diſputemus, ut qui de Veritate potius quam de Victoriâ ſimus ſolliciti.

Queritur primo Vir Clariſſimus, quod ſeſe una cum Doctiſſimis Viris *Borello*, & *Morlando*, tanquam Cordis Motum cum pondere inerti conferentem, injuſte perſtrinxerim. Ego certe, cum prius notaſſem Motum quendam Sanguinis & Arteriarum ex Cordis Vi oriri, dixi tandem ſciri non poſſe Cordis Potentiam quanta ſit, niſi Motus hujuſce quantitatem cognitam teneamus: Motum vero quemlibet cum pondere quieſcente comparari non magis poſſe, quam Lineam cum Rectangulo. Quibus verbis id ſignificare volui, Doctiſſimos Viros non quidem diſerte Motum Cordis cum pondere quieſcente comparare, ſed ipſos, cum Cordis Potentiam per pondus exponerent, nullam oſtendiſſe rationem, quâ Motûs quantitas ex Cordis Potentiâ oriundi poſſet æſtimari. Ex hâc Objectione, ſi recte aſſequor mentem Viri Clariſſimi, ita ſeſe expedire conatur. Cordis Potentia in preſſione conſiſtit, eamque æquabiliter in Sanguinem impendit, eodem prorſus modo, quo Gravitatis vis deorſum pondus impellit, & actione perpetuâ in motum accelerat. Proinde, cum Cordis Potentia ponderi per Corollarium *Newtonianum* definito æqualis eſt, ea Motum eundem durante Syſtole in Sanguinem imprimet, quem pondus iſtud eodem tempore cadendo per Gravitatis Vim comparabit. Ita vero cum mentem ſuam exponit Vir Cl. ſublatum iri penitus Objectionem iſtam noſtram confitemur; ſi nimirum Cordis Potentia prædicto ponderi æqualis ſit, eademque conſiſtat in æquabili preſſione per totam Syſtolen continuatâ. Atqui ex duabus iſtis Propoſitionibus poſteriorem neutiquam probare conatur Vir Doctiſſimus, ſed Hypotheſeωs loco ponit; quamvis nos rationibus quibuſdam adductis contrariam Sententiam conati ſumus veriſimiliorem reddere; nempe, quod Cordis Potentia nequaquam æquabiliter agat

in

in Sanguinem per totam Syſtolen, ſed cum totas vires exiguâ temporis particulâ collegerit, inde uno impetu in Sanguinem irruat, eumque ex Ventriculis expellat, eo modo quem in Diſſertatione noſtrâ Epiſtolari fuſius expoſuimus. Priorem vero Propoſitionem, etiam conceſſâ Viro Cl. iſtâ Hypotheſi, falſam eſſe mox demonſtrabimus.

Corollarii *Newtoniani* ſenſum quod attinet, nolumus Lectori moleſtiam nimiam faceſſere, cum neque putemus ejus intereſſe, uter *Newtoni* mentem rectius acceperit; neque ita perſpicue ſententiam ſuam expoſuerit Doctiſſimus Adverſarius, quin periculum ſit, ne aliquem ei ſenſum affingamus, quem ipſe forſitan, ſi poſſet adhuc ſe defendere, foret repudiaturus. Id vero adnotaſſe operæ pretium erit, quod cum loquatur *Keillius* de Vi quâ ex Orificio aliquo aqua exprimitur, *Newtonus* nullum omnino verbum in iſto Corollario poſuerit, quo Aqua per Vim aliquam exprimi ſignificetur; ſed pondus ſolum determinaverit æquale iſti Vi, quâ totus Aquæ effluentis Motus generari poteſt, ſive quod Gravitatis Vi cadendo Motum comparare poteſt Motui aquæ eodem tempore effluentis æqualem.

Quod autem Corollarium illud, ſi non male intellexerit Vir Cl. certe non ſatis apte uſurpârit, facile perſpiciet Lector Eruditus, qui animum adverterit, quid interſit diſcriminis inter effluxum aquæ ex foramine in fundo vaſis ſemper pleni, quomodo à *Newtono* conſideratur in eo Corollario, & effluxum Sanguinis ex Corde in Aortam. In caſu enim priori aqua jam totam velocitatem comparavit, & per datum temporis ſpatium æquabiliter effluit ex foramine. At Cordis Vis per Hypotheſin *Keillianam*, applicatur Sanguini in Ventriculo quieſcenti, & eum primo temporis momento velocitate infinite parvâ verſus Aortam propellit; continuatâ vero æquabili preſſione tandem ei finitam ve-

locitatem imprimit, eamque perpetim auget, donec omnem Sanguinem ex Ventriculo expulerit.

Rurſum in caſu *Newtoniano* conſideratur Motus non quidem totius aquæ Cataractâ contentæ, quæ omnis in motu conſtituta eſt, & diverſa velocitate verſus exitum tendit, ſed aquæ ſolum in ipſo foramine poſitæ & jam exilientis. Vis autem Cordis toti Sanguinis moli Ventriculo contentæ Motum imprimit, totamque Aortam verſus propellit.

Denique negamus pondus quinque unciarum, à Viro Cl. determinatum, poſſe eam Motus quantitatem durante Cordis Syſtole per Gravitatis Vim comparare, quam Cordis Potentia producit, conceſſâ etiam ei Hypotheſi iſtâ, quod Cordis Potentia in æquabili preſſione conſiſtat. Per hanc enim Hypotheſin erit Motus à Ventriculi ſiniſtri Potentiâ productus, ex Calculo noſtro [*Tranſ.* Numb. 359. p. 932, 934.] æqualis Motui Ponderis Octodecim librarum circiter, quod ſingulis minutis ſecundis longitudinem uncialem percurrat. Motus autem, quem pondus quinque unciarum durante Cordis Syſtole, ſi tollatur omnis Arteriarum & Sanguinis præcedentis reſiſtentia, ſive decimâ parte minuti ſecundi, per Gravitatis Vim comparabit, æquabitur fere Motui Ponderis duodecim librarum, quod ſuprapoſitâ velocitate moveatur. Quod ſi cui libuerit adſumptâ hâc Hypotheſi verum pondus definire, quod Cordis Potentiæ æquale eſt, is poſito Calculo eliciet pondus unciarum circiter ſeptem cum ſemiſſe. Hoc enim durante Syſtole Cordis eundem fere Motum cadendo comparabit, quem producit ipſa Cordis Potentia.

Sed inquiet forſitan aliquis diſcrimen modo expoſitum inter Motum à *Keilliano* pondere acquiſitum, & Motum ex Potentia Cordis oriundum inde proficiſci potuiſſe, quod forte minus accuratæ fuerint poſitiones illæ, quibus Characteres Algebraicos in Calculo noſtro ad numeros

revo-

revocavimus. Cui dubio ut occurramus, & oſtendamus ſimul nos longe majus diſcrimen inventuros fuiſſe, niſi contigiſſet ut poſitiones iſtæ *Keillio* faverent; operæ pretium erit caſum aliquem ſimpliciorem adſumere, quo data moles aquæ, per datum orificium, dato tempore, per vim aliquam ſive preſſionem æquabilem exprimatur, quæ ſunt conditiones ab Adverſario poſitæ ad Potentiam Cordis definiendam.

In eo autem caſu demonſtrabimus neque Motum aquæ effluentis, neque Motum toti tandem moli aquæ per Vim illam impreſſum, Motui aquæ in Corollario *Newtoniano*; neque Vim eam ſive preſſionem, ponderi per iſtud Corollarium definito, æquari. Quod ſi præſtare licuerit, corruat funditus neceſſe eſt tota demonſtratio *Keilliana*.

Adſumemus igitur Cylindrum aquæ datum, tubo Cylindrico infinitæ longitudinis contentum; eritque pro orificio iſta ſectio tubi ad quam pertingit utralibet aquæ ſuperficies, alteri autem ſuperficiei Vis applicabitur ope Emboli eâdem Diametro cum ipſo tubo. Perfluat jam dato tempore data quævis aquæ quantitas per dictam ſectionem tubi; tum alia quantitas æqualis per foramen pari Diametro factum in fundo vaſis, quod more *Newtoniano* uſque plenum conſervatur: & primo loco diſpiciamus, utrum pares futuri ſint in utroque caſu Motus aquæ effluentis.

*Exponatur tempus effluxus aquæ per rectam A C, velocitas autem æquabilis, qua aqua effluit ex foramine in fundo vaſis per rectam A B. Unde moles aquæ effluentis ex foramine, cum ſit in ratione temporis & velocitatis conjunctim, exponetur per Rectangulum A B C D; & Motus ejuſdem exponetur per ſolidum Parallelepipedon, ex eodem Rectangulo ducto in altitudinem A B, quippe qui ſit in ratione compoſita ex rationibus molis & velocitatis.

* *Vide Fig.* II.

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In caſu altero, ubi aqua per tubum Cylindricum fluit, tempus, ut prius, exponetur per eandem rectam A C; velocitas autem aquæ erit in ratione temporis, quippe cùm vis adhibita, ex Hypotheſi, in datam aquæ molem æquabiliter agat, & proindè repræſentabitur per rectam mutabilem F G, rectæ A F, ſive temporis ab initio effluxus, proportionalem. Molecula autem aquæ, particulâ temporis F H prædictam Sectionem præterfluens, exponetur per Rectangulum ex ipſâ F H ductâ in exponentem velocitatis F G; vel ſi evaneſcere intelligatur rectula F H, per Trapezium F G I H, & moles aquæ toto tempore A C præterfluens ſignificabitur per Triangulum rectangulum A C E. Et quoniam ex Hypotheſi moles iſta moli aquæ in caſu priore effluenti æqualis eſt, erit Triangulum A C E æquale Rectangulo A B D C; unde C E, ſive velocitas acquiſita in fine temporis A C, dupla erit velocitatis C D ſive A B, quâ aqua ex foramine in fundo vaſis effluebat. Motus autem aquæ particulâ temporis F H præterlabentis, cum ſit in ratione molis & velocitatis conjunctim, exponetur per Priſma evaneſcens, quod fit ex Trapezio F G I H ducto in velocitatem F G: Unde totus Motus aquæ toto tempore A C præterfluentis exponetur per Pyramidem, cujus baſis eſt Quadratum rectæ C E, cujuſque altitudo perpendicularis eſt ipſa A C. Quæ Pyramis cum ſit ad Parallelepipedon caſu priore definitum, ut 4 ad 3, erunt quoque Motus aquæ effluentis in utroque caſu in eâdem ratione, & proinde inæquales, quod primo loco demonſtrandum ſuſceperamus.

Proximum eſt, ut oſtendamus Motum tandem impreſſum toti aquæ tubo contentæ non eſſe æqualem Motui in exemplo primo determinato. Hic autem, cum tota iſta moles aquæ per poſitiones ſupra ſcriptas neutiquam definita ſit, adſumemus eam æqualem moli expoſitæ per Rectangulum A B C D, quæ in caſu

Vide Fig. II.

primo

primo effluit ex foramine, quæque in ſecundo ſectionem dictam præterfluit. Unde cum totus Motus ei tandem impreſſus ſit in ratione molis & velocitatis in fine acquiſitæ, idem exponetur per Parallelepipedon ex Rectangulo A B D C ducto in rectam C E. Hoc autem eſt ad Parallelepipedon, primo caſu definitum, ex eodem Rectangulo & rectâ C D, ut altitudo C E ad altitudinem C D, ſive in ratione duplâ. Porro, cum molem aquæ tubo contentæ per quodvis aliud Rectangulum, loco Rectanguli A B C D, exponere licuiſſet, patet inde Motum hunc poſſe quamlibet rationem ad Motum primo caſu definitum obtinere, & idcirco nequaquam eidem eſſe æqualem. Quod erat ſecundo loco demonſtrandum.

Supereſt, ut oſtendamus Vim in hoc caſu adhibitam ponderi per Corollarium *Newtonianum* definito non eſſe æqualem. Hæc autem Vis & vis Gravitatis agens in iſtud pondus, cum ambæ ſint æquabiles, erunt in ratione Motuum ex iiſdem dato tempore productorum. Quos cum inæquales eſſe modo demonſtratum ſit, erunt illæ Vires itidem inæquales. Quod erat demonſtrandum poſtremo.

Pergit Vir Cl. ad alterum illud vitium, quod ego in ejus ſolutione reprehenderam, nempe quod velocitatem Sanguinis ex Corde effluentis æquabilem poſuerit, quam inſigniter inæqualem fieri à me demonſtratum eſt. Negat autem ſe æquabilem velocitatem Sanguini tribuiſſe, ſed pro ſummâ diverſarum omnium velocitatum velocitatem **mediam** uſurpaſſe. Præterea nondum ſatis ſibi conſtare dicit, utrum æqualis vel inæqualis ſit Sanguinis ejecti velocitas, ſed quæ pro æquali velocitate ſtat ratio, eam ſibi firmiorem videri. Utrum vero, qui velocitatem Sanguinis inventurus molem Sanguinis expulſi ad orificium Aortæ applicat, nullâ factâ mentione neque diverſarum velocitatum, neque velocitatis

tatis mediæ, velocitatem Sanguinis æquabilem ponat, penes æquum Lectorem ſit Judicium. Idem quoque facile æſtimabit, utrum Vis aliqua ſive preſſio fluido in vaſe quieſcenti applicata, quæ eſt Hypotheſis Viri Doctiſſimi, id fluidum primo temporis momento, eâdem velocitate quâ in fine, propulſura ſit.

Poſtquam ita ſatisfactum putat Vir Cl. iis Objectionibus, quas contra priorem ſuam Methodum attuleram, jam ad alteram illam faciliorem vindicandam accedit. In hac Ego animadverteram Virum Cl. adſumere iſtam Propoſitionem, quod Vires Cordis in diverſis Animalibus ſint in ratione ponderum, item ponere velocitatem Sanguinis ex ſectâ Iliacâ Arteriâ profluentis æqualem ei, quâ Sanguis ex Corde in Aortam emittitur; quas ambas poſitiones falſas eſſe nobis demonſtratum eſt. Vitium poſterius non defendit Vir Cl. prius vero tuetur *Borelli* & aliorum Doctorum Virorum auctoritate, qui aſſumptionem iſtam ſæpius uſurparunt. Ita quidem, & nos ejuſmodi aſſumptionem in *Borello* reprehendimus, neque valet cujuſquam auctoritas contra legitimam demonſtrationem. Supereſt ergo Viro Cl. ad examen revocanda noſtra demonſtratio. Hanc autem fallaci quodam Principio inniti putat, quo cum omnia Theoremata noſtra ſuperſtructa ſint, communi ruinâ omnia involvit. Ait enim me ponere, quod Ventriculi Cordis, tanquam ſolidum corpus datâ velocitate motum, in Sanguinem impingunt, eoque ictu Motus ſui partem eidem communicant. Quam Hypotheſin Motui neque Sanguinis, neque Cordis, neque Aeris ex Pulmone expreſſi, competere cenſet Vir Clariſſimus.

Quod Pulmonem attinet, quoniam hoc obiter attingere voluit Vir D. agnoſco me conſideraſſe Pulmonem inter contrahendum tanquam datâ velocitate impingentem in Aerem contentum, idque conſulto feciſſe profiteor.

Quum

Quum enim tum *Bellinus*, tum alii multi Viri Doctiſſimi, quos inter eminet Cl. Adverſarius, multa protulerint de Vi illâ, quâ Aer inter exſpirandum in Sanguinem Pulmones præterfluentem agit, ejuſque moleculas diſſolvit; quam ſolutionem ipſo exſpirationis initio cenſent accidere; mihi propoſitum erat hanc ipſorum ſententiam ad trutinam revocare. Videbam autem, quod, ſi aerem per Vim æquabilem ſive preſſionem expelli ſtatuerem, Motus aeri à Pulmone impreſſus initio exſpirandi, ſive reactio aeris in Pulmonem, adeoque in Sanguinem præterfluentem, pro quantitate infinite parvâ habenda erat, adeoque nihil omnino eorum effectuum, quæ ipſi adſcribebantur, præſtare poterat. Ita vero ſi feciſſem, jure queſturos putabam *Bellini* ſequaces, quod inique ſecum ageretur; quippe cum rejiceretur ipſorum ſententia propter demonſtrationem ex Hypotheſi arbitrariâ & eâdem omnium adverſiſſimâ deductam. Malui igitur ex illâ Hypotheſi demonſtrationem deducere, quæ omnium maxime ipſis faveret, maximamque Motus quantitatem exſpirandi initio aeri tribueret. Hæc autem erat, quâ ponebatur Pulmo initio exſpirationis datâ velocitate in Aerem impingere.

Cæterum in Potentiâ Cordis definiendâ iſtam quidem Hypotheſin, quâ ipſius Ventriculi, omni impetu momento temporis concepto, tanquam ſolidum corpus datâ velocitate præditum, in Sanguinem irruunt, primo loco propono, tanquam omnium ſimpliciſſimam, ex eâque ſolutionem deduco. Atqui deinde conſidero tum eam Hypotheſin, quâ Ventriculi Cordis Motum omnem ſuum particulâ temporis admodum parvâ concipiunt, quæque mihi veri ſimillima videtur, tum ipſam Hypotheſin *Keillianam*, atque alias infinitas, iiſque omnibus ſolutionem meam accommodo. Adeo ut, ſive iſtud Principium incertum & fallax, ſive verum & ſtabile

reperiatur, nihil exinde ſolutionis noſtræ certitudini detrahatur.

Non tamen videmus aliquid argumenti allatum, quo minus iſtam poſitionem nobis adhibere, pari jure atque Viro Cl. contrariam illam de Vi ſive preſſione uſurpare licuerit. Nihil ſane ſpatii inter parietes Ventriculorum & Sanguinem intercedere non diffitemur, & tamen quare res ictu peragi nequeat nondum liquet. Certe, ſi Cubo Globum contingenti ictus imprimatur, Cubus partem Motus ſibi impreſſi Globo communicabit pari facilitate, ac ſi ſpatium inter eos interceſſerit.

At hæc ſunt corpora ſolida, & ubi de fluidorum Motu agitur, longe alia res eſt. Diſcrimen ſane inter ictus corporum ſolidorum, & actionem ſive ſolidi in fluidum, ſive fluidi in ſolidum, fuſius exponit Vir Cl. quod diſcrimen cum me minus advertiſſe cenſeat, ex eo fonte fluere pronunciat quicquid Erroris in meis Propoſitionibus continetur. Ego vero differentiam iſtam ut recte traditam a Viro Cl. lubens admitto, & aio me communem illam doctrinam neutiquam ignoraſſe, cum nihil frequentius in Mechanicis ſcriptoribus occurrat, ſed caſus quoſdam novos expoſuiſſe, quibus ea doctrina cum adhiberi nequiret, alia erat ineunda ratio atque hactenus fuerat uſurpata. Ea tribus verbis abſolvi poteſt. Nam, ut exemplo facillimo utamur, quieſcere ponatur Cylindrus aquæ datæ longitudinis in dato tubo, & moveatur per iſtum tubum Cylindrus alius ſolidus pari diametro, ac datâ velocitate in Cylindrum aqueum impingat. Quid inde futurum eſt? Nempe totus Cylindrus aquæ eo ictu in motum ciebitur, pari ratione, ac ſi fuiſſet & ipſe ſolidus Cylindrus: alter vero Cylindrus Motus ſui partem momento temporis deperdet, & ambo Cylindri communi velocitate per tubum deferentur. Simili modo res eveniet, ſi Cylindrus aqueus per tubum fluens Cylindro ſolido quieſcenti

centi impegerit. Quod ſi Cylindrus aqueus datâ velocitate per tubum feratur, eique occurrat Cylindrus ſolidus aliâ velocitate, ita ut quantitates Motuum Cylindri aquei & ſolidi utrinque pares ſint, jam momento temporis deſtruetur utriuſque Cylindri Motus, pariter ac ſi duo ſolida corpora æquali Motu prædita ſibi mutuo occurrant. Caſus magis compoſitos quoſcunque ex diſſertatione noſtrâ de Motu Aquarum fluentium facile eruet Lector Eruditus, idemque ſimul videbit, quomodo id fieri poſſit, quod Adverſarium Cl. præcipue torſiſſe videtur, nempe, quod Sanguinem toto impetu ex Ventriculo ruentem ſiſti poſſe docuerim, occurrente in contrarium corpore ſolido datâ Motus quantitate prædito.

Quod autem nos amicè admodum hortatur Vir Candidiſſimus, ut ſepoſitâ noſtrâ de Vaſorum ictu Hypotheſi, & Vi preſſuræ, quâ Naturam uti cenſet, pro Principio adhibitâ, Theoremata alia conſtruamus; id profecto, niſi gravi morbo impeditus perfunctorie prorſus evolviſſet noſtram Diſſertationem, dudum à nobis præſtitum animadvertere potuiſſet. Quum enim ponimus Motum Cordis in ratione temporis augeri, eâdem utique Hypotheſi utimur, ac ſi Vim preſſionis adhibeamus. Hoc autem poſito, Motum ex Cordis Potentiâ oriundum determinavimus, duplo ſcilicet majorem quam ubi Ventriculorum ictu res peragitur. Calculum vero ipſum, ut ſatis facilem & priori noſtro ſimilem, Lectori reliquimus inſtituendum. Quæ autem ſequuntur Theoremata & in iis Theorema quintum, quod rejiciendum ſtatuit Vir Cl. tanquam ex Hypotheſi de Ventriculorum ictu deductum, neutiquam pendent ex iſtâ Hypotheſi, ſed ex ipſâ Hypotheſi Doctiſſimi Adverſarii pari facilitate demonſtrantur.

Nequaquam dubitamus, quin ipſe Vir Cl. quid iſta veri habeant, ſi in vivis adhuc ageret, pro ſuâ ſagaci-

rate facile perſpecturus foret; jam vero, quoniam egregium illud Rei Medicæ Lumen amiſimus, eadem aliis Eruditis perpendenda ſimul proponimus & dijudicanda. Tibi præſertim, Vir Doctiſſime, cujus auctoritatem & ille plurimi fecit, & nos præcipuam habemus, Judici ſimul integerrimo & maxime idoneo, totam iſtam diſputationem lubentiſſime ſubjicimus.

III. *Methodus Differentialis* Newtoniana *Illuſtrata.* *Authore* Jacobo Stirling, *è Coll. Balliol.* Oxon.

ARithmeticæ pars præcipua conſiſtit in inveniendâ in numeris quantitate quâcunque determinatâ; cum vero quantitatum & numerorum natura non patiatur ut omnes quantitates exhibeantur in numeris accurate, neceſſe habemus ad Approximationes confugere. Hoc eſt, ubi quantitatum valores mathematice accurati neqeunt obtineri, quærendi ſunt ii qui ab accuratis diſtant minus datâ quâvis differentiâ.

Quicquid hâc de re à Veteribus ad nos pervenit, vel eſt particulare, ut Methodus eorum reducendi Æquationes Quadraticas; vel ſaltem uſibus generalibus male deſtinatum, ut Methodus Exhauſtionum. *Vieta* quidem primus erat qui aliquid generale in hâc arte aſſequutus eſt: quippe invenit methodum reducendi Æquationes Rationales, quæ ſolæ tunc in uſu erant. In hâc acquievêre omnes Geometræ ex ejus temporibus uſque ad ea *Newtoni.* Hic ex Interpolationibus primo pervenit ad Series: quas poſtea ad reductionem Æquationum omnium omnino generum univerſaliter applicuit. Hæc autem methodus procedit per quantitatum naſcentium & evaneſcentium rationes primas & ultimas, ſeu ſi ita loqui liceat, per quantitatum coincidentium

cidentium differentias infinite parvas. Sed & ulterius promovit *Newtonus* hanc methodum; docuitque qua ratione approximandum ſit ad quantitates quæ determinantur per regularem ſeriem terminorum, non per Æquationem ut vulgo fit. Atque ſic poſuit fundamenta calculi hujus Differentialis, qui procedit per quantitatum differentias cujuſcunque magnitudinis: ideoque eſt methodo Serierum univerſalior. Per haſce artes *Newtonianas* univerſa doctrina Approximationum reducitur ad ſolutionem Problématis, *Invenire Lineam Geometricam quæ per data quotcunque puncta tranſibit.* Ex hujus inquam ſolutione inveniuntur radices Æquationum quarumcunque, & etiam quantitates quarum relationes ad alias datas per nullas Æquationes hactenus notas poſſunt exprimi. Exiſtimo igitur *Newtonum* perduxiſſe methodum Approximandi ad ſummum perfectionis faſtigium; dum ex unico ſimpliciſſimo principio totam hanc doctrinam longe lateque patentem deducit. Quapropter credendum eſt animum *Newtoni* non ſatis perſpectum fuiſſe iis, qui ejus methodos appellant particulares, & alias tanquam ſuas & ſolas genuinas atque generales venditant, quæ aliæ non erant quam Corollaria facillima à *Newtonianis.*

Author noſter, in Epiſtola ad *Oldenburgum, Octob.* 24. 1676. data, mentionem fecit de methodo expeditâ ducendi Lineam Parabolicam per data quotcunque puncta; qua dixit ſe uſum fuiſſe ubi Series ſimplices non ſunt ſatis tractabiles. Et hanc methodum primo publicavit in Lemmate quinto Libri tertii *Principiorum.* Atque in Lectionibus publicis, circa idem tempus quo dicta Epiſtola ſcripta eſt, *Cantabrigiæ* habitis, expoſuit modum generalem determinandi Curvas cujuſcunque generis quæ tranſibunt per totidem data puncta quot earum natura patitur. Hæ Lectiones ſub titulo *Arithmeticæ Univerſalis* anno 1707. publicatæ ſunt, ubi habetur

betur methodus exemplis illuſtrata in ſectionibus Conicis. Anno vero 1711. tandem prodiit, inter alios ejuſdem Authoris tractatus, ipſa Methodus Differentialis plenius quam ante expoſita, cum fundamento ejus demonſtrato

Archimedes in methodo Exhauſtionum, *Cavallerius* in methodo Indiviſibilium, & *Walliſius* noſter in Arithmetica Infinitorum, poſuerunt fundamenta doctrinæ de determinanda quantitate quæſitâ per locum quem obtinet inter terminos in data Serie: at qua ratione approximandum eſſet ad valores quantitatum ſic determinatarum. horum nemo docuit; Hoc primus & ſolus perfecit *Newtonus:* atque exinde haud parum ampliata eſt univerſa Analyſis. Nam ſicut ante hoc inventum, ea Problemata Arithmetica ſola pro ſolutis habebantur, ubi relatio quantitatis quæſitæ ad alias datas definiebatur Æquatione, jam pro ſolutis habenda ſunt non minus ea, in quibus quantitas quæſita locum datum ſortitur inter terminos datæ Seriei; ſiquidem numeri deſiderati non minus accurate obtinentur per Methodum Differentialem, quam per extractionem Radicum: hiſce vero habitis, parum intereſt quomodo ad eos deventum eſt. Et experientia multiplex docuit, quod plurima Problemata ad Æquationes ægre deducuntur, dum ad methodum Differentialem facillime. Qualis eſt ex multis aliis toties decantata Circuli Quadratura; quam tam perfectam, mea opinione, *Walliſius* in Arithmetica Infinitorum exhibuit quam *Archimedes* illam Parabolæ.

Propoſitio

Propositio.

Invenire Lineam Parabolicam quæ transibit per extrema Ordinatorum quotcunque æquidistantium.

Casus Primus.

Designent A, $A2$, $A3$, $A4$, $A5$, $A6$, $A7$, $A8$, $A9$, &c. Ordinatas æquidistantes insistentes Abscissæ in dato angulo. Collige earum differentias B, $B2$, $B3$, $B4$, $B5$, $B6$, $B7$, $B8$, &c. harumque differentias C, $C2$, $C3$, $C4$, $C5$, $C6$, $C7$, &c. harumque differentias D, $D2$, $D3$, $D4$, $D5$, $D6$, &c. harumque differentias E, $E2$, $E3$, $E4$, $E5$, &c. harumque F, $F2$, $F3$, $F4$, &c. Et sic porro. Differentiæ autem colligi debent auferendo

ferendo priores ſemper de poſterioribus. Hoc eſt ponendo $B = A2 - A$, $B2 = A3 - A2$, $B3 = A4 - A3$, $B4 = A5 - A4$, $B5 = A6 - A5$, &c. Tum $C = B2 - B$, $C2 = B3 - B2$, $C3 = B4 - B3$, $C4 = B5 - B4$, &c. deinde $D = C2 - C$, $D2 = C3 - C2$, $D3 = C4 - C3$, &c. Et ſimiliter ſunt omnes differentiæ ſequentes colligendæ. Vel ſint $\alpha, \beta, \gamma, \delta, \varepsilon, \zeta, \eta$, &c. æquales A, $A2$, $A3$, $A4$, $A5$, $A6$, $A7$, &c. Eritque $A = \alpha$, $B = \beta - \alpha$, $C = \gamma - 2\beta + \alpha$, $D = \delta - 3\gamma + 3\beta - \alpha$, $E = \varepsilon - 4\delta + 6\gamma - 4\beta + \alpha$, $F = \zeta - 5\varepsilon + 10\delta - 10\gamma + 5\beta - \alpha$, $G = \eta - 6\zeta + 15\varepsilon - 20\delta + 15\gamma - 6\beta + \alpha$, &c. In hiſce valoribus numerales Coefficientes ipſorum $\alpha, \beta, \gamma, \delta, \varepsilon$ &c. generantur ut in dignitatibus integris Binomii $1 - z|^0$, $1 - z|^1$, $1 - z|^2$, $1 - z|^3$, $1 - z|^4$, &c. Scribendo numeros 1, 2, 3, 4, 5, &c. in Serie $1 \times \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} \times \frac{n-3}{4} \times \frac{n-4}{5} \times$ &c. ſucceſſive pro n. Sit jam PQ quælibet Ordinata reliquis intermedia, & AP ejus diſtantia ab Ordinata prima A appelletur z, tum erit

$$PQ = A + B \times \frac{z}{1} + C \times \frac{z}{1} \times \frac{z-1}{2} + D \times \frac{z}{1} \times \frac{z-1}{2} \times \frac{z-2}{3} + E \times \frac{z}{1} \times \frac{z-1}{2} \times \frac{z-2}{3} \times \frac{z-3}{4} + F \times \frac{z}{1} \times \frac{z-1}{2} \times \frac{z-2}{3} \times \frac{z-3}{4} \times \frac{z-4}{5} + G \times \frac{z}{1} \times \frac{z-1}{2} \times \frac{z-2}{3} \times \frac{z-3}{4} \times \frac{z-4}{5} \times \frac{z-5}{6} + \text{&c.}$$

Adeoque

Adeoque ſignum ipſius z mutandum eſt, quando PQ cadit ad alteras partes Ordinatæ primæ, ut pq.

Caſus Secundus.

Sit jam $A5$ Ordinata in medio omnium; pone $A = B4 + B5$, $B = D3 + D4$, $C = F2 + F3$, $D = H + H2$, &c. & $a = C4$, $b = E3$, $c = G2$, $d = I$, &c. id eſt, ſi ſint $A6 = \alpha$, $A7 = \beta$, $A8 = \gamma$, $A9 = \delta$, &c. $A4 = x$, $A3 = \lambda$, $A2 = \mu$, $A = \nu$, &c. Pone $A = \alpha - x$, $B = \beta - 2\alpha + 2x - \lambda$, $C = \gamma - 4\beta + 5\alpha - 5x + 4\lambda - \mu$, $D = \delta - 6\gamma + 14\beta - 14\alpha + 14x - 14\lambda + 6\mu - \nu$, &c. $a = \alpha - 2A5 + x$, $b = \beta - 4\alpha + 6A5 - 4x + \lambda$, $c = \gamma - 6\beta + 15\alpha - 20A5 + 15x - 6\lambda + \mu$, $d = \delta - 8\gamma + 28\beta - 56\alpha + 70A5 - 56x + 28\lambda - 8\mu + \nu$, &c. Et dicatur $A5P$, z, tum erit

$$PQ = A5 + \frac{Az + azz}{1.2} +$$
$$\frac{2Bz + bzz}{1.2} \times \frac{zz - 1}{3.4} +$$
$$\frac{3Cz + czz}{1.2} \times \frac{zz - 1}{3.4} \times \frac{zz - 4}{5.6} +$$
$$\frac{4Dz + dzz}{1.2} \times \frac{zz - 1}{3.4} \times \frac{zz - 4}{5.6} \times \frac{zz - 9}{7.8} +$$
$$\frac{5Ez + ezz}{1.2} \times \frac{zz - 1}{3.4} \times \frac{zz - 4}{5.6} \times \frac{zz - 9}{7.8} \times \frac{zz - 16}{9.10} +$$
&c.

Caſus Tertius.

Sint jam $A4$, $A5$, Ordinatæ duæ in medio omnium. Pone $A = \frac{A4 + A5}{2}$, $B = \frac{C3 + C4}{2}$, $C = \frac{E2 + E3}{2}$, $D = \frac{G + G2}{2}$

$\frac{G+G_2}{2}$, &c. $a = B4$, $b = D3$, $c = F2$, $d = H$, &c.
Vel ſint $A5 = \alpha$, $A6 = \beta$, $A7 = \gamma$, $A8 = \delta$, &c. $A4 = \varkappa$, $A3 = \lambda$, $A2 = \mu$, $A = \nu$, &c. Deinde erunt $2A = \alpha + \varkappa$, $2B = \beta - \alpha - \varkappa + \lambda$, $2C = \gamma - 3\beta + 2\alpha + 2\varkappa - 3\lambda + \mu$, $2D = \delta - 5\gamma + 9\beta - 5\alpha - 5\varkappa + 9\lambda - 5\mu + \nu$, &c. Et $a = \alpha - \varkappa$, $b = \beta - 3\alpha + 3\varkappa - \lambda$, $c = \gamma - 5\beta + 10\alpha - 10\varkappa + 5\lambda - \mu$, $d = \delta - 7\gamma + 21\beta - 35\alpha + 35\varkappa - 21\lambda + 7\mu - \nu$, &c. Et ſit O punctum medium inter $A4$, $A5$, atque appelletur OP, z; eritque Ordinata

$$PQ = \frac{A + az}{4^0} + \frac{3B + bz}{4^1} \times \frac{4zz-1}{2\cdot 3} + \frac{5C + cz}{4^2} \times \frac{4zz-1}{2\cdot 3} \times \frac{4zz-9}{4\cdot 5} + \frac{7D + dz}{4^3} \times \frac{4zz-1}{2\cdot 3} \times \frac{4zz-9}{4\cdot 5} \times \frac{4zz-25}{6\cdot 7} + \frac{9E + ez}{4^4} \times \frac{4zz-1}{2\cdot 3} \times \frac{4zz-9}{4\cdot 5} \times \frac{4zz-25}{6\cdot 7} \times \frac{4zz-49}{8\cdot 9} + \text{&c.}$$

In hiſce duobus etiam caſibus z eſt negativa, quando Ordinata PQ cadit ad alteras partes initii Abſciſſæ. Et in omnibus tribus caſibus diſtantia communis Ordinatarum ponitur unitas.

Omnes tres caſus demonſtrantur facillime per calculum. In caſu primo pro PQ ſcribo ſucceſſive α, β, γ, δ, ε, &c. & pro z interea 0, 1, 2, 3, 4, &c. quæ ſunt longitudines Abſciſſæ ordine ſequentes; & provenient æquationes

$\alpha = A$, $\beta = A + B$, $\gamma = A + 2B + C$, $\delta = A + 3B + 3C + D$, $\varepsilon = A + 4B + 6C + 4D + E$, &c.

$\beta - \alpha$

$\beta - \alpha = B$, $\gamma - \beta = B + C$, $\delta - \gamma = B + 2C + D$,
$\varepsilon - \delta = B + 3C + 3D + E$, &c.

$\gamma = 2\beta + \alpha = C$, $\delta - 2\gamma + \beta = C + D$, $\varepsilon - 2\delta + \gamma$
$= C + 2D + E$, &c.

$\delta - 3\gamma + 3\beta - \alpha = D$, $\varepsilon - 3\delta + 3\gamma - \beta = D + E$, &c.

$\varepsilon - 4\delta + 6\gamma - 4\beta + \alpha = E$, &c.

Hæ Æquationes, capiendo earum differentias, nullo labore reſolvuntur, uti videre eſt. Et dant eoſdem ipſorum A, B, C, D, &c. valores, qui antea poſiti ſunt in ſolutione. Et ad eundem modum demonſtrantur caſus duo reliqui.

Harum trium ſerierum unaquæque converget ad valorem Ordinatæ $P\mathcal{Q}$, ubi Ordinatarum datarum differentiæ ſunt juſtæ magnitudinis. At ubi non convergunt, aliæ artes adhibendæ ſunt. Sed impræſentiarum de hujus Propoſitionis uſu pauca adjiciamus.

Deſignent α, β, γ, δ, ε, ζ, η, θ, $\varkappa$, λ, &c. terminos quoſcunque æquidiſtantes, quorum differentiæ ſunt perexiguæ; & relationes quas inter ſe obtinent definientur quamproxime per Æquationes ſequentes, quæ oriuntur capiendo differentias & differentias differentiarum continuò, & ponendo eas æquales nihilo.

$\alpha - \beta = 0$

$\alpha - 2\beta + \gamma = 0$

$\alpha - 3\beta + 3\gamma - \delta = 0$

$\alpha - 4\beta + 6\gamma - 4\delta + \varepsilon = 0$

$\alpha - 5\beta + 10\gamma - 10\delta + 5\varepsilon - \zeta = 0$

$\alpha - 6\beta + 15\gamma - 20\delta + 15\varepsilon - 6\zeta + \eta = 0$

$\alpha - 7\beta + 21\gamma - 35\delta + 35\varepsilon - 21\zeta + 7\eta - \theta = 0$

$\alpha - 8\beta + 28\gamma - 56\delta + 70\varepsilon - 56\zeta + 28\eta - 8\theta + \varkappa = 0$

$\alpha - 9\beta + 36\gamma - 84\delta + 126\varepsilon - 126\zeta + 84\eta - 36\theta + 9\varkappa - \lambda = 0$.

&c.

Hæc

Hæc Tabula in uſum reſervanda eſt, ut conſulatur quoties opus ſit. Quod autem hæ Æquationes vel obtinent accurate, vel ad verum approximant, ubi differentiæ terminorum ſunt parvæ, patet ex demonſtratione caſus primi Propoſitionis.

Aſſumatur quælibet Series $\frac{1}{101}, \frac{1}{102}, \frac{1}{103}, \frac{1}{104}, \frac{1}{105}, \frac{1}{106}$, &c. Et quæratur terminus qui ſtat proximus ante $\frac{1}{101}$: patet quod ille eſt $\frac{1}{100}$; videamus ergo qualem hæc methodus exhibebit eundem. Repræſentet α terminum quæſitum, eritque

$\frac{1}{101} = \beta = 0099,0099,0099,0,$
$\frac{1}{102} = \gamma = 0098,0392,1568,7,$
$\frac{1}{103} = \delta = 0097,0873,7864,1,$
$\frac{1}{104} = \varepsilon = 0096,1538,4615,4,$
$\frac{1}{105} = \zeta = 0095,2380,9523,8,$
$\frac{1}{106} = \eta = 0094,3396,2264,2.$

Æquatio. { 1ma, 2da, 3tia, 4ta, 5ta, 6ta } dat α { 0099,0099,0099,0, 0099,9805,8629,3, 0099,9994,3455,0, 0099,9999,7824,8, 0099,9999,9895,8, 0099,9999,9993,1. }

Patet ergo quod hæc methodus continue approximat. Si terminorum differentiæ fuiſſent minores, valores acceſſiſſent citius ad verum, & contra tardius quando differentiæ ſunt majores. Hinc ſi in Tabulis numericis deſit terminus, poteſt is per hanc methodum inſeri.

Hoc modo etiam prodeunt ipſiſſimæ Series Specioſæ, quæ per alias methodos prodire ſolent. Proponatur $\overline{1+zz}|^{-1}$ Ordinata Curvæ quadrandæ: Ea eſt prima in ſerie regulari $\overline{1+zz}|^{-1}$, $\overline{1+zz}|^{0}$, $\overline{1+zz}|^{1}$, $\overline{1+zz}|^{2}$, $\overline{1+zz}|^{3}$, &c. Ordinatarum, quæ omnes præter primam dant ſuas areas z, $z+\frac{1}{3}z^3$, $z+\frac{2}{3}z^3+\frac{1}{5}z^5$, $z+\frac{3}{3}z^3+\frac{3}{5}z^5+\frac{1}{7}z^7$, &c. conſtituentes novam ſeriem cujus primus terminus erit Area quæſita: quæ ideo invenietur ponendo pro eâ α, & pro reliquis in ſuo Ordine β, γ, δ, ε, &c. Prima Æquatio dat $\alpha = z$, ſecunda $\alpha = z - \frac{1}{3}z^3$, tertia $\alpha = z - \frac{1}{3}z^3 + \frac{1}{5}z^5$, quarta $\alpha = z - \frac{1}{3}z^3 + \frac{1}{5}z^5 - \frac{1}{7}z^7$, &c.

&c. Eſt ergo univerſim area quæſita $z - \frac{1}{3}z^3 + \frac{1}{5}z^5 - \frac{1}{7}z^7 + \frac{1}{9}z^9 - \frac{1}{11}z^{11}$ &c. Eſtque hæc Series arcus ad Tangentem z, in circulo radium habente unitati æqualem. Eam invenit *Jacobus Gregorius* noſter, & cum *Collinio* communicavit initio anni 1671. à quo, mediante *Oldenburgo* ad *Leibnitium* delata eſt.

Sit jam &c, e, d, c, b, a, P, α, β, γ, δ, ε, &c. Series utrinque excurrens in infinitum, ubi dantur omnes termini præter P in medio omnium. Sit $A = \alpha + a$, $B = \beta + b$, $C = \gamma + c$, $D = \delta + d$, $E = \varepsilon + e$, &c. atque erit

$$P = \frac{A}{2} + \frac{A-B}{6} + \frac{5A-8B+3C}{60} + \frac{7A-14B+9C-2D}{140} + \frac{42A-96B+81C-32D+5E}{1260} + \frac{66A-165B+165C-88D+25E-3F}{2772} + \frac{429A-1144B+1287C-832D+325E-72F+7G}{24024} + \text{&c.}$$

Inveſtigatur hæc Series ex Æquationibus, excerpendo alternas in quibus numerus terminorum eſt impar. Nam earum differentiæ relinquent terminos in hac Serie; quæ itaque ad libitum produci poteſt.

Sit $\overline{1+z}|^{-1}$ Ordinata Hyperbolæ, & quæratur Area ejus quæ jacet ſupra Abſciſſam z, quando ea evadit unitas. Hæc Ordinata eſt media in Serie Ordinatarum

rum, &c. $\overline{1+z}|^{-5}$, $\overline{1+z}|^{-4}$, $\overline{1+z}|^{-3}$, $\overline{1+z}|^{-2}$, $\overline{1+z}|^{-1}$, $\overline{1+z}|^{0}$, $\overline{1+z}|^{1}$, $\overline{1+z}|^{2}$, $\overline{1+z}|^{3}$ &c. æquidiſtantium, hinc inde excurrente in infinitum. Adeoque Areæ ab hiſce Ordinatis genitæ conſtituent ſeriem conſimilem, cujus medius terminus erit Area quæſita; quæ proinde obtinebitur per Seriem modo expoſitam. Quando z eſt unitas, ut in caſu præſente, areæ curvarum evadunt &c. $\frac{15}{64}$, $\frac{7}{24}$, $\frac{3}{8}$, $\frac{1}{2}$, & 1, $\frac{3}{2}$, $\frac{7}{3}$, $\frac{15}{4}$, &c. Hinc eſt $A = 1 + \frac{1}{2} = \frac{3}{2}$, $B = \frac{3}{2} + \frac{3}{8} = \frac{15}{8}$, $C = \frac{7}{3} + \frac{7}{24} = \frac{21}{8}$, $D = \frac{15}{4} + \frac{15}{64} = \frac{155}{64}$, &c. Hiſce in Serie ſubſtitutis, prodit P, id eſt, area Hyperbolæ, $\frac{3}{4} - \frac{3}{48} + \frac{3}{480} - \frac{3}{4480} +$ &c. id eſt, $\frac{3}{4} - \frac{A}{4\cdot3} - \frac{2B}{4\cdot5} - \frac{3C}{4\cdot7} - \frac{4D}{4\cdot9} - \frac{5E}{4\cdot11} -$ &c. Ubi jam A, B, C D, &c. more *Newtoniano*, deſignant terminos in ſuo ordine ab initio. Calculum appono.

TERMINI.

Affirmativi	*Negativi.*
7500,0000,0000,0000,0	0625,0000,0000,0000,0
62,5000,0000,0000,0	6,6964,2857,1428,5
7440,4761,9047,6	845,5086,5800,8
97,5586,9130,8	11,3818,4731,9
1,3390,4086,1	1585,7062,8
188,7745,5	22,5708,7
2,7085,0	3260,2
393,4	47.5
5,7	7
+7563,2539,3930,7494,1	−0631,7821,3370,8041,1

Summam negativam ſubducens ab affirmativâ, habeo pro Areâ, id eſt, pro Logarithmo Hyperbolico Binarii, numerum 6931,4718,0559,9453.

Pro

Pro conſtructione Tabularum quarumvis numericarum percommoda eſt Series quæ ſequitur. Deſignent &c. $e, d, c, b, a, \alpha, \mathfrak{C}, \gamma, \delta, \varepsilon$, &c. terminos alternos in Serie utrinque ſerpente in infinitum; Pone $A = \alpha + a$, $B = \mathfrak{C} + b$, $C = \gamma + c$, $D = \delta + d$, $E = \varepsilon + e$, &c. Et terminus inter α & a erit

$$\frac{A}{2} +$$

$$\frac{1}{1} \times \frac{A-B}{2^4} +$$

$$\frac{1 \cdot 3}{1 \cdot 2} \times \frac{2A-3B+C}{2^7} +$$

$$\frac{1 \cdot 3 \cdot 5}{1 \cdot 2 \cdot 3} \times \frac{5A-9B+5C-D}{2^{10}} +$$

$$\frac{1 \cdot 3 \cdot 5 \cdot 7}{1 \cdot 2 \cdot 3 \cdot 4} \times \frac{14A-28B+20C-7D+E}{2^{13}} +$$

$$\frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} \times \frac{42A-90B+75C-35D+9E-F}{2^{16}} +$$

$$\frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9 \cdot 11}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} \times \frac{132A-297B+275C-154D+54E-11F+G}{2^{19}} +$$

&c.

Hæc Series ſequitur ex caſu tertio Propoſitionis, ponendo $z = 0$. Coefficientes numerales literarum ſic producuntur; exempli gratiâ, in quarto termino coefficiens literæ penultimæ C eſt 5; pone $5 + 1 = n$, & numeri qui proveniunt ex multiplicatione terminorum $1 \times \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} \times \frac{n-3}{4} \times \frac{n-4}{5} \times$ &c. erunt 1, 6, 15, 20, &c. Horum differentiæ 5, 9, 5, ſunt numeri quæſiti. Atque adeo Series ad libitum produci poteſt.

Datis Logarithmis numerorum 46, 48, 50, 52, 54, 56, 58 & 60; invenire Logarithmum numeri 53, qui conſiſtit in medio omnium. Pone $l, 52 + l, 54 = A = 3,4483,9710,34$, $l, 50 + l, 56 = B = 3,4471,5803,13$.

$l, 48$

$l, 48 + l, 58 = C = 3,4446,6923,08$, $l, 46 + l, 60 = D = 3,4409,0908,19$. Hiſce valoribus in Serie ſcriptis, primi quatuor termini dabunt 1,7242,2586,96 pro Logarithmo numeri 53. Et eadem ratione invenire licet quemvis alium intermedium.

In Conſtructione ergo Tabularum ſufficit primo quærere aliquos terminos in debitis diſtantiis, nam reliqui poſſunt hoc modo interſeri. Etenim continuo ſunt intercalandi termini primo inventi, uſque dum perventum fuerit ad ultimos qui deſiderantur. Hoc modo habebitur tota Tabula ex datis paucis terminis ſub initio pro fundamento operationis. Sed non convenit ut termini quos primo quærimus, ſint omnes per totam Tabulam æquidiſtantes; nam ſi omittimus alternos ubi eorum differentia eſt maxima, poſſumus alibi per ſaltum omittere duos, tres, viginti aut forte plures terminos. Numerus autem terminorum inter duos datos conſiſtentium, qui omittuntur, debet ſemper eſſe aliquis ſequentium 1, 3, 7, 15, 31, 63, &c. dummodo volumus inſerere eos per hanc Seriem; hoc vero neutiquam incommodabit opus.

Poſſunt autem pro Praxi termini in unam ſummam colligi, ut factum vides in hac Tabella. Prima expreſſio eſt primus terminus; ſecunda eſt ſumma primi & ſecundi; tertia eſt ſumma primi, ſecundi & tertii: & ſic porro.

2	$\frac{A}{2}$
4	$\frac{9A-B}{16}$
6	$\frac{150A-25B+3C}{256}$
8	$\frac{1225A-245B+49C-5D}{2048}$
10	$\frac{39690A-8820B+2268C-305D+35E}{65536}$

Sic

Sic datis aliquibus terminis alternis, intermedii confeſtim dabuntur per haſce expreſſiones, nullâ ratione habitâ naturæ Tabulæ particularis. Nam hæ regulæ ſunt eædem in omnibus. Areæ curvarum ſunt proxime æquales areis Parabolicæ figuræ quæ tranſit per extrema Ordinatarum ſuarum. Sed quoniam laborioſum nimis eſſet ſemper recurrere ad Parabolam, computavi Tabulam ſequentem, quâ Areæ directe exhibentur ex datis Ordinatis.

1	$\frac{A}{1}R$
3	$\frac{A+4B}{6}R$
5	$\frac{7A+32B+12C}{90}R$
7	$\frac{41A+216B+27C+272D}{840}R$
9	$\frac{989A+5888B-928C+10496D-4540E}{28350}R$
11	$\frac{16067A+106300B-48525C+272400D-260550E+427368F}{598752}R.$

Hic numerus Ordinatarum eſt impar, *A* eſt ſumma primæ & ultimæ, *B* ſecundæ & penultimæ, *C* tertiæ & antepenultimæ; & ſic porro, uſque dum deventum ſit ad eam in medio omnium, quæ per ultimam literam in quâque expreſſione repræſentatur. *R* eſt baſis ſeu pars Abſciſſæ inter primam & ultimam Ordinatam interceptæ. Expreſſiones ſunt Areæ contentæ inter Curvam, baſin & Ordinatas hinc inde extremas. Tabulam pro pare numero Ordinatarum non appoſui, quoniam Area cæteris paribus ex impare earum numero accuratius definitur.

Quæratur area quæ generatur ab Ordinatâ $\overline{1+zz}|^{-1}$ & jacet ſupra Abſciſſam *z* quando ea evadit unitas. In

$\overline{1+zz}|^{-1}$; pro z ſcribe $\frac{0}{10}, \frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}, \frac{5}{10}, \frac{6}{10}, \frac{7}{10}, \frac{8}{10}, \frac{9}{10}, \frac{10}{10}$; & prodibunt undecim Ordinatæ $1, \frac{100}{101}, \frac{25}{26}, \frac{100}{109}, \frac{25}{29}, \frac{4}{5}, \frac{25}{34}, \frac{100}{149}, \frac{25}{41}, \frac{100}{181}, \frac{1}{2}$. Hinc eſt $A = 1 + \frac{1}{2} = \frac{3}{2}$, $B = \frac{100}{101} + \frac{100}{181} = \frac{28100}{18281}$, $C = \frac{25}{26} + \frac{25}{41} = \frac{1675}{1066}$, $D = \frac{100}{109} + \frac{100}{149} = \frac{25800}{16241}$, $E = \frac{25}{29} + \frac{25}{34} = \frac{1575}{986}$, $F = \frac{4}{5}$. Hiſce valoribus ſubſtitutis in ultimâ expreſſione, & unitate pro R, invenies aream eſſe 785398187. Juſtus eſt hic numerus in ſeptimâ figurâ, in octavâ verum ſuperans Binario.

Si undecim Ordinatæ non dent aream ſatis exactam, erige plures; & concipe aream diviſam eſſe in plures partes, quarum quamque ſeorſum quærens habebis pro lubitu juſtam.

Valor ipſius $\overline{1+z}|^n$ exprimi poteſt per quamcunque trium ſerierum ſequentium.

$$\overline{1+z}|^n = 1 + z \times \frac{n}{1} + z^2 \times \frac{n}{1} \times \frac{n-1}{2} + z^3 \times \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} + z^4 \times \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} \times \frac{n-3}{4} + z^5 \times \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} \times \frac{n-3}{4} \times \frac{n-4}{5} + \&c.$$

$$\text{Vel } \overline{1+z}|^n = 1 + R \times \frac{n}{1} + R^2 \times \frac{n}{1} \times \frac{n+1}{2} + R^3 \times \frac{n}{1} \times \frac{n+1}{2} \times \frac{n+2}{3} +$$

$R^4 \times$

$$R^4 \times \frac{n}{1} \times \frac{n+1}{2} \times \frac{n+2}{3} \times \frac{n+3}{4} +$$
$$R^5 \times \frac{n}{1} \times \frac{n+1}{2} \times \frac{n+2}{3} \times \frac{n+3}{4} \times \frac{n+4}{5} + \text{&c.}$$

posito scilicet $R = \frac{1+Q}{Q}$. Vel

$$\overline{1+Q}|^n = 1 +$$
$$\frac{2+n+1\times Q}{\overline{1+Q}|^1} \times Q \times \frac{n}{1.2} +$$
$$\frac{4+n+2\times Q}{\overline{1+Q}|^2} \times Q^3 \times \frac{n}{1.2} \times \frac{nn-1}{3.4} +$$
$$\frac{6+n+3\times Q}{\overline{1+Q}|^3} \times Q^5 \times \frac{n}{1.2} \times \frac{nn-1}{3.4} \times \frac{nn-4}{5.6} +$$
$$\frac{8+n+4\times Q}{\overline{1+Q}|^4} \times Q^7 \times \frac{n}{1.2} \times \frac{nn-1}{3.4} \times \frac{nn-4}{5.6} \times \frac{nn-9}{7.8} +$$
$$\frac{10+n+5\times Q}{\overline{1+Q}|^5} \times Q^9 \times \frac{n}{1.2} \times \frac{nn-1}{3.4} \times \frac{nn-4}{5.6} \times \frac{nn-9}{7.8} \times \frac{nn-16}{9.10}$$
$$+ \text{&c.}$$

Primæ duæ Series demonstrantur per Casum primum Propositionis Nam si $\overline{1+Q}|^0$, $\overline{1+Q}|^1$, $\overline{1+Q}|^2$, $\overline{1+Q}|^3$, $\overline{1+Q}|^4$, *&c.* designent Ordinatas totidem æquidistantes in Parabolicâ figurâ. erit $\overline{1+Q}|^n$ ejusdem Ordinata, cujus distantia à $\overline{1+Q}|^0$ est n. Et sic prodit Series prima. At si in alia Parabola $\overline{1+Q}|^0$, $\overline{1+Q}|^{-1}$, $\overline{1+Q}|^{-2}$, $\overline{1+Q}|^{-3}$, $\overline{1+Q}|^{-4}$, *&c.* sint æquidistantes Ordinatæ, erit $\overline{1+Q}|^n$ Ordinata in eadem, cujus distantia à $\overline{1+Q}|^0$ est $-n$; sic proveniet Series secunda. Sit jam in tertia Parabola &c. $\overline{1+Q}|^{-4}$, $\overline{1+Q}|^{-3}$, $\overline{1+Q}|^{-2}$, $\overline{1+Q}|^{-1}$, $\overline{1+Q}|^0$, $\overline{1+Q}|^1$, $\overline{1+Q}|^2$, $\overline{1+Q}|^3$, $\overline{1+Q}|^4$, *&c.* Series Ordinatarum

æquidiſtantium hinc inde progrediens in infinitum, eritque in eadem $\overline{1+z}|^n$ Ordinata, diſtantiâ n à termino medio $\overline{1+z}|^0$ remota. Et ſic provenit Series tertia per Caſum Secundum Propoſitionis. Prima abrumpit quando eſt n integer & affirmativus, ſecunda quando eſt n integer & negativus, & tertia in caſu utroque abrumpit. Per harum quamque radices numerales commode evolvuntur in Series. Tertia reliquis multo citius convergit: ejus terminus ſecundus adhiberi poteſt pro correctione, ubi fit extractio per repetitionem calculi.

Halleius in ſua methodo conſtruendi Logarithmos, ex prima harum ſerierum demonſtrat Seriem *Mercatoris* pro Quadratura Hyperbolæ. Sit ejus Ordinata $\overline{1+z}|^{-1}$, vel $\overline{1+z}|^{n-1}$, exiſtente n numero infinite parvo; unde per methodos Quadrandi, area quæ jacet ſupra Abſciſſam z, id eſt, Logarithmus numeri $1+z$, erit $\frac{\overline{1-z}|^n-1}{n}$. Eſt vero per primam Seriem $\overline{1+z}|^n = 1 + \frac{n}{1}z + \frac{n}{1}\times\frac{n-1}{2}z^2 + \frac{n}{1}\times\frac{n-1}{2}\times\frac{n-2}{3}z^3 + \&c.$ adeoque in caſu præſente, ubi eſt n infinite parvus, eſt $\overline{1+z}|^n = 1 + \frac{n}{1}z - \frac{n}{2}z^2 + \frac{n}{3}z^3 - \frac{n}{4}z^4 \&c.$ quo ſubſtituto in valore areæ, ea prodit $z - \frac{1}{2}z^2 + \frac{1}{3}z^3 - \frac{1}{4}z^4 + \frac{1}{5}z^5 - \&c.$ quæ eſt Series *Mercatoris*.

Similiter per Seriem ſecundam prodit hæc regula; Sit datus numerus $1+z$, pone $R = \frac{z}{1+z}$, eritque ejus Logarithmus $R + \frac{1}{2}R^2 + \frac{1}{3}R^3 + \frac{1}{4}R^4 + \frac{1}{5}R^5 + \&c.$

Per Seriem tertiam provenit ſequens regula. Sit quilibet numerus R, pone $z = \frac{\overline{R-1}|^2}{2R}$, eritque ejus Logarithmus

garithmus $\frac{RR-1}{2R} - \frac{1}{3}Az - \frac{2}{5}Bz - \frac{3}{7}Cz - \frac{4}{9}Dz - \frac{5}{11}Ez$ — &c. Ubi A, B, C, D, E, &c. more *Newtoniano* deſignant terminos Seriei ſicut ab initio. Hæc Series, ut ea ex qua deducitur, reliquis duabus multis vicibus celerius approximat: eſtque eadem generalius expreſſa quam, ex fundamento haud abſimili, pro inventione Logarithmi Binarii prius dedimus.

Methodus inveniendi valores Serierum Arithmeticarum utcunque tarde convergentium.

In aliquibus Seriebus ſumma terminorum haberi nequit niſi ad pauciſſima figurarum loca, dummodo præter ſimplicem eorum additionem aliæ artes non adhibeantur. Proponatur jam Series quælibet cujus termini omnes iiſdem ſignis afficiuntur, & quorum proximi continue tendunt eſſe inter ſe æquales; quales ſunt ſequentes $\frac{1}{1.2} + \frac{1}{3.4} + \frac{1}{5.6} + \frac{1}{7.8} +$ &c. $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} +$ &c. Collige ſummam aliquot terminorum ſub initio, ii proxime addendi ſint $\alpha, \beta, \gamma, \delta, \varepsilon, \zeta$, &c. In numeris proximis ſit $r = \frac{\alpha\gamma - \beta\beta}{\alpha\beta - 2\alpha\gamma + \beta\gamma}$, & quantitatum $\alpha \times \frac{\alpha + r\beta}{\alpha - \beta}$, $\alpha + \beta \times \frac{\beta + r\gamma}{\beta - \gamma}$, $\alpha + \beta + \gamma \times \frac{\gamma + r\delta}{\gamma - \delta}$, $\alpha + \beta + \gamma + \delta \times \frac{\delta + r\varepsilon}{\delta - \varepsilon}$, $\alpha + \beta + \gamma + \delta + \varepsilon \times \frac{\varepsilon + r\zeta}{\varepsilon - \zeta}$, &c. differentiæ ſint a, b, c, d, e, &c. Deinde in numeris proximis ſit $s = \frac{ac - bb}{ab - 2ac + bc}$, & ipſorum $a \times \frac{a + sb}{a - b}$, $a + b \times \frac{b + sc}{b - c}$, $a + b + c \times \frac{c + sd}{c - d}$, $a + b + c + d \times \frac{d + se}{d - e}$, &c. differentiæ ſint A, B, C, D, &c. & ſit $t = \frac{AC - BB}{AB - 2AC + BC}$: atque ſic procede

cede quoad libuerit. Tum erit $\alpha + \beta + \gamma + \delta + \varepsilon + \&c. = \alpha \times \frac{\alpha + r\beta}{\alpha - \beta} + a \times \frac{a + sb}{a - b} + A \times \frac{A + sB}{A - B} + \&c.$ atque ultra duos primos terminos hujus novæ Seriei raro opus erit progredi.

Ut si desideretur valor Seriei $\frac{1}{1.2} + \frac{1}{3.4} + \frac{1}{5.6} + \frac{1}{7.8} + \&c.$ collige primos 21 terminos, quorum summam reperio fore 6813,8410,1885. Termini proxime addendi sunt $\alpha =$,0005,2854,1226, $\beta =$,0004,8309,1787, $\gamma =$,0004,4326,2411, $\delta =$,0004,0816,3265, &c. Hinc fit $r = 1$ proxime, & $\alpha \times \frac{\alpha + r\beta}{\alpha - \beta} =$,0117,6449,6282, $a =$ —,0000,0017,5096, $b =$ —,0000,0014,7410, $c =$ —,0000,0012,4986, &c. Unde $s = \frac{1}{3}$ prope, & $a \times \frac{a + sb}{a - b} =$ —,0000,0141,8111, quem propter signum negativum subduco ab $\alpha \times \frac{\alpha + r\beta}{\alpha - \beta}$, & remanet, 0117,6307, 8171: hic additus summæ primo inventæ 6813,8410, 1885, dat pro summa totius Seriei numerum 6931, 4718,0056, qui justus est in nonâ decimali; at ante duas hasce correctiones summa erat justa in primâ figura sblâ. Si animus sit propius scopum attingere, pergendum erit ad approximationes sequentes. Si termini Seriei diversa habeant signa, conjungendi sunt, ut omnes eadem tandem habeant, ut in Serie $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} -$ &c. conjunctis terminis ea evadit $\frac{2}{1.3} + \frac{2}{5.7} + \frac{2}{9.11} + \frac{2}{13.15} + \&c.$ Sed hic notandum est quod differentiæ $a, b, c, d, e,$ &c. ut & $A, B, C, D,$ &c. colligi debent subducendo quantitates antecedentes de subsequentibus. Et in omnibus hujusmodi Seriebus si $p, q, r,$ repræsentent tres terminos ordine sequentes, p primum,

mum, q ſecundum, r tertium, & rectangulum $\frac{p+r}{2} \times q$ non ſit majus pr, valor Seriei erit infinite magnus: at magnitudinis ſemper finitæ ubi accidit contrarium. Poteſt hæc regula nonunquam fallere, ubi termini p, q, r parum diſtant ab initio Seriei, at ſi conſiſtant inter eos ab initio aliquantum remotos, evadet regula certiſſima.

Ad alia Serierum genera debent aliæ regulæ adhiberi. Sit Series regularium Polygonorum Circulo Incriptorum, exiſtente Radio unitate.

$H = 2,0000,0000,0000,000 \mid 4$
$G = 2,8284,2712,4746,190 \mid 8$
$F = 3,0614,6745,8920,718 \mid 16$
$E = 3,1214,4515,2258,051 \mid 32$
$D = 3,1365,4849,0545,938 \mid 64$
$C = 3,1403,3115,6954,752 \mid 128$
$B = 3,1412,7725,0932,772 \mid 256$
$A = 3,1415,1380,1144,299 \mid 512$

Dicatur jam ultimum Polygonum A, penultimum B, antepenultimum C, & reliqua in ſuo ordine retrorſum $D, E, F,$ &c. atque area Circuli quæſita erit $A + \frac{A-B}{3} + \frac{4A-5B+C}{3 \cdot 15} + \frac{64A-84B+21C-D}{3 \cdot 15 \cdot 63} + \frac{4096A-5440B+1428C-85D+E}{3 \cdot 15 \cdot 63 \cdot 255} +$ &c. Ubi ſi pro A, $B, C, D, E,$ &c. ſcribantur proprii valores, primi quatuor termini dabunt 3,1415,9265,3589,790 pro area Circuli. Hæc autem Series eſt generalis, ex natura Circuli neutiquam dependens: applicabilis eſt quotieſcunque numerorum approximantium differentiæ priores ſunt poſteriorum quaſi quadruplæ. Factores in Denominatoribus ſunt dignitates integræ numeri 4 unitatibus minutæ:

nutæ: quibus datis, coefficientes literarum in diverſis terminis formantur ex multiplicatione continua numerorum 1, $\frac{n}{3}$, $\frac{n-3}{15}$, $\frac{n-15}{63}$, $\frac{n-63}{255}$, &c. Ubi pro n ſubſtituendus eſt ultimus Factorum in Denominatore.

Ultima quantitatum $x-1$, $2\sqrt[2]{x}-2$, $4\sqrt[4]{x}-4$, $8\sqrt[8]{x}-8$, $16\sqrt[16]{x}-16$, &c. æqualis eſt Logarithmo numeri x. Pro x ſcribe 2, & per repetitam extractionem radicis quadratæ exibunt numeri

$M = 1,0000,0000,0000,0000.$
$L = 8284,2712,4746,1901.$
$I = 7568,2864,0010,8843.$
$H = 7240,6186,1322,0613.$
$G = 7083,8051,8838,6214.$
$F = 7007,0875,6931,7337.$
$E = 6969,1430,7308,8294.$
$D = 6950,2734,2438,7611.$
$C = 6940,8641,2851,8363.$
$B = 6936,1658,4759,4014.$
$A = 6933,8182,9699,9493.$

Dicatur ultimus numerorum A, penultimus B, & ſic retro, atque Logarithmus quæſitus erit $A + \frac{A-B}{1} + \frac{2A-3B+C}{1.3} + \frac{8A-14B+7C-D}{1.3.7} + \frac{64A-120B+70C-15D+E}{1.3.7.15}$ + &c. Primi quinque termini dant 6931,4718,0559, 9457 pro Logarithmo Hyperbolico Binarii. Et quomodo hæc Series procedit in infinitum facile colligitur ex eo quod de priore diximus: eſtque etiam univerſalis, proprietates Hyperbolæ minime reſpiciens.

Extenditur quoque Methodus hæcce Differentialis ad Reſolutionem Æquationum & alia quamplurima quorum hic non fit mentio. Continetque fundamenta Serierum generaliſſima; ut in Reductione Æquationum Irrationalium & Fluxionalium brevi forſan monſtrabo.

IV. *An*

IV. *An account of ſome Experiments made on the 27th day of* April, 1719. *to find how much the Reſiſtance of the Air retards falling Bodies. By* J. T. Deſaguliers, LL. D. & F. R. S.

I Took 12 Balls (ſix of which were ſolid Leaden Globes of about 2 Inches Diameter; three hollow Glaſs Balls of about 5 Inches Diameter; and three light Paſtboard hollow Globes of about the ſame Diameter) and having carried them to the upper Gallery in the Lanthorn, on the Dome of St. *Paul*'s Church, I cauſed them to fall down by two at a time, in the following manner;

Firſt, a Leaden Ball and a Glaſs Ball.

Secondly, a Leaden Ball and a Glaſs Ball.

Thirdly, a Leaden Ball and a Glaſs Ball.

Then I let fall in the ſame manner the three other Leaden Balls, each with a Paſtboard Ball.

After that, having the Leaden and Paſtboard Balls brought up again, I repeated the Experiment twice more with a Leaden and Paſtboard Ball: then I made the Experiment twice more with a Paſtboard Ball alone, to ſee how long it would be in falling.

Upon the whole it appeared that the Leaden Balls were a very little longer than $4\frac{1}{2}$ Seconds in falling; the two largeſt of the Glaſs Balls 6 Seconds, and the Paſtboard Balls $6\frac{1}{2}$ Seconds.

The height of the Gallery, from whence the Bodies fell, was 272 Foot above the Pavement of the Church (then cover'd with Boards) upon which they fell.

The times of the Falls were taken two ways above, *viz.* with a Wheel-Chronometer, which meaſures a

ſmall part of Time accurately, nearer than to a quarter of a Second (made and contriv'd by Mr. *George Graham*, an ingenious Clock-maker) and with an ½ Second Pendulum: And the differences of Time between the fall of the Leaden Balls and the other Balls were taken below, by the Preſident, *Martin Folkes* Eſq; *F.R.S.* and another Perſon, who all agreed in their Obſervations of the Time, which they made each with an half-Second Pendulum.

The following Table gives the Marks, Weights, and Diameters of the ſeveral Balls, in three Columns.

Leaden Balls	*Troy Weight.* *l. oz. d.*	*Diameters in Inches and Decimals.*
1 *c*	2 : 1 : ½	2 , 1
2 *c*	1 : 11 : 4	1 , 99
3 *c*	1 : 11 : 12	2 , 0
4 *c*	1 : 11 : 12	2 , 0
5 *c*	1 : 11 : 12	2 , 0
6 *c*	1 : 10 : 0	1 , 98
Paſtboard Balls.		
A	0 : 3 : 6	5 , 5
B	0 : 1 : 14	5 , 1
C	0 : 1 : 17	5 , 1
Glaſs Balls.		
D	0 : 3 : 13½	3 , 9
E	0 : 5 : 3½	5 , 42
F	0 : 6 : 0½	5 , 55

N.B. The Polar and Equatorial Diameters of the Glaſs Balls being different, I have ſet down a Mean Diameter for each of them; the true Diameters are thus, of D 4 & 3,8. of E 5, 6 and 5,25. of F 5,7 & 5,4 Inches.

The

The particular Experiments are as follows.

Experiment I. Fall of 1*c* and D.
c fell by the Pendulum in $4\frac{1}{2}''$.
The Fall of D was ſo near it, that the Difference was not taken either above or below.

Experiments II. Fall of 2*c* and E.
2*c* fell by the Chronometer in $5''$, by the Pendulum in $4\frac{1}{2}''$.
Time of the fall of E not taken above.
The Difference taken below $1\frac{3}{4}''$.

Experiment III. Fall of 3*c* and F.
3*c* fell by Chronometer in $4\frac{1}{2}''$, by Pendulum in $4\frac{1}{2}''$.
F fell in Six Seconds.
Difference taken below was $1\frac{1}{2}''$.

Experiment IV. Fall of 4*c* and A.
4*c* fell by Chronometer in $4\frac{3}{4}$, by Pendulum in $4\frac{1}{2}$.
A fell in $6\frac{1}{2}$ Seconds.
Difference taken below $= 2''$.

Experiment V. Fall of 5*c* and B.
We made no Obſervation above nor below.

Experiment VI. Fall of 6*c* and C.
6*c* fell by Chronometer in $4\frac{3}{4}''$, by Pendulum in $4\frac{3}{4}''$.
C not taken above.
Difference below $= 2\frac{1}{4}''$.

Experiment VII. Fall 1 *c* and B.
1 *c* fell by Chronometer in $4\frac{3}{4}''$, by Pendulum in $4\frac{3}{4}''$.
B not taken above.
Difference taken below $2\frac{3}{8}''$

Experiment VIII. Fall of 5*c* and A.
5*c* fell by Pendulum in $4\frac{3}{4}''$.
A fell foul and ſo was not obſerv'd at all.
Difference taken below 2″.

Experiment IX. Fall of B alone.
by the Chronometer in $6\frac{1}{2}''$, by the Pendulum in $6\frac{1}{2}''$.

Experiment X. Fall of C alone
by the Chronometer in $6\frac{1}{2}''$ by the Pendulum in $6\frac{1}{2}''$.

By *Galileo*'s Theory the Lead, which was $4\frac{1}{2}''$ in falling, muſt fall 4 Foot the firſt $\frac{1}{2}''$, or 16 Feet the firſt Second, which amounts to 324 Feet in $4\frac{1}{2}''$. But as the Sound of the Ball (as it ſtruck the Bottom) by which we reckon'd our Time, had 272 Feet to move, we muſt abate a $\frac{1}{4}$ of a Second nearly, (ſuppoſing Sound to move one Mile in $4\frac{1}{2}''$) which will take away 35 Feet, that the Body muſt have fallen in the laſt $\frac{1}{4}$ of a Second, and reduce the number of Feet to 289: ſo that the Lead will have only fallen 17 Feet ſhort of the Theory, which muſt be attributed to the Reſiſtance of the Air.

The large Glaſs Ball in the 6 Seconds of its Fall, wou'd in a *Vacuum* go thro' 576 Feet: but taking away the laſt $\frac{1}{4}$ of a Second or 47 Feet, for motion of Sound, it muſt only fall 529 Feet in *Vacuo*. Now ſince it fell but 272, there have been 257 Feet taken off from the Fall by the Air's Reſiſtance.

Likewiſe the Paſtboard Ball in $6\frac{1}{2}$ Seconds muſt have fallen 676 Feet: but deducting the laſt quarter of a Second or 51 Feet for the motion of the Sound, there remains only 625 Feet for its fall in *Vacuo*. But as it fell only 272 Feet, we muſt allow a Retardment of 353 Feet for the Reſiſtance of the Air.

At

At a mean we may call the weight of the Glaſs Ball 5 *oz. Troy*, and its Diameter 5 Inches and $\frac{1}{2}$; and the weight of the Paſtboard Ball 2 Ounces *Troy*, and a little more than 5 Inches Diameter.

The Lead Balls all fell within near a Foot of one another, and made an impreſſion in the Boards of about $\frac{1}{3}$ of their Depth.

The Barometer ſtood at 30, 1 Inches, and the Mercury was very Convex, and therefore inclined to riſe ſtill.

A further Account of Experiments made for the ſame purpoſe, upon the 27th Day of July *laſt. By the ſame.*

HAving found by our former Experiments, that thin Glaſs Balls, and even Balls of paſted Paper, were too heavy to make ſo conſiderable a Difference between the time of their Fall and the fall of Leaden Balls, that it might be eaſily Obſerv'd; I contrived a way to make dryed Hogs Bladders perfectly round, by blowing them (when moiſt) within a ſtrong Spherical Box of *Lignum Vitæ*, and letting them dry in the ſaid Box before I took them out: which I did by opening the Box that ſcrew'd in the middle, and had a hole in the Pole of one of its Hemiſpheres to let the Bladder paſs thro', in order to tye it after blowing; and ſome few ſmall holes all over the Box, that in blowing no Air might be confin'd between the inſide of the Box and the Bladder, ſo as to hinder it from putting on a Spherical Figure. Beſides I took off the ends of the *Ureters*, the Fat and a great deal of the

upper

upper Coats of the Bladders, before I blowed them in the Box, to render them ſtill lighter.

The Bladders I uſed were ſome of the thinneſt I cou'd find ready blown at a *Druggiſts*, which I moiſtned in Water, taking care to leave none in the inſide. I choſe thoſe rather than Green ones, which in drying wou'd have ſtuck ſo faſt to the inſide of the Box, that it wou'd ſcarce have been poſſible to have got them out without tearing.

Having prepared five Bladders in the manner aforeſaid, (which I have deſcribed the more fully to direct any body elſe that ſhou'd be willing to try the like Experiments) I took them up to the upper Gallery in the Lantern on the Top of the *Cupola* in St. *Pauls* Church; and there by a Contrivance, which I ſhall juſt now deſcribe, I let them fall by one at a time, together with a Leaden Ball of about 2 Inches Diameter, and weighing 2 *l. Troy:* and I took notice of the time of the Fall of each Bladder, knowing by former Experiments that the Balls are about $4\frac{1}{4}$ Seconds, or a little longer time, in falling the ſame Height, which is 272 Feet.

The following Table, conſiſting of five Columns, gives in the firſt, the Marks of the Bladders; in the next their Diameters; in the third their Weights in Grains *Troy*; in the fourth the times of their Fall in Second Minutes of time; and in the fifth, the difference of Time between the Falls of the Leads and of each Bladder, taken below by the *Preſident*, Dr. *Halley*, Dr. *Jurin*, *Martin Folkes* Eſq; and Mr. *George Graham* the Clock-maker. The Time was taken above with Mr. *Graham*'s Chronometer, (formerly deſcribed); and below with the ſame Inſtrument, and three half Second Pendulums, all which agreed very well together.

The

The Experiments having been made twice over, the Table is twice ſet down; and thoſe Experiments in which the Bladders fell ſtreight down, and the moſt regularly, have this Mark before them (*).

Marks.	*Diameters in Inches.*	*Weight in Grains Troy*	*Time of the whole Fall.*	*Diff. between the Lead and Bladder.*
A	5,3	128	$19\frac{3}{8}''$	$14\frac{2}{8}$ Seconds.
*B	5,193	156	$17\frac{1}{4}$	$12\frac{3}{4}$
C	5,33	$137\frac{1}{2}$	$18\frac{3}{4}$	$14\frac{5}{8}$
D	5,26	$97\frac{1}{2}$	$22\frac{1}{8}$	$17\frac{6}{8}$
*E	5,02	$99\frac{1}{8}$	$21\frac{5}{8}$	17
*A			$19''$	$14\frac{1}{2}$
B			$18\frac{5}{8}$	$14\frac{1}{4}$
*C			$18\frac{3}{8}$	14
D			24	$19\frac{1}{8}$ $6\frac{1}{8}$
E			$21\frac{1}{4}$	$16\frac{1}{8}$

The Diameters and Weights may be relyed upon, being taken the Day that the Experiments were made, and the Day after; but the Diameters and Weights taken 10 Days before, not agreeing with theſe, I have left them out. For the Bladders by drying had loſt of their Weight, and altered their Diameters.

As the Necks of the Bladders in drying ſhrink, ſo as to open a little, they muſt be blown before each Experiment. And for the manner of letting them fall exactly in the ſame Inſtant of time, it is deſcribed by Figure II, in which

A, A, A A, is the Hole through which the Bodies fell: 1, 2, is a Board laid over the Hole. G, D, D is another Board fixt to the firſt Board by the two Wood-Screws D, D, with a Pulley G at the other end of it, over the Hole. W is a two Pound Ball of Lead faſtned

faſtned to a ſtrong Thread, which going over the Pulley is ſtretched horizontally from G to the Nails F; to which it is faſtned, ſo as to be about a quarter of an Inch above the Board.

B is one of the Bladders, hanging with the Neck or heavieſt part downwards, by means of a Loop of fine Thread as E H, which goes over the Horizontal Thread G E F. Now when with a pair of Sciſſars the Thread of the Lead (which in all is but one Foot long) is cut juſt at E, before the Loop of the Bladder, the Lead pulling away the String the Loop of the Bladder ſlips off the remaining Thread F E, and begins to fall exactly in the ſame Inſtant as the Lead: But if the Thread ſhould be cut between E and F, as the Lead falls its Thread might give the Bladder an oblique Direction.

He that obſerves the time either with a Pendulum or Chronometer may take it very exactly, by ſeeing the motion of the Sciſſars as they cut the Thread.

N. B. As the Diameters of the Bladders were taken by wrapping a Thread twice round them, and ſomething muſt be allowed for the thickneſs of the Thread; I have here under ſet down the Diameters of the Bladders, as corrected by that Allowance. *Viz.* A 5,28 Inches; B 5,19; C 5,30; D 5 $\frac{1}{4}$; and E juſt 5 Inches in Diameter.

The Bladder E was rough, with ſeveral Wrinkles and inequalities, which made it be longer in falling than it ought to have been, according to its Diameter and Weight.

A Pail of Water thrown down met with ſuch a Reſiſtance in falling 272 Foot thro' the Air, that it was all turn'd into Drops like Rain.

F I N I S.

ERRATA. *Phil. Tranſ.* N°. 357. Page 848. l. 22. lege ab *h* 11′ 32″. N°. 359. p. 932. l. 17. lege $t = 0''$,1. p. 937. l. 5, 6. lege reſtituantur. Et Syſtole Arteriarum cum Cordis Diaſtole duratione convenit. N°. 361. p. 1005. l. 16. read, *proof of the falſity of the Opinion.*

PHILOSOPHICAL TRANSACTIONS.

For the Months of *Novemb.* and *Decemb.* 1719.

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II. *An Account of ſome new Experiments, relating to the Action of Glaſs Tubes upon Water and Quickſilver.* By James Jurin, M. D. Reg. Soc. & Coll. Med. Lond. Soc.

III. *Part of a Letter from Dr.* Rich. Richardſon, *to Dr.* Will. Sherard, R.S.S. *giving a relation of a wonderful Fall of Water from a Spout, upon the Mores in* Lancaſhire.

IV. *An Account of the* Phænomena *of a very extraordinary* Aurora Borealis, *ſeen at* London *on* November 10. 1719. *both Morning and Evening. By Dr.* Edmond Halley. R. S. Secr.

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VII. *An Account of another very conſiderable* Aurora Borealis *obſerved at* Streatham *in* Surry, *by Mr.* Thomas Hearne; *and Communicated by Coll.* Francis Nicholſon, R. S. S.

VIII. *Nuperæ Obſervationes Aſtronomicæ cum Regia Societate communicatæ.*

Jos: Banks

I. *A Letter of Mr.* Joſeph Williamſon *Watchmaker, to the Publiſher, wherein he aſſerts his Right to the curious and uſeful Invention of making Clocks to keep Time with the Suns Apparent Motion.*

HAving been inform'd of a *French* Book lately publiſhed, wherein the Author ſpeaks of making Clocks to agree with the Sun's apparent Motion; and ſuppoſeth that it was a thing never thought of by any before himſelf: I was therefore willing by the advice of ſome of my Friends, to write this ſhort Account of what I have performed in that matter my ſelf.

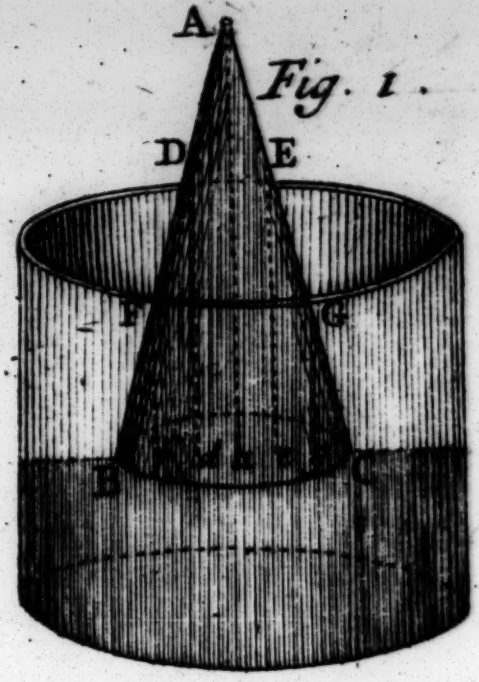
And in the firſt place I muſt take notice of the Copy of a Letter in this Book, wrote by one *P. Kreſa* a *Jeſuit*, to one Mr *Williamſon*, Clockmaker to his *Imperial Majeſty*; of a Clock found in the late King *Charles* the *Second* of *Spain*'s Cabinet, about the Year 1699 or 1700. which ſheweth both equal and apparent Time according to the Tables of Equation; and which went 400 Days without winding up. This I am well ſatisfied is a Clock of my own making; for about ſix Years before that time, I made one for Mr. *Daniel Quare*, for whom I then wrought moſtly, which agrees with the Deſcription he gives of it, and went 400 Days as he ſaith. This Clock Mr. *Daniel Quare* ſold, ſoon after it was made, to go to the ſaid King *Charles* the *Second* of *Spain*: and it was made ſo that if the *Pendulum* was adjuſted to the Sun's mean Motion, the Hands would ſhew Equal Time on two fixed Circles, on one the Hour, and on the other the Minute. But there were other two moveable Circles of the ſame kind, that moved

moved forwards and backwards, as the time of the Year required; on which the ſame Hands ſhewed Apparent Time likewiſe, according to the Equation Tables. This Method the Author owns he knew of, and applyed the ſame Motion to Pocket Watches 12 or 14 Years ago, which I confeſs I never did; being well ſatisfied that Watches with Springs and Ballances are very unfit to ſhew the minute difference, as it increaſeth and decreaſeth, between equal and apparent Time.

Soon after this Clock was ſent to *Spain*, I made others for Mr. *Quare* which ſhewed Apparent Time by lengthning and ſhortning the *Pendulum*, in lifting it up and letting it down again, by a Rowler ſomewhat in the form of an *Ellipſis*, through a ſlit in a piece of Braſs, which the Spring at the Top of the Pendulum went through. By this means every vibration of the *Pendulum* would agree to a Second of Time of the Suns apparent Motion; that Rowler which lifted up the *Pendulum*, and let it down again, being continually moving about all the Year; ſo that it may ſeem very ſtrange that this Author never heard of it, ſo many Years after they were made: For one of thoſe, and not the firſt, made with the riſing and ſetting of the Sun, Mr. *Quare* ſold to the late King *William*, and it was ſet up at *Hampton-Court* in his Life time, where it hath been ever ſince. This contrivance of lengthning and ſhortning the *Pendulum*, I thought of ſeveral Years before I made any of them. Since then I have made others for Mr. *Quare* likewiſe, which ſhewed the difference between equal and apparent Time according to the Equation Tables, by a Hand moving both ways from the top of a Circle; on one ſide ſhewing how much a Clock keeping equal Time ought to be faſter than the Sun, on the other ſide how much ſlower.

But

But theſe Clocks that I then made to agree with the Sun's Apparent Time, were done according to the Equation Tables, which I found not to agree very exactly with the Sun's apparent Motion: neither can any other be made to keep equal Time that will gain and loſe all the Year agreeable to the ſaid Tables: for though the Tables themſelves may be true, yet ſome difference in Motion does proceed, in both ſorts of Clocks, from Cold and Heat altering the length of their *Pendulums*. This difference by ſome Obſervations I have made, I ſuppoſe to be about the $\frac{1}{100}$ part of an Inch in the length of a *Pendulum* vibrating Seconds, which will alter the Motion of the Clock about 12 Seconds in 24 Hours. But to make my Clocks made for keeping Apparent Time to go as exact as poſſible, I made a Table my ſelf by Obſervation: For obſerving the Sun, as often as it was to be ſeen, when it came on the Meridian, for ſeveral Years together, always ſetting down the Difference between its coming to the Meridian and the Time by a Clock I had adjuſted as well as I could to equal Time, and always taking notice how much my Equal-Time Clock gain'd or loſt at the end of every Year, I compleated my Table in the Year 1711. Since then I have made a conſiderable many of theſe Clocks, ſeveral of which I ſold to Perſons of great Note and Ingenuity; and in particular one I made about five or ſix Years ſince for the Right Honourable the Lord *Parker*, at preſent Lord High Chancellor of *Great Britain*; and all of them have given good content to thoſe that bought them. So that I think I may juſtly claim the greateſt right to this contrivance of making Clocks to go with Apparent Time; and I have never yet heard of any ſuch Clock ſold in *England*, but what was of my own making, though I have made of them ſo long.



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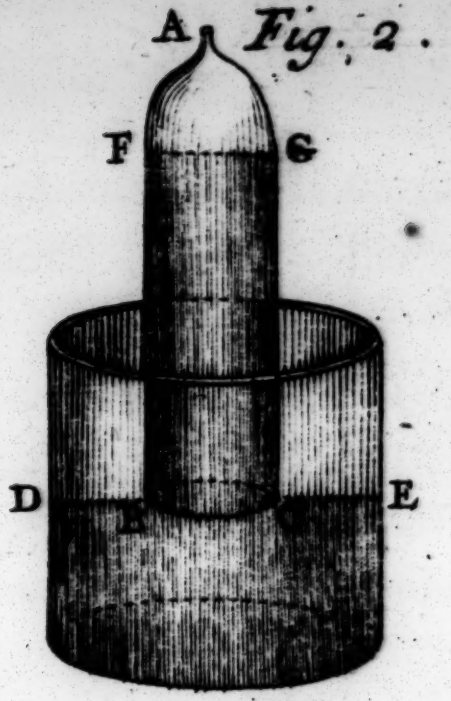
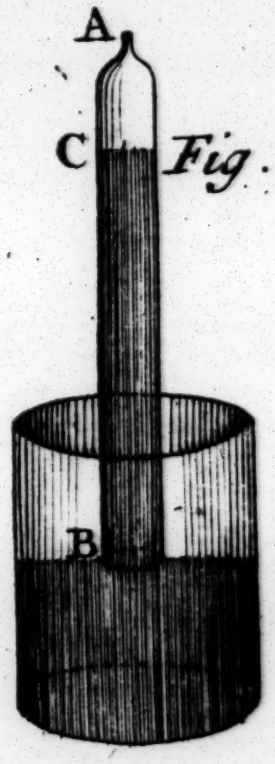


Fig. 2.



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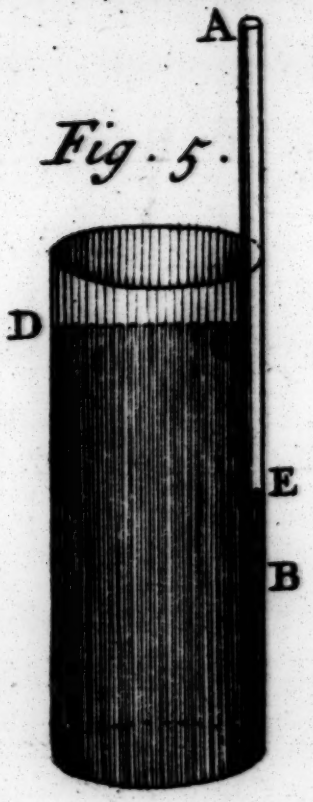

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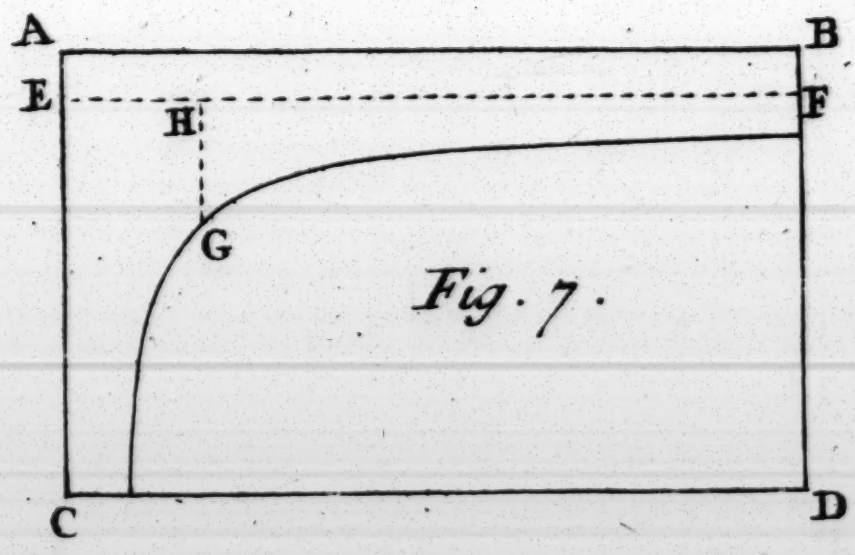


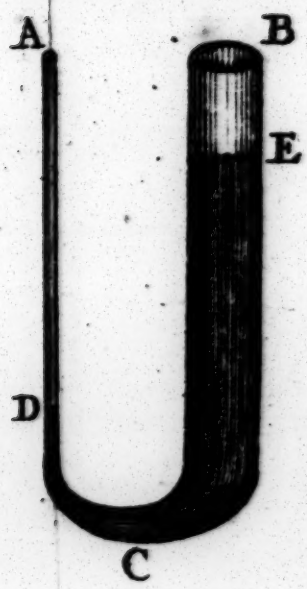
Fig. 7.

Fig. 10.

A *Fig. 5*

C

Fig. 6.



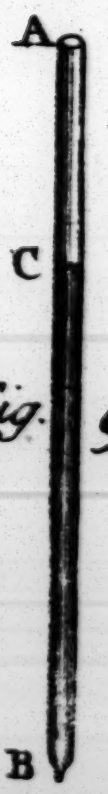


Fig. 9

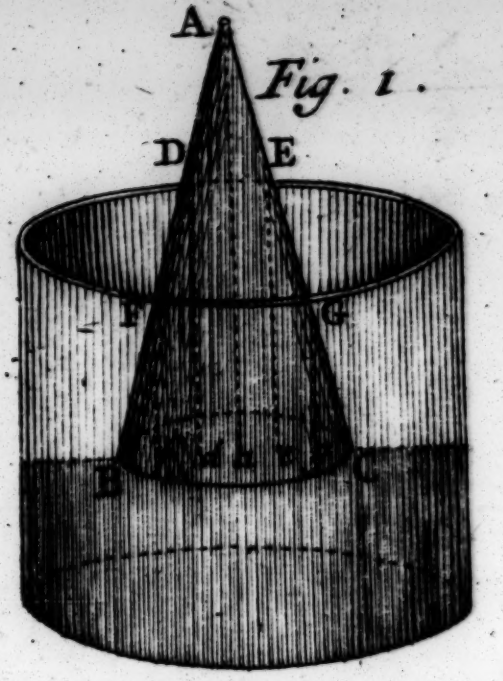


Fig. 1.

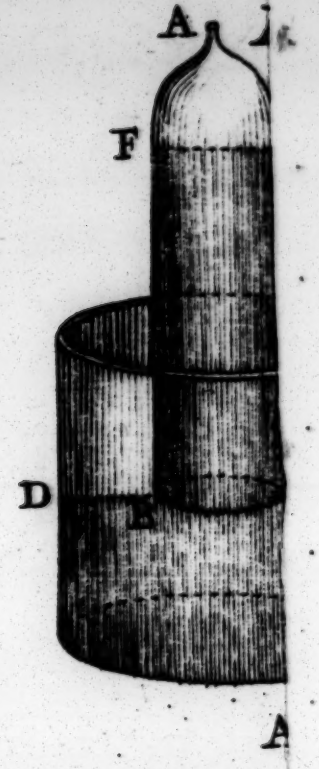
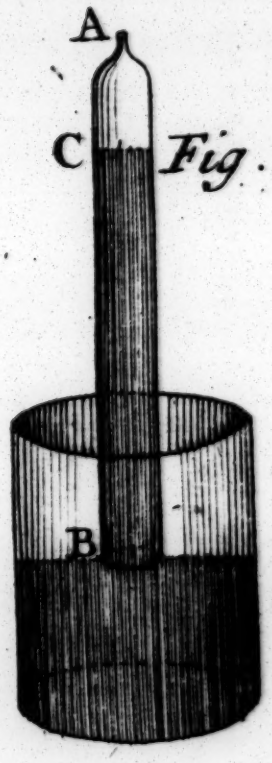


Fig. 5.



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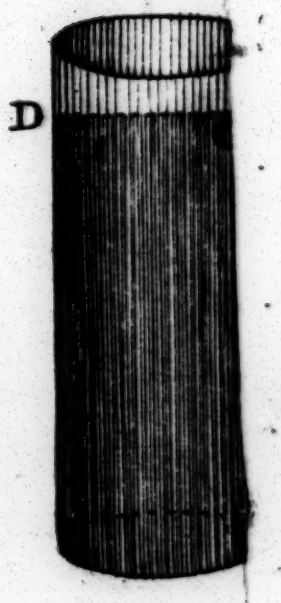


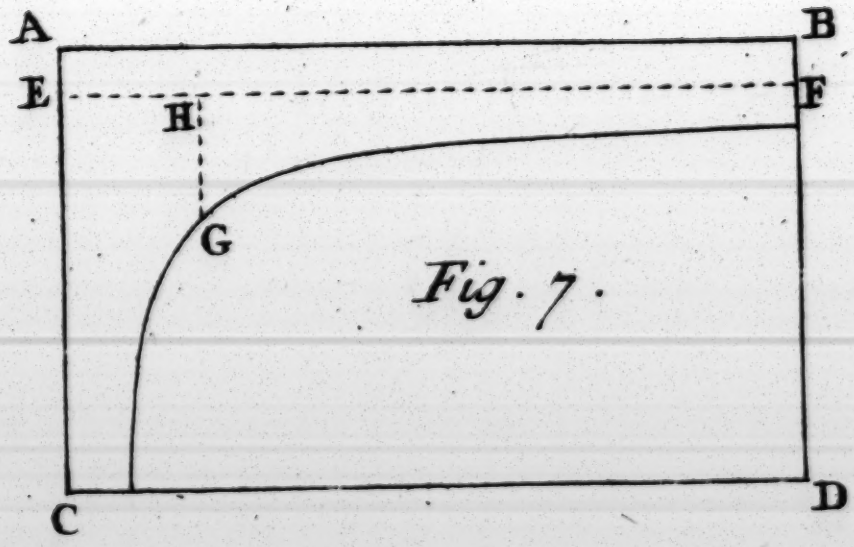
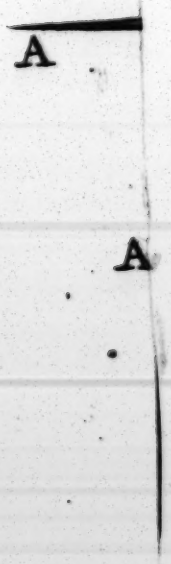


Fig. 7.



II. *An Account of ſome new Experiments, relating to the Action of Glaſs Tubes upon Water and Quickſilver.* By James Jurin, M.D. Reg. Soc. & Coll. Med. Lond. Soc.

IN a * Diſcourſe formerly preſented to the *Royal Society*, I maintain'd, that the Suſpenſion of Water in a Capillary Tube was owing to the Attraction of a ſmall annular ſurface on the inſide of the Tube, which touch'd the upper part of the Water. Among the ſeveral Experiments made uſe of to prove this Aſſertion, was that of a Glaſs Funnel of ſeveral Inches Diameter, having its ſmall end drawn out into a very fine Tube, which Funnel being inverted and fill'd with Water, the whole quantity of Water therein contain'd was ſuſtain'd above the Level by the Attraction of that narrow *Annulus* of Glaſs, with which the upper ſurface of the Water was in contact.

Soon after that Diſcourſe was printed, came out a Book publiſh'd by a very Learned and Ingenious Member of this Society, in which that Experiment was accounted for in the following Manner.

If there be a Funnel, as A B C, Fig. 1. *full of Water, and whoſe wide end ſtands in a Veſſel of Water as B C; and the Top of the Funnel A ends in a Capillary Tube open at A, the whole Water will be ſuſtain'd: the Pillar A a by the Attraction of the Circle of Glaſs within the Tube immediately above it; and all the reſt of the Pillars of Water, as F f, D d, E e, G g, &c. in ſome meaſure by the Attraction of the parts of the Glaſs above them, as F,*

* *Philoſ. Tranſact.* N°. 355.

D, E, G: And that the ſmall Pillars or Threads of Water D d, and E e, do not ſlide down to F f, and G g, and ſo go quite down, ſeems to be owing to their Coheſion with the Pillar A a, which is ſuſtain'd by the Capillary Tube A: For if you break off the ſaid Tube at D E, the whole Water will preſently ſink down.

As this Solution was very different from what I had before given, and the Reputation of that Gentleman, whoſe great Knowledge in Experimental Philoſophy is generally known, was ſufficient to give weight to any of his Opinions, I thought my ſelf under an Obligation to examine his account of the Experiment, in order either to demonſtrate its inſufficiency, or to retract my own Solution. Accordingly at the next meeting of the Society, I produced the following Experiment.

The Funnel, A F G B C, *Fig* 2. whoſe lower part B C F G, was Cylindrical to a conſiderable height, and whoſe top was drawn out into a fine Tube at A, being fill'd with Water to the height B F, ſo that the ſurface of the Water F G, did not reach to the arched part of the Funnel, I touch'd the end A with a wetted Finger, whereby a ſmall quantity of Water being inſinuated into the Capillary Tube at A, the Water contain'd in the Funnel was ſuſpended above the Level of the Water in the Ciſtern D E, as in the former Experiment.

In this Experiment it is manifeſt, that the little Columns, into which we may ſuppoſe the Cylinder of Water, F G B C, to be divided, are no way ſuſtain'd by the Attraction of the arched part of the Glaſs above them, ſince they have no contact with it. Nor is there any ſuch middle Pillar of Water, which, by its contact with the Tube at top, is both ſuſtain'd it ſelf, and helps to ſupport the Pillars about it. Upon the

Sup-

ſuppoſition of which two Particulars that Gentleman's Solution was founded.

This Experiment may be thus accounted for. The Cylinder of Water F G B C, by its weight balances a part of the preſſure of the Atmoſphere, which is incumbent on the Water in the Ciſtern, and endeavours to force that Cylinder upwards. The reſt of that preſſure is balanced by the Spring of the Air, A F G, which is included between the Cylinder of Water F G B C, and the little Column of Water in the Capillary A. But, as this Air by its Spring preſſes equally every way, it muſt balance as much of the preſſure of the Atmoſphere upon the little Column of Water at A, as it does of that upon the Water in the Ciſtern. The remainder of the preſſure of the Atmoſphere upon the Column of Water at A is ſuſtain'd by the force with which that Column adheres to the Capillary Tube, which therefore does exactly balance the weight of the Cylinder of Water F G B C, and is the real, though not the immediate, cauſe of its Suſpention.

The experiment ſucceeds in the ſame manner when a Column of Quickſilver is raiſed into the Funnel, inſtead of the Column of Water F G B C, the top of the Tube being touch'd with a wet Finger as before. But then the height of the Quickſilver in the Funnel muſt be as much leſs than that of the Water, as its Specifick Gravity is greater.

I proceed now to acquit my ſelf of a Promiſe I made in the Diſcourſe abovemention'd, of examining whether the Experiments therein contain'd would ſucceed *in Vacuo*; and whether Water could be ſuſpended in a wide Tube by means of a Capillary at Top, at a greater height, than what it can be rais'd to by the Preſſure of the Atmoſphere.

In order to this, I boil'd ſome Water, and afterwards purged it of its Air by means of the Air-pump; which being done, thoſe Experiments all ſucceeded in the exhauſted Receiver, in the ſame manner as in the open Air.

The 13*th* Experiment in particular was made with a Tube of about 35 Inches in length, and a quarter of an Inch Diameter, the top of it being drawn out into a fine Capillary. Which being fill'd with Water purged of its Air, as before mention'd, the whole quantity continued ſuſpended in the exhauſted Receiver.

This plainly ſhews, that the ſucceſs of that Experiment does not depend upon the Preſſure of the Air, ſince the ſmall quantity of Air left in the Receiver was by no means capable of ſuſtaining the Water at ſo great a height, and conſequently that the height, at which Water may be ſuſpended in this manner, is not limited by that Preſſure.

But here I muſt not omit taking notice of a conſiderable Difficulty, which preſents it ſelf to thoſe who attentively conſider this Experiment. In order to make which the better appear, it will be proper to obſerve what happens, when a ſimple Capillary Tube is fill'd with Water purged of Air, and inclos'd in the exhauſted Receiver.

In this caſe the whole Column of Water contained in the Tube A C B, *Fig.* 3*d.* is ſuſpended by the Attraction of the *Annulus* at the top of the Tube, A. And though that *Annulus* does not immediately act upon any part of the Water, except what is either contiguous to it, or ſo near as to be within the Sphere of its Attraction, which extends but to a very ſmall diſtance; yet it is impoſſible, that any other part of the Water, as for inſtance that at C, ſhould part from the Water above it and ſink down, becauſe its deſcent is oppos'd by

by the attraction of the contiguous *Annulus* at C. For this, being equal to the upper *Annulus* at A, is capable of ſuſtaining a Column of Water of the length A B, and conſequently is more than ſufficient for ſupporting the Column of Water below it, C B From which it is plain, that no part of the Water contain'd in the Tube can poſſibly deſcend, unleſs the upper part, aſſiſted by the weight of the Water below it, be ſufficient to overcome the Attraction of the *Annulus* of Glaſs at A.

But in ſuch a compound Tube as that made uſe of in our Experiment, *Fig.* 4*th* A C B, the caſe is very different, and it does not eaſily appear, why in a *Vacuum* any part of the Water in the wider part of the Tube, as for Example at C, ſhould not leave that which is above it, and deſcend; ſince the *Annulus* at C is by much too wide to ſuſtain a Column of Water of ſo great a length as C B,

The beſt anſwer I can give to this difficulty is, that the Coheſion between the Water contain'd in the Capillary and that below it, is ſufficient to balance the weight of the Column ſuſpended. But how far this Coheſion may depend upon the Preſſure of a Medium ſubtile enough to penetrate the Receiver, is worthy of Conſideration. For though ſuch a Medium will pervade the Pores of the Water, as well as thoſe of the Glaſs, yet it will act with its intire Preſſure upon all the ſolid Particles, if I may ſo call them, of the ſurface of the Water in the Ciſtern; whereas ſo many of the ſolid Particles of the Water in the Tube, which happen to lie directly under the ſolid Particles of the Water above them, will thereby be ſecur'd from this Preſſure; and conſequently there will be a leſs Preſſure of this Medium upon any ſurface of the Water in the Tube below the Capillary, than upon an equal ſurface of the Water in the Ciſtern. So that the Column of Water ſuſpended

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in the Tube may be ſuſtain'd by the difference between thoſe two Preſſures. This Explication ſeems to be favour'd by the following Experiments, which may all be accounted for in the ſame manner, though I ſhall anon mention another Cauſe, which contributes to the Succeſs of the firſt and ſecond.

The firſt I ſhall mention, is the famous Experiment of the ſuſpenſion of Mercury purged of Air, to the height of 70 or 75 Inches, in the *Torricellian Tube*, in the open Air. To which we may add the ſuſtaining of Mercury likewiſe purged of Air within the exhauſted Receiver, as related by that Learned and Succeſsful Promoter of Natural Knowledge, Monſ. *Papin*, in his *Continuation du Digeſteur*. I forbear to mention the ſuſpenſion of Water purged of Air, in the *Vacuum*, which he deſcribes in the ſame Book; becauſe there is little difference between that Experiment and our own abovementioned, the very top of the arched part of his Tube, which top we may ſuppoſe as ſmall as we pleaſe, ſupplying the place of the fine Capillary at the top of our Tube. But we muſt not omit the Experiments made by the famous Monſ *Huygens*, and deſcribed by him in *Phil. Tranſact.* No. 86. of the cohering of poliſh'd Plates with a conſiderable force in the exhauſted Receiver; as likewiſe of the running of Water and Mercury, when purged of Air, through a Siphon of unequal Legs in the *Vacuum*: All which he accounts for from the ſame Principle, and much in the ſame manner, as we have uſed for explaining the Experiment above.

As to the Exiſtence of ſuch a Medium, I ſhall content my ſelf to refer to what has been ſaid by our *Illuſtrious Preſident* in the Queries at the latter end of the laſt Edition of his *Opticks*: and as I have lately had the Honour to entertain the Society with ſome Experiments upon Quickſilver, which were exactly the

reverſe

reverſe of thoſe made by Dr. *Taylor*, the late Mr. *Hawksbee* and my ſelf, upon Water; by which I am now enabled to throw this whole affair into a little Syſtem by it ſelf, I ſhall beg leave to lay it down in the following Propoſitions, the Proof of which is contain'd in the Experiments annext.

PROPOSITION I.

The Particles of Water attract one another.

This, I think, is now univerſally acknowledged, and therefore needs no Demonſtration; the Sphericity of the drops of Rain, and the running of two drops of Water into one another upon their contact, manifeſtly proving it.

PROPOSITION II.

The Particles of Quickſilver attract one another.

This is likewiſe manifeſt from the Spherical Figure, into which a drop of Mercury forms it ſelf upon a Table; and from two of them immediately running together, as ſoon as they come to touch.

PROPOSITION III.

Water is attracted by Glaſs.

This plainly appears from all the Experiments, that we have ſhewn upon this Subject.

PROPOSITION IV.

Quickſilver is attracted by Glaſs.

Experiment I. If a ſmall Globule of Quickſilver be laid upon a clean Paper, and be touched with a piece of clean Glaſs; upon drawing the Glaſs gently away,

away, the Quickſilver will adhere to it, and be drawn away with it. And if the Glaſs be lifted up from the Paper, the Quickſilver will be taken up by it; in the ſame manner as a piece of Iron is drawn up by the Loadſtone, and will ſtick to the Glaſs by a plain Surface of a conſiderable breadth, in proportion to the bulk of the drop, as manifeſtly appears by an ordinary Microſcope. Then if the Glaſs be held a little obliquely, the drop of Mercury will roll ſlowly upon its Axis along the under ſide of the Glaſs, till it comes to the end, where it will be ſuſpended as before.

Exp. 2*d.* If a pretty large drop of Mercury be laid upon a Paper, and two pieces of Glaſs be made to touch it, one on each ſide; upon drawing the Glaſſes gently from each other, the drop of Mercury will adhere to them both, and will be viſibly drawn out from a globular to an oval Shape; the longer Axis paſſing through the middle of thoſe Surfaces, in which the drop touches the Glaſſes.

PROPOSITION V.

The Particles of Water are more ſtrongly attracted by Glaſs, than by one another.

This manifeſtly appears from the riſing of Water in ſmall Tubes above the Level. For when the Water begins to riſe into a Capillary Tube, all the Particles of Water, which touch the ſmall *Annulus* at the bottom of the Tube, muſt have quitted the contact of the other Water, and have riſen contrary to their Gravity, to come into contact with the Glaſs. After the ſame manner the other Experiments of Dr. *Taylor*, Mr. *Hawksbee* and my ſelf, upon this Subject, are eaſily explicable. For upon a careful Examination, it will be found in

in them all, that ſome parts of the Water quit the contact of the other Water, and join themſelves to the Glaſs.

PROPOSITION VI.

The Particles of Quickſilver are more ſtrongly attracted by one another, than by Glaſs.

Exp I. *Fig.* 5. If a ſmall Tube as A B, open at both ends, be dipt into a Glaſs Veſſel fill'd with Mercury, and be held cloſe to the ſide of the Veſſel, that the riſe of the Mercury within it may appear; the Mercury will partly enter into the Tube, but will ſtand within it at ſome depth, as C E, below the Surface of the Quickſilver in the Veſſel, C D; and this depth will always be reciprocally as the Diameter of the Tube.

In this Experiment a Column of Quickſilver of the height C E endeavours to force the Mercury higher into the Tube; and as Glaſs has been already prov'd to attract Quickſilver, the Attraction of the annular Surface on the inſide of the Tube, which is contiguous to the upper part of the Mercury, will likewiſe conſpire to farther its aſcent. What oppoſes the aſcent of the Quickſilvet, is the Power, by which that part of it, which endeavours to riſe into the Glaſs, is drawn back by the Attraction of the other Mercury, with which it is in contact laterally, and this does not only balance the Attraction of the Glaſs, but likewiſe the weight of the Column of Mercury of the height C E, and conſequently this Attraction is conſiderably ſtronger than the Attraction of the Glaſs.

The cauſe therefore, that ſuſpends the weight of the Column of Mercury C E, being the difference between the Attraction of the annular Surface of the Tube at E, and that of an equal Surface of the Quickſilver in the Ciſtern, from which the Mercury, that endeavours to

riſe into the Tube, muſt recede, in order to unite it ſelf to ſuch an Annulus of the Glaſs, will always be proportional to that annular Surface, or to the Diameter of the Tube. And ſince the Column ſuſtain'd muſt be proportional to the Cauſe that ſuſpends it, that Column muſt likewiſe be as the Diameter of the Tube. But the Column ſuſpended is as the Square of the Diameter of the Tube and the height C E conjointly; from which it follows, that the height C E muſt be as the Diameter of the Tube reciprocally, as it is found to be by Experiment.

The Experiment of the Aſcent of Water above the Level in a Capillary Tube, is juſt the Reverſe of this.

Exp. II. *Fig.* 6. Quickſilver being poured into the inverted Siphon A C B, one of whoſe Legs A C is narrower than the other C B; the height C E, at which the Mercury ſtands in the wider Leg C B, is greater than the height C D, at which it ſtands in the narrower Leg C A.

On the contrary, Water ſtands higher in the narrower Leg, than in the wider.

Exp. III. *Fig.* 7. A B C D repreſents a rectangular plane of Glaſs, which makes one ſide of a wooden Box. On the inſide of this is another Glaſs plane of the ſame ſize, which at the end A C is preſt cloſe to the former, and opens to a ſmall Angle at the oppoſite end B D. When Mercury is pour'd into this Box to any height as C E, it inſinuates it ſelf between the two Glaſs planes, and riſing to different heighrs between the Glaſſes, where the opening is greater or leſs, it forms the common Hyperbola C G F; one of whoſe Aſymptotes E F is the line on which the Surface of the Mercury in the Box touches the inner Glaſs; the other is the line A C, in which the Planes are join'd. This Hyperbola being carefully examined by Mr. *Hawksbee*

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and my ſelf, the Rectangle E H G, whereſoever taken, proved always equal to it ſelf, to as great an accuracy as could be expected, when the Planes were opened to any conſiderable Angle: But when the opening was very ſmall, the inequalities of the Planes, though the beſt I could procure, bearing a greater proportion than before to the diſtance between them, occaſion'd a ſenſible variation. Which, by the way, I take to be the reaſon, why the Ordinates found by the late Mr. *Hawksbee*, in examining the Curve produced in a contrary ſituation, upon dipping two Glaſs Planes ſo join'd into Spirit of Wine, do not anſwer to thoſe of the Hyperbola.

Exp. IV. *Fig.* 8. A B is a perpendicular Section through two Glaſs Planes join'd at A, and open'd to a ſmall Angle at B. C repreſents a pretty large drop of Mercury, the larger the better, which being made to deſcend as far as C, by holding the Planes in an erect poſture, with the end A downwards, retires from the contact of the Planes to D, upon inclining the Planes towards an horizontal Situation; and the diſtance C D becomes greater or leſs, as the Planes are more or leſs inclin'd towards the Horizon.

A drop of any Oily or Watery Liquor moves the contrary way, as has been ſhewn by the late Mr. *Hawksbee*.

Exp. V. *Fig.* 9. A B is a Tube open at both ends, and a Foot or two in length, whoſe lower part is drawn out into a fine Capillary at B. This Tube being fill'd with Mercury, the whole Column of Quickſilver will be ſuſtain'd in it, provided the Capillary Tube at B be ſufficiently ſmall. But if the Mercury in the end B be ſuffer'd to touch any other Mercury, it runs all out of the Tube. If, without letting it touch any other Mercury, a ſmall part of the end B be broken off,

the Mercury will run out, till it comes to ſome leſſer height as B C, at which it will again ſtop, the height B C being nearly in a reciprocal proportion to the Diameter of the ſmall end of the Tube.

The Seventh Experiment in *Phil. Tranſ.* N°. 355 is the Reverſe of this.

Exp. VI. *Fig.* 10. Is the ſame in ſubſtance with the former, but made with a large Glaſs Funnel A B, inſtead of a Tube.

The Reverſe of this in Water is the thirteenth Experiment in the ſame Tranſaction.

In all theſe Experiments it is eaſily ſeen, that the Effect is owing to the difference between the two Attractions, by which Mercury tends to Glaſs and to its own body; they being always oppoſed to one another, ſo that a particular Explication is no way neceſſary. But perhaps it may ſave ſome little trouble to the Reader, to remove the following Objection, which will readily occur to him.

In the Experiments brought to demonſtrate the fourth Propoſition, the Globule of Mercury adheres to the Glaſs in a plane Surface, which cannot be done without encreaſing the Surface of the Globule, and conſequently removing ſome of its Particles from the contact of one another. If therefore they tend more ſtrongly to one another than to the Glaſs, why do they not recede from the Glaſs, and aſſume a figure perfectly Spherical, that they may all have the greateſt poſſible contact with each other?

To this we may anſwer, that the Power, by which Mercury is attracted either by Glaſs, or by other Mercury, is proportional to the attracting Surface; and therefore, though, *cæteris paribus*, the tendency of Mercury to Glaſs is not ſo ſtrong as its tendency to other Mercury, yet in this caſe a much greater number of Mer-

Mercurial Particles coming into contact with the Glaſs, than what recede from the contact of one another, it is no wonder, that the Attraction of the Glaſs prevails, and cauſes the Globule to adhere to it. For the number of Mercurial Particles which loſe their contact with the other Mercury, is no more than what makes up the difference of Surface, which ariſes from changing the figure of the Drop: whereas the Particles, which by this means come to adhere to the Glaſs, are all thoſe that conſtitute the plane Surface, in which the Globule touches it.

Which Conſideration ought likewiſe to be apply'd to the Suſpenſion of Quickſilver in Glaſs Tubes, either at extraordinary heights in the open Air, or at leſſer heights in a *Vacuum*, as above mention'd. For the top of the Tube being Spherical, or nearly ſo, it will be found, that the contact of the Mercury with the extremity of the Tube, is to the contact with other Mercury, which would be gain'd by its leaving the Top of the Tube and deſcending a very ſmall ſpace, in a *Ratio* infinitely great; and conſequently that the contact of the Mercury with the top of the Tube is one cauſe of its Suſpenſion.

Coroll 1*ſt*. From this Propoſition it appears, that in a Barometer made with a narrow Tube, the Quickſilver will never ſtand at ſo great a height as in a wider. Which accounts for the *Phænomenon* ſo often mention'd in the Yearly Hiſtory of the Royal Academy of Sciences at *Paris*, by Monſ. *De la Hire*; that in the Barometer, which he conſtantly made uſe of for his annual Obſervations, the Quickſilver did not riſe ſo high, as in another he kept by him, by about three Lines and a half, which is near a third of an Inch our Meaſure: For he tells us, that the Tube of his Barometer is very ſmall. So that there is no need to have recourſe to any

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peculiarity either in the Quickſilver or the Glaſs of which that Tube was made; or to an unperceived remnant of Air left in the Tube, from ſome of which cauſes that Effect and ſome others of the ſame kind were imagined to proceed.

Cor. 2d. In a Barometer made with a ſmall Tube, the Mercury will riſe and fall irregularly. For, as the height of the Mercury depends partly upon the Diameter of that part of the Tube that touches the upper Surface of the Mercury, it is plain, that the unavoidable inequalities in the Diameter of the Tube will be more conſiderable, in reſpect to the whole Diameter; and conſequently will affect the height of the Mercury more in a ſmall Tube than in a wider. And this I take to be the reaſon, why it is ſo very difficult, not to ſay impoſſible, to make two Barometers, which ſhall exactly agree in the height of the Quickſilver in all conſtitutions of the Air, eſpecially if the Tubes be very narrow. This irregularity is ſtill more conſiderable in the Pendent Barometer, in which the Quickſilver moves through a large ſpace, in order to make a ſmall alteration in the length of the Column ſuſpended: The ſame conſideration is eaſily extended to thoſe Levels, that depend upon the riſing of Mercury to the ſame height in the oppoſite Legs of a bent Tube; an Inſtrument of which kind has been lately offer'd for the ſervice of the Publick. And as the effect is juſt contrary in Levels made with Water or Spirit of Wine, due regard ought to be had to this Property in the conſtruction of thoſe Inſtruments, by making the Tubes ſufficiently wide, in order to diminiſh the Error as much as poſſible.

III. *Part of a Letter from Dr.* Rich. Richardſon, *to* Will. Sherard, LL. D. *&* R. S. S. *giving a relation of a wonderful Fall of Water from a Spout, upon the Mores in* Lancaſhire.

I Had an opportunity when in *Lancaſhire* of viſiting a ſecond time a vaſt breach in the Ground, which was made by a Spout, which fell upon *Emott-more*. The account I took of it when I firſt ſaw it, I put into Writing; and upon a ſecond Inſpection, finding it to be pretty exact, I thought a Tranſcript of it, would not be ungrateful to you, which you may communicate to your Friends, and make what uſe of it you pleaſe. You may depend on it as true.

Tho' our printed Voyages of ſeveral parts of the World furniſh us with frequent accounts of damage done at Sea by Spouts of Water, yet ſuch rarely happening at Land, induc'd me to take the following Relation of a remarkable one, which fell on *Emott-more*, nigh *Coln* in *Lancaſhire*, on *Tueſday* the 3*d* of *June*, 1718. about ten in the Morning: when ſeveral Perſons who were imploy'd in digging *Peat* nigh the place where this Accident happen'd, upon a ſudden were ſo terrify'd with an unuſual noiſe in the Air, that they left their Work and ran Home, which was about a Mile from the Place: But to their great ſurpriſe they were intercepted by Water; for a ſmall Brook in the Way was riſen above Six Foot Perpendicular in a few Minutes time, and had overflown the Bridge.

It is to be obſerv'd, that there was no Rain at that time on *Emott-more*, only a Miſt, which is very frequent

quent upon thoſe high Mountains in Summer time. There was a great Darkneſs in the Place where the Water fell, without either Thunder or Lightning. (as I had my Information from an Eye Witneſs) The Meadows at *Wicolae* were ſo much floated that the like had not been ſeen in ſeveral Years before, tho' there it was a very bright Day.

Upon this account, I went to view the Place where the Water fell; tho' I believ'd this Inundation might proceed from an eruption of Water out of the ſide of the Mountain; ſuch being not unfrequent, where Lead or Coal have been Dug, but neither have ever been ſought for here. Upon approaching the Place, I was ſtruck with unſpeakable Horror, the Ground was torn up to the very Rock, where the Water fell, which was above Seven Foot deep, and a deep Gulf made for above half a Mile, and vaſt heaps of Earth caſt up on each ſide of it, ſome pieces remaining yet above twenty Foot over, and ſix or ſeven Foot thick. About ten Acres of Ground were deſtroy'd by this Flood. The firſt Breach where the Water fell is about ſixty Foot over, and no appearance of any Eruption, the Ground being firm about it, and no Cavity appearing. I muſt not forget to mention, that the Ground on each ſide the Gulf was ſo ſhaken, that large *Chaſmes* appear'd at above 30 Foot diſtance, which a few Days after I obſerv'd the Shepherds were filling up, leaſt their Sheep ſhould fall into them.

IV. *An*

IV. *An Account of the* Phænomena *of a very extraordinary* Aurora Borealis, *ſeen at* London *on* November 10. 1719. *both Morning and Evening.* *By Dr.* Edmond Halley. R. S. Secr.

UPON *Tueſday, November* 10. 1719. in the Morning, *Jupiter* applying to the Second in the Wing of *Virgo*, I got up about 5 of the Clock to obſerve him, and having had the Satisfaction to ſee my Calculus perfectly well anſwer the Heavens, I found certain white Streaks in the Sky, ſeeming nearly Perpendicular; which whilſt I conſidered them ſeemed inſtantly to vaniſh, and ſoon after others came as inſtantaneouſly in their room. I began to imagine that this was likely to be ſome part of the *Phænomena* of the *Aurora Borealis*. But there appearing nothing like that luminous Arch which we have of late ſo often ſeen in the *North*, I knew not what to think; till looking up towards the *Zenith*, I perceived an entire *Canopy* of ſuch kind of white *Striæ*, ſeeming to deſcend from a white Circle of faint Clouds, about 7 or 8 degrees in Diameter, which Circle ſometimes would vaniſh on a ſudden, and as ſuddenly be renewed. I obſerved that the Center of this place of Concourſe was not preciſely in the *Zenith*, but rather 14 degrees to the Southwards thereof; which I was well enabled to eſtimate by a Star, which on each return thereof ſhewed its ſelf about the Center of the Circle. This Star is the 33*th* Star of the *Great Bear* in *Tycho*'s Catalogue, whoſe diſtance from the Pole at this time is 52 ½ degrees, and which about half an hour paſt Five that Morning paſt the Meridian, ſo that thoſe Rays centred very nearly on the Meridian it ſelf. It

a very entertaining Sight, till ſuch time as the Day-break began to obſcure theſe Lights, which were but faint, though ſufficiently diſtinguiſhable. They came none of them lower than to about 30 or 40 degrees of Altitude, and ſeem'd not to have aſcended from the Horizon. The Sky was perfectly Serene and Calm, which ſeems to be one of the concomitant Circumſtances attending the *Aurora Borealis*, of which this was certainly a Species. For the Night following a Neighbour gave me notice of a ſtrange ſtreaming of Lights ſeen in the Air, which thereupon I attended from the Hours of 9½ to 11, when a Fog came ſo thick as to put an end to my Proſpect. But during that whole time there aſcended out of the *E. N. E.* and *N. E.* a continued ſucceſſion of whitiſh *Striæ*, ariſing from below; and after changing as it were into a ſort of luminous Smoke, paſt over head with an incredible ſwiftneſs, not inferiour to that of Lightning; and as it paſt, in ſome part of its Paſſage, ſeemed as it were guilded, or rather as if the ſmoke had been ſtrongly illuminated by a blaze of Fire below. Some of the *Striæ* would begin high in the Air, and a whole ſet of them ſubordinate to one another, like Organ Pipes, would preſent themſelves with more rapidity than if a Curtain had been drawn from before them; ſome of which would die away where they firſt appeared, and others change into a luminous Smoke, and paſs on to the Weſtwards with an immenſe Swiftneſs And I am of opinion, that had it not been for the Moon, then ten Days old and very bright, this for the time would have been reckoned as conſiderable an Appearance as that of the 6*th* of *March*, 1716.

V. *A Relation of the ſame Appearance, ſeen at* Cruwys Morchard *in* Devonſhire. *Being part of a Letter to* Sam. Cruwys, *Eſq*; R. S S. *and by him Communicated to the* Royal Society.

YOU have doubtleſs been ſurprized again afreſh by the wonderful Lights which have been ſeen ſeveral times of late like thoſe deſcribed in the *Philoſophical Tranſactions*; ſeen on *March* 6. 1716.

Monday the 26 of *October*, between 7 and 8 in the Evening, I ſaw ſome ſmall appearance of it, *viz.* 3 or 4 large Coruſcations in form of Pyramids, of reddiſh Colour inclining to Yellow, which roſe about 50 degrees above the Horizon, and continued but few Minutes. But the North part of the Hemiſphere was very bright and red all the Evening both before and after, till Ten, if not longer.

Tueſday, Novemb. 10. Theſe Lights were ſeen again about 4 in the Morning, of which ſome ſay (to uſe their own Expreſſions) that the Element opened ſometime at one place, then at other; from whence came great ſhining Lights that continued a while, and then went away by degrees, and the Holes cloſed up again. This continued till Day break.

The Evening following coming from *Tyverton* about half an hour after Eight, I ſaw the North part of the Horizon very light and reddiſh (notwithſtanding the Moon being about 10 Days old, was then in or paſt the Meridian, and ſhone very bright) In a ſhort time the ſtreaming luminous Rays began to appear very plain, ſome in one ſhape, ſome another; many of them

like Cones or Pyramids, but moſt of them badly terminated; ſome of which mounted very high, almoſt to the Zenith, to which place, or near, they all or moſt, ſeemed to point. Shortly after there appeared a long Streak of about 30 Degrees, parallel to the Horizon and about 15 or 20 diſtant from it, and about 2 or 3 broad, but badly terminated and of a fiery red Colour: which ſent out ſome of the ſame ſtreaming Beams towards the Zenith. About 6 or 7 Minutes after there appeared (ſomewhat ſudden) a Circular Figure like an Iris, but twice as broad, of a pale Colour. The Eaſt part was terminated by the Horizon at full Eaſt, if not ſomething to the South, and the Weſt End about North Weſt; the upper part of its Arch being 50 or 60 Degrees high, great numbers of luminous Rays darted from it upward and downwards, (or elſe paſſing croſs it from the Horizon) at oblique Angles pointing to the Zenith, eſpecially from the North Eaſt part. This continued, as near as I can gueſs (by the diſtance I rode) about 8 or 9 Minutes, when it divided and diſappeared. After an Interval of 3 or 4 Minutes, another Iris-like Figure appeared, (of a Colour (as it ſeemed) paler than any of the ſtreaming Lights had been) whoſe Diameter was leſs than that of the former, and ſhewed more than its Semicircle above the Horizon, the upper part of its Arch approaching near the Zenith. I could not obſerve any Rays to paſs from, (or a croſs) this as from the other. The Centre of this laſt was much more to the Weſt than that of the firſt. After the continuance of a Minute or two, it began to break in the upper part of its Arch and ſhining Particles being ſent out from both its broken Ends towards the Zenith, (to which they were near before) or rather a little beyond it to the South or South Weſt, they there formed a ſort of *Corona*, curving and bending ſomewhat like Flames reverberated on the Arch

of

of an Oven: tho' this expreſſeth it but badly, yet I know not how to deſcribe it better. It ſeemed to me and others to be finely tinged with various Colours, Red-Yellow and Blueiſh, *&c.* and ſent out every way from it (except South and South-weſt) long flame-coloured Rays. After this had continued about two Minutes, its ſhining Light abated, and it left behind it for ſome Minutes, ſomething like a whitiſh Cloud (like in Colour to what the Light on the 19*th* of *March* laſt left behind it, after the fiery Particles were extinguiſhed, but thinner).

N. B. All this while the Moon ſhone very bright, from which this *Corona* was not very far diſtant, perhaps not twenty Degrees, to the North Eaſt. After this there continued to be ſent up many fiery Coloured or Yellowiſh ſtreaming Lights, ſometimes more, ſometimes leſs, now here, now there, all along the North part of the Hemiſphere, but moſtly from the North North Eaſt. All this while ſomething like ſmall whitiſh Clouds (which to me ſeemed to move towards the Zenith, or to point a little more Southward, but diſappear'd as they approached the Moon) were carried very ſwiftly, and at very ſhort Intervals, moſtly coming from the Eaſt and North Eaſt, but many alſo from North and North Weſt. We took but little notice of this at firſt, ſuppoſing it had been nothing but the reflection of the other Lights, or the ſhadows of the Clouds (whereof the North parts were pretty full) as the ſtreams of Light paſt behind them: But at laſt we obſerved that when the Lights at any time abated, theſe kind of Clouds continued to fly as ſwift and frequent as ever. This I ſaw till Twelve or One next Morning: many others ſaw it next Morning till almoſt break of Day, when it appeared much more red and fiery than it was in the Evening; the Moon perhaps being

being then ſet. Some People obſerved tall Cones to ariſe in the Eaſt, and to be carryed to the Weſt pretty ſwiftly in an erect Poſition, but I ſaw them not. It has been repreſented here in all ſorts of Appearances, Armies, Battles, *&c.* and has put abundance of People in diſmal Frights: But I had not an Imagination ſtrong enough for it, *&c.*

Novemb. 16. *Will. Maunder.*

VI. *A further relation of the ſame Appearance as ſeen at* Dublin, *communicated to the Publiſher by an unknown Hand.*

IT is with pleaſure that I now give you the trouble of reading the enſuing Account of the ſurprizing Lights which on *Tueſday* the tenth of *November* we ſaw in the Northern Semicircle of our *Horizon.*

The Afternoon was very Calm and Serene; about ſix in the Evening the Sky was ting'd with a ſtrange kind of Light, and ſome Streams began to project from the North and North Eaſt. One of them aroſe about N by E. and was nearly a Subtenſe of an Arc between that and S. W. by Weſt; it was a little curvated toward the Sun, and what I ſaw of it (for the North part of the *Horizon* was conceal'd by Houſes) very much reſembled the tail of a Comet: About the ſame time there was one or two which aroſe in the Eaſt, aſcending obliquely ſo as to leave the *Zenith* ſeveral Degrees to the Northward,

Theſe *Striæ* continu'd to appear and diſappear alternately till toward Eight in the Evening; they were *Pyramidal*, and their *Vertices* frequently projected ſeveral Degrees to the South of our *Zenith.*

Between

Between nine and ten I was agreeably ſurpriz'd with a kind of Coruſcation, or Flaſhing, that ſhew'd it ſelf between twenty and ſixty Degrees from the *Zenith*, in the South or South by Weſt; and which from four or five, ſometimes from more places at once, darted with a Velocity not much inferior to that of Lightning; and by interfering with each other produc'd a beautiful Tremour or Undulation in that ſubtile *Vapour*, which I cannot better illuſtrate, than by comparing it to the Beams of the Sun, reflected on a Ceiling from the Surfaces of two or three Baſons of Water: Theſe *Waves* of Light were only viſible at the inſtant of Coruſcation, and were of a pale whitiſh Colour, ſomewhat reſembling the flaſhes produced by the violent agitation of Quickſilver in an Exhauſted Receiver; but ſo ſtrong that a Gentleman who about that time was in a Room by himſelf, without a Candle, aſſur'd me he took it for common Lightning: Thus it continued inceſſantly for more than an Hour, during which time ſeveral lucid *Areas*, like little Clouds, diſcover'd themſelves in the pure Sky, and after they had continu'd about five or ſix ſecond Minutes, as near as I could gueſs, would inſtantaneouſly diſappear; moſt of them pretty much reſembl'd a very thin white Smoke or Vapour illuminated by the Full Moon.

About three quarters paſt Ten, this Vapour was almoſt ſpent, or by a brisk Gale at South by Weſt diſpers'd and driven to the Northward; at which time, between the Weſt and North, a vaſt body of it, like a very bright Flame-colour'd *Crepuſculum*, ſeem'd to be fix'd: From this Baſis ſeveral Beams or *Striæ* of ſhining matter were at uncertain intervals, emitted; and tho' it was not ſo ſenſible to the Eaſtward of the North, yet ſeveral mighty Pillars were alſo ejected from thence; One, which if I miſtake not, aroſe directly under

the

the Pole, was, above all others that had preceded it, both as to its Magnitude and Denſity ſo ſurprizing, that I'm perſuaded the ſmalleſt Print might have been read by the Light thereof, had not that of the Moon, which ſhone very bright, pretty much effac'd it: 'twas ting'd with a kind of Yellow and Violet Colour. In about two or three Minutes it died away, and was ſucceeded by others of an inferior Order: It was now about a quarter paſt Eleven of the Clock, and nothing but repeated *Phaſes* of the ſame Spectacle offering themſelves to View; the Vibrating Motion had ceaſed; the Vapour ſhewed it ſelf no longer in lucid *Areas*; the ſtreams of Light were not ſo frequent, and thoſe more languid than before; and the bright *Aurora* having ſetled nearer the *Horizon*, I concluded the *Scene* was at an End, and accordingly gave over the queſt of new *Phænomena*, with only obſerving that about N. E. there appear'd ſome Clouds that reflected an unuſual kind of reddiſh Light. Others, who thro' a Principle of Fear ſat up longer than I did, repreſent the End with very ſurprizing Circumſtances; but as it eſcap'd the Eyes of thoſe who were beſt qualified to oblige the World with an Hiſtory of it, ſo I deſpair of adding any thing that may be ſatisfactory: and there were no doubt many Circumſtances of Weight that I did not obſerve; for the wonderful Variety this *Phænomenon* afforded, and the frequency and ſuddenneſs of its Alterations, made it impoſſible for the Eye of any ſingle Perſon to trace it.

On *Tueſday* the 24*th* of *November* we had the ſame *Phænomena* repeated, tho' not with the ſame Variety: About a quarter paſt ten at Night, a vaſt Body of ſhining matter was collected between N. W. by Weſt, and N. by E. in the form of the Segment of a Circle, whoſe Center was about 25 or 30 Degrees below the *Horizon*;

Horizon; from its *Periphery* a few ſhort *Pyramidal* Streams, of the ſame luminous Vapour, aſcended by a ſlow and nearly uniform Motion, and were exceeding rare ſo as not to efface the ſmalleſt of the fix'd Stars; and in a Minute or two vaniſh'd: It was very remarkable that the Light which that Collection of Vapour emitted was ſo great, that in the otherwiſe very dark Night, I cou'd thereby (at three quarters paſt Ten) read the Title of the laſt *Philoſ. Tranſact.* which then happen'd to lye on my Desk; and at four or five Yards diſtance ſee the ſmalleſt Books in my Study.

VII. *An Account of another very conſiderable* Aurora Borealis *obſerved at* Streatham *in* Surrey, *by Mr.* Thomas Hearne; *and Communicated by Coll.* Francis Nicholſon, R. S. S.

HAving ſeen Dr. *Halley*'s Account of the Coruſcations in the Morning and Evening of *Novemb.* 10*th.* (and the Letter annexed to it from *Devonſhire*) I had the Pleaſure to find the Obſervations made upon that Appearance very agreeable to what I had my ſelf obſerved the Evening of that Day; and to what I did not at that time obſerve, but had an opportunity of obſerving in the Night of *Dec.* 11. I believe much more plainly than Dr. *Halley* had in the Night of *November* 10.

Dec. 11*th.* About one a Clock at Night (or rather in the Morning of *Dec.* 12*th*) I was called to obſerve Coruſcations which appeared of a much different Colour, and in a very different manner from any I had before ſeen.

The ſtreams of Light that darted upwards from the Horizon ſeemed to be at conſiderably a greater diſtance, but not at all in leſs quantity than thoſe of *Nov.* 10*th*. But their meeting in a Point near the *Zenith*, and there forming a kind of *Canopy*, was what was particularly remarkable in the *Manner* of the Coruſcations now different from thoſe of *Nov.* 10.

The ſtreams of Light roſe from the Horizon only towards the North, and on each hand towards N. Eaſt and N. Weſt: But near the *Zenith* a Canopy was formed of ſtreams of Light meeting in a Point, not only from thoſe Quarters, but alſo from the South, *&c.* Only to thoſe Points they extended downwards from the *Zenith* but a little Way, and were neither in ſo great quantity nor quite ſo bright as thoſe Northwards. At firſt I thought the Point in which the Streams met was exactly the *Zenith*, but upon obſerving it ſomething longer, I found it was not ſo, but a few Degrees to the South of the *Zenith*. The ſtreams of Light near the *Zenith*, which formed this Canopy, were of a pretty bright Colour, and in great Quantity, and darted very ſwiftly.

On each ſide of the N. towards E. and W. but not exactly in the N. it ſelf (at leaſt when I ſaw it) from about 10 or 15° to 40 or 50° above the Horizon, the Streams were of a glowing red Colour, whereas all that I had ever ſeen before were very pale. The redneſs was like that of a burnt Brick, and neareſt of any thing I have ſeen to the Colour, which remained for a few Minutes, like that tract through which the Meteor paſſed in the Spring.

The Streams appeared of this fierce Colour when I firſt ſaw the Coruſcations, and continued ſo for ſome time, till the redneſs by degrees wearing off, in about $\frac{1}{4}$ of an Hour they appeared of the uſual Paleneſs,

when

when I left them ſtill forming a Canopy near the *Zenith*, as is above deſcribed.

The Air was very Calm and Serene, not a breath of Wind ſtirring; as I remember it was alſo *Nov.* 10*th.*

The Moon was now a Day or two older than it was on *Nov.* 10*th.* and a good deal further to the W. than when I ſaw the Coruſcations that Night being then near full South. She had now round her what is commonly called a Burr larger than ordinary, and ſeveral very lucid Clouds at a little diſtance.

VIII. *Nuperæ Obſervationes Aſtronomicæ cum Regia Societate communicatæ.*

CUM in Num. harum *Tranſactionum* 357^mo^. Obſervationes nonnullas Planetarum ac Lunæ conſervari digniſſimas in unum congeſſimus, ac probante Societate noſtra edidimus; liceat paucula folia hujuſmodi collectionibus in ſequentibus quotannis conſignari. Nuperæ autem quas habemus Obſervationes hæ ſunt.

1718. *October* 10°. mane, applicabatur *Jupiter* ad Fixas Teleſcopicas, quarum loca, occaſione primæ apparitionis Cometæ anni 1680, (de quâ vide *Phil. Tranſ.* N°. 342) ſedulo inquiſivit Rev. D. *Pound*, ac nuper verificata nobiſcum communicavit, una cum accuratâ obſervatione tranſitûs Jovis juxta eas hac vice, ac deinde alterâ *Febr.* 11°. ſtatim ab oppoſitione Solis & Jovis. Ineunte autem *Januario* 1719. loca ſtellarum ſic ſe habuere.

	Long.	Lat. Bor.		Long.	Lat. Bor.
d ♑	29°.59′.43″	1. 7.50	*a* ♍	0°.25′.41″	1°.28′.54″
e ♍	0. 6.13	1.10.18	*x* ♍	0. 5.43	0.51.56
c ♍	0. 3.13	0.32.50			

Ubi notandum ſtellas *d* & *e* eandem præcisè hoc ſeculo ſortiri declinationem, *x* vero exiguam eſſe ſtellulam in priore deſcriptione ob parvitatem omiſſam.

Jam *Octob.* 9°. 17ʰ. 50′ T. æq. Jovis limbus orientalis attigit lineam ſtellas *e* & *c* jungentem, ſimul centrum ejus diſtabat ab *e* 21′. 20″ & à *c* 16′. 25″. ſtatimque aberat à *d* 19′. 35″. Parvula *x* Jovi proxima latuit, luce ejus obumbrata.

Decemb. 11°. 18ʰ. 30. T. æq. *Saturni* centrum diſtabat à *μ Libræ Bayero*, 28′. 32″, & Fixâ Borealius erat 4′. 31″. Hinc concluſit D. *Pound* Obſervator *Saturni* locum ♍ 10°. 41′. 10″, cum Lat. Boreal. 2°. 16′. 43″.

1719 *Feb.* 11°. 6ʰ. 56′½ T. æq. *Jovis* retrogradi centrum diſtabat à ſtella *d* ſuperius deſcriptâ —— 10′. 42″

6. 58¼ Idem centrum diſtabat ab *e* —— 6. 7

9. 37⅓ Iterum diſtantia capta à *d* —— 10. 9

9. 43½ Iterum ab *e* —— 6. 11

9. 49½ Jovis centrum diſtabat ab *a* —— 25. 21

9. 58½ Idem centrum à parvula *x* —— 24. 38

Circa Horam ſeptimam *Jovis* limbus orientalis attigit lineam per *x* & *e* productam; *Jupiter* itaque tunc habuit ♍ 0°. 6 cum Latitudine Boreal. 1°. 16′. 30″. Deinde,

Feb. 13°. 8ʰ. 0′. T. æq. Declinatio centri Jovis, Micrometro menſurata, Borealior erat eâ ſtellæ utriuſque *d* & *e* 11′. 37″, & 8ʰ. 20′ eadem differentia inventa eſt 11′. 36″. Horâ vero 8ʰ. 48′ centrum Jovis diſtabat ab *e* 17′. 40″.

Apr. 22°. 10ʰ. 45′. T. æq. *Saturni* centrum ſequebatur *μ Libræ* 4″½ Temp. ſive 1′. 8″ Aſc. Rectæ. Micrometro autem Borealior inventus eſt Fixâ 35′. 25″. Stella autem in *Catalogo Britannico* tunc habuit, ♍ 10°. 16′. 8″. Lat. Bor. 2°. 3′. 54″.

Maii 16°. 8ʰ. 00′ T. æq. ♍ ſequebatur *Cor Leonis* 1′. 34″½ Aſcenſionis rectæ; Borealior autem erat ſtellâ illâ 0′. 41″½. Temporis, hoc eſt, 10′. 7″ Arcus cœleſtis.

Eâdem nocte, 15ʰ. 18′ T. app. Obſervavit D. *Stephanus Grey Martem*, ratione Aſcenſionis rectæ, ſequi ſtellam *in Cauda Capricorni* orientalem 16′. 15″; ſimul non niſi 0′. 11″. auſtralior erat quam Fixa.

Junii 7°. 10ʰ. 15′. T. app. *Jupiter* directus iterum reverſus eſt ad ſtellas Teleſcopicas prædictas, & tum ſequebatur ſtellam *d.* 0′. 35″ Aſcenſionis rectæ, & 10ʰ. 30′ diſtabat fixa à limbo *Jovis* proximo 4′. 18″.

Poſtridie *Junii* 8°. 10ʰ. 20′, *Jupiter* ſequebatur ſtellam alteram *e* 1′. 30″ Aſcenſionis rectæ, ac ſtatim diſtantia limbi *Jovis* proximi à ſtella capta eſt Micrometro 7′. 30″.

Julii 5. 8ʰ. 26′. T. app. Conjungebantur arctè *Jupiter* & *Venus*, quæ tum Borealior præcedebat *Jovem* ſecundum Aſcenſionem rectam 1′. 20″: Centrorum autem diſtantia ex decies repetitis media, capta eſt 13′. 36″. Hæc tria *Londini* obſervata communicavit harum Scientiarum eximius Cultor D. *Martinus Folkes*, R. S. Soc.

Aug. 3. 12ʰ. 20′ T. æq. *Mars* pene Acronychus ſequebatur ſtellam τ *Aquarii Bayero* 10′. 58″ Temporis, ſive 2°. 44′. 57″ Aſcenſionis Rectæ. Erat autem fixâ *Mars* Borealior 0′. 36″ tantum; unde conceſſo loco ſtellæ *Britannico* fit locus *Martis* obſervatus ♐ 7°. 10′. 10″ cum latitudine Auſtrali 6°. 38′. 10″.

Aug. 10°. 11ʰ. 50 T. æq. *Mars* ſequebatur fixam minorem quæ præcedit τ *Aquarii* 1°. 39′. 30″ ratione Aſcenſionis rectæ; Auſtralior vero quam fixa 10′. 42″.

Aug. 16°. 7ʰ. 18′. T. æq. *Spica Virginis* præcedebat *Veneris* centrum 5″¼ ſecundis temporis, ſive 1′. 20″ Aſcenſionis rectæ, auſtralior Planetâ 18″¼ temp. ſive 4′, 35″.

Aug. 17°. *Mars* pridie Acronychus ac Terris proximus obſervatus eſt ad duas ſtellulas contiguas, Parallaxis

ejus

ejus inveſtigandi gratiâ, juxta methodum à D. *Caſſino*, in libro de Cometa anni 1680, exhibitam, Unde *Martis* Parallaxin eruere in *Tranſact.* proximâ conabimur. Harum vero ſtellularum borea tum temporis locum habuit ♐ 3°. 5′. 50″ cum Latitudine auſtrali 6°. 6′¼: altera vero Auſtralior habuit ♐ 3°. 5′. 30″, cum Lat. Auſt. 6°. 10′¼ proximè. Horâ vero 10ʰ. 40′. T. æq. Auſtralem ſequebatur *Mars* 41′ min. 40″ Aſcenſionis rectæ, câque adhuc Auſtralior erat 7′. 50″.

Sept. 18. 9ʰ. 20′. T. æq. *Mars* viſus eſt præcedere ſtellam in *Catalogo Britannico* Aquarii 53ⁱᵃᵐ. 3′. 45″ Temporis, ſive 56′. 24″. Aſcen. Rectæ; ſimulque Stella Borealior erat limbo Martis boreo, non niſi unâ Planetæ diametro. Locus ſtellæ ♒ 29°. 57′½ Lat. Auſt. 4°. 48′½.

Octob. 30. Veſperi 5ʰ. 45′. T. app. *Mars* proximus ſtellis duabus contiguis ad *h* ♒ *Bayero*, quæ ſunt ♒ 73ⁱᵃ & 74ᵗᵃ *Catal. Brit.* Præterierat rectam per eaſdem ductam, eratque angulus ad *Martis* centrum ad ſenſum rectus: Borea vero ſtellarum eandem habuit declinationem cum limbo Planetæ auſtrino. 5ʰ. 53′ diſtantia ſtellæ à centro *Martis* 2′. 30″. 5ʰ. 56′ centrum *Martis* diſtabat à tertiâ & Auſtraliore ad *h*, ſive 75ᵗâ Aquarii, 17′. 04″. 6ʰ. 18′ diſtantia centri à Boreâ ſive 73ⁱâ erat 3′. 5″. Hinc concludere licet *Martem*, horâ 3ʰ. 30′ proximè, ſtellæ Boreæ conjunctum fuiſſe, eamque uno tantum minuto ad Boream reliquiſſe. Fixæ autem locus è *Catalogo Britannico* tunc erat ♐ 10°. 29′. 00″ cum Lat. Auſt. 1°. 40′¼. 74ᵗᵃ vero habuit ♐ 10°. 29′. 50″ cum Lat. Auſt. 1°. 44′¼.

Novemb. 16°. 19ʰ. 18′. T. æq. *Venus* præcedebat *Lancem Libræ* Auſtrinam 3′. 13″ Temp. ſive 48′. 23″ Aſcen. Rect. ſimulque fixâ borealius erat centrum *Veneris* 7′. 45″. *Venus* quaſi Stationaria apud Nodum ejus Aſcendentem.

Decemb.

Decemb. 3°. 19ʰ. T. æq. *Saturnus* præcedebat tertiam ad ♎ *Libræ*, ſive *Libræ* 29ᵃᵐ. *Cat. Brit.* 0′. 46″ Temp. ſive 11′. 32″ Aſc. Rect. Erat autem fixâ Auſtralior 15′. 29″. differentiâ per Micrometrum captâ. Unde Saturni locus ♏ 20. 25¼ cum Lat. Bor. 2°. 5′¼.

Obſervationes Lunæ & Eclipſium.

In dicta *Tranſact.* N°. 357. *pag.* 852. Obſervationem dedimus Eclipſeos Lunaris anno 1717 *Martii* 15° P. M. St. vet. apud *Cambridg Nov-Anglorum* habitam; apud nos vero ob Nubes inconſpicuam. Deſiit autem Eclipſis ibidem 11ʰ. 42′½, neque alia ejus tum temporis ſuppetebat obſervatio. Poſtea vero arte Nauticâ & induſtriâ inter primos inſignis Dom. *Candler* Navarcha Regius, ex *America* attulit & nobiſcum communicavit ejuſdem Eclipſeos phaſes *Limæ Peruviæ* à *Dom. Petro Peralta*, Mathematico Regio multis titulis claro, obſervatas, Typiſque ibidem impreſſas. Initium autem Eclipſis ponit *Limæ* 8ʰ. 41′. 8″. Finem vero 11ʰ. 19′. 55″. Simul laudatus D. *Candler* propriam obſervationem, ad Inſulam quam *Virgine Gorda* vocant captam, conceſſit, Ibi deſiit Eclipſis 12ʰ. 13′. P. M. Fine per cælum ſudum diſtinctè viſo. Poſtremo inter Acta Regiæ Scientiarum Academiæ *Pariſienſis* iſtius anni, comperimus duas & quidem ſatis conformes hujus Eclipſeos obſervationes, alteram à D. *Caſſino*, alteram à D. *de la Hire* in Obſervatorio Regio captas: Hic Initium æſtimavit 13ʰ. 54′. Finem vero certius 16ʰ. 38′. 10″. At Ille Initium 13ʰ. 55′ & Finem 16ʰ. 38′. 25″. Maxima obſcuratio huic 7⅓ Dig. illi 7½ Dig.

Hinc ex Fine, in ſingulis locis ut videtur accuratius ſumpto, proveniunt Longitudinum differentiæ inter *Pariſios & Limam* 5ʰ. 18′. 20″, Inter *Pariſios* & *Cambridg* 4ʰ. 55′.

$4^h.\ 55'.\ 50''$. Inter *Pariſios* & Inſulam *Virgine Gorda* $4^h.25'.20''$. E quibus ſi $9'.40''$ ſubduxeris, provenient Longitudines ad occaſum *Londini*, nempe *Limæ* $77°.\ 10'$. *Cambridg Nov Anglorum* $71°\frac{1}{2}$, ac denique Inſulæ *Virgine Gorda* $63°.\ 55'$; unde Inſularum adjacentium ſitus Geographici certo corrigi poterint.

Altera Lunæ Eclipſis ejuſdem anni *Septembris* nono Veſperi, ab iiſdem obſervatoribus & D. *Maraldo Pariſiis* conſpecta eſt. Finem *Londini* obſervavimus in ædibus Societatis Regiæ $7^h.\ 26'$. *Pariſiis* vero D. *Caſſino* Finis $7^h.\ 34'.\ 50''$, D. *Maraldo* $7^h.\ 35'.\ 30''$, & D^no *De la Hire* $7^h.\ 34'.\ 15''$. Simul D. *Wurtzelbaur Noribergæ* eundem Finem vidit $8^h.\ 10'.\ 45''$ Hinc confirmantur Meridianorum differentiæ *Londinum* inter & *Pariſios*, præſertim ex obſervatione D. *Maraldi*, nempe $9'.\ 30''$; uti & inter *Londinum* & *Noribergam* $44'.\ 45''$, quantam ſæpius olim experti ſumus. Porro quinto die poſt Eclipſin, *Septembris* 14^to veſperi, Luna occultavit *Palilicium Pariſiis*, obſervantibus ſigillatim DD. *Maraldo* & *Deliſle* Juniore. Evanuit autem ſtella è regione Maculæ *Grimaldi* ſive *Paludis Mareotidis*, Hora $9^h.\ 11'.\ 35''$. Emerſit autem è limbo Lunæ obſcuro $10^h.\ 3'.\ 55''$. Hujus Occultationis obſervatio *Londini* habetur *pag.* 853. *Phil. Tranſact.*

Obſervationes illas, in quibus Temp. æq. *adhibetur, Rev. D^o* Pound *acceptas debemus. Tubo autem quindecimpedali captæ pro certiſſimis habendæ ſunt.*

AN

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